

Two-neutron halo in light drip-line nuclei from the low-energy limit of neutron-neutron interaction

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○ Carbon isotopes

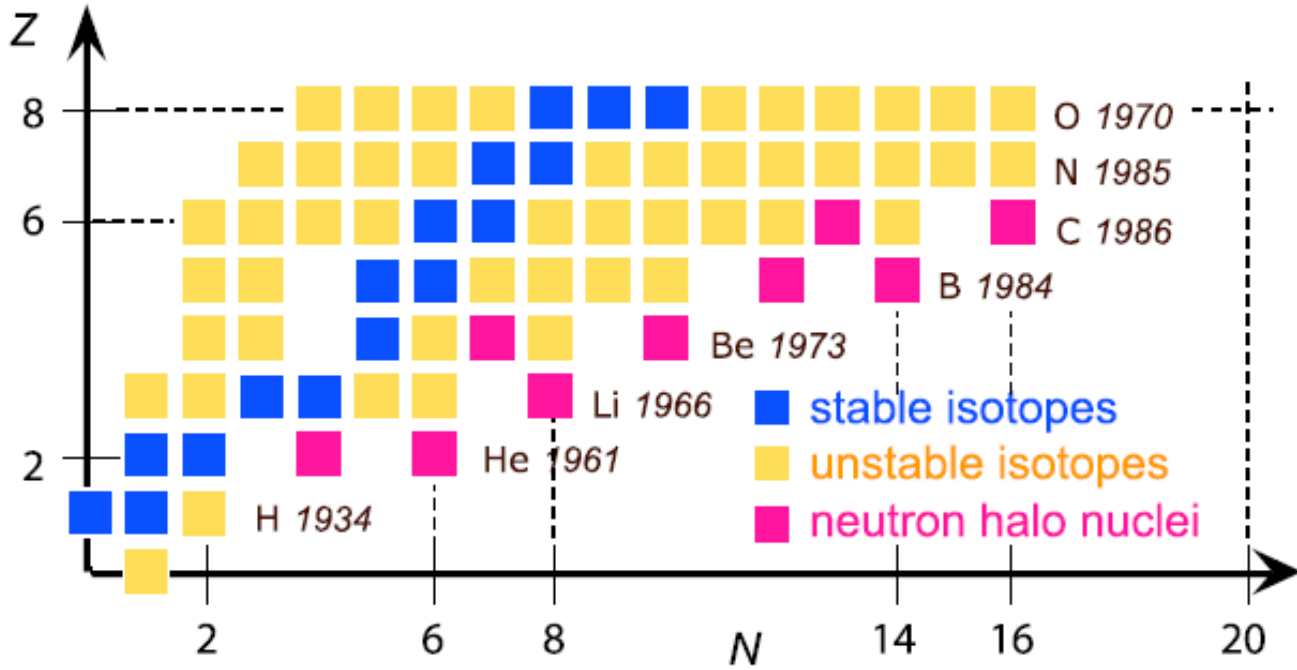
^{21}C = unbound

^{22}C = drip-line nucleus & 2n-halo nucleus

How ^{22}C become bound?

Neutron-neutron interaction plays an important role!

Mechanism of forming two-neutron halo from the low-energy limit of n-n interaction



1. 3-body model for core+2n system with low-energy limit of n-n interaction

2. Application to ^{24}O

S_{2n} and matter radius

3. Application to ^{22}C

▪ Closed-core approximation for ^{20}C : $1p^{10} \nu 1d_{5/2}^6$

▪ Correlated-core model for ^{20}C : mixing of $2s_{1/2}$ -orbit

S_{2n} vs. neutron halo radius

density of halo neutron

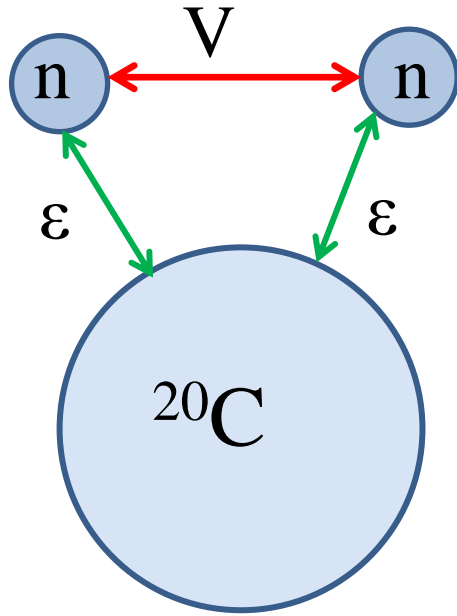
S_{2n} vs. $\langle v_{nn} \rangle$: condition for ^{21}C to be unbound

→ halo radius is estimated to be 6-7 fm, which is small compared with previous estimations

Spectra of ^{22}C for closed and correlated-cores

4. Comments on Efimov states

$${}^{22}\text{C} = {}^{20}\text{C} + \text{n} + \text{n}$$



$${}^{21}\text{C} = {}^{20}\text{C} + \text{n} : \text{unbound}$$

$$\varepsilon > 0$$

$$E_{\text{nn}} = 2\varepsilon + V$$

$$= 2(\varepsilon + V) - V$$

$$< 0 \Leftrightarrow {}^{22}\text{C} : \text{bound}$$

$$H = t_1 + u(\vec{r}_1) + t_2 + u(\vec{r}_2) + v_{\text{nn}}(\vec{r}_1 - \vec{r}_2)$$

$$h_1 = t_1 + u(\vec{r}_1); \quad u(r) = \text{Woods-Saxon potential}$$

$$\{h_1 + h_2 + v_{\text{nn}}(\vec{r}_1 - \vec{r}_2)\}\phi(\vec{r}_1)\phi(\vec{r}_2) = E_{\text{nn}}\phi(\vec{r}_1)\phi(\vec{r}_2), \quad S_{2\text{n}} = -E_{2\text{n}}$$

$$\phi(\vec{r}_1) = \phi(\vec{r}_2) \equiv \phi(\vec{r})$$

$$\varepsilon = \langle \phi(\vec{r}) | h(\vec{r}) | \phi(\vec{r}) \rangle$$

$$\{h + w(\vec{r})\}\phi(\vec{r}) = (E_{\text{nn}} - \varepsilon)\phi(\vec{r})$$

$$w(\vec{r}) = \langle \phi(\vec{r}') | v_{\text{nn}}(\vec{r} - \vec{r}') | \phi(\vec{r}') \rangle$$

$$E_{\text{nn}} = 2\varepsilon + V$$

$$V = \langle \phi(\vec{r}) | w(\vec{r}) | \phi(\vec{r}) \rangle = \langle \phi(\vec{r})\phi(\vec{r}') | v_{\text{nn}}(\vec{r} - \vec{r}') | \phi(\vec{r})\phi(\vec{r}') \rangle$$

$$E_{\text{nn}} - \varepsilon = \varepsilon + V \equiv -S_{1\text{n}}$$

$$S_{2\text{n}} = -2(\varepsilon + V) + V = 2S_{1\text{n}} + V$$

$$\phi(r) \propto \exp(-\eta r) / r$$

$$\eta = \sqrt{2m | E_{\text{nn}} - \varepsilon |} / \hbar = \sqrt{m | -S_{2\text{n}} + V |} / \hbar$$

n-n interaction in the low energy limit:

$$a_{\text{nn}} = -18.9 \pm 0.4 \text{ fm}, \quad r_{\text{nn}} = 2.75 \pm 0.11 \text{ fm}$$

$$v_{\text{nn}}(r) = -v_0 \exp(-(r/r_0)^2), \quad r_0 = 1.795 \text{ fm}$$

[R. Machleidt, Phys. Rev. C 63 \(2001\) 024001;](#)

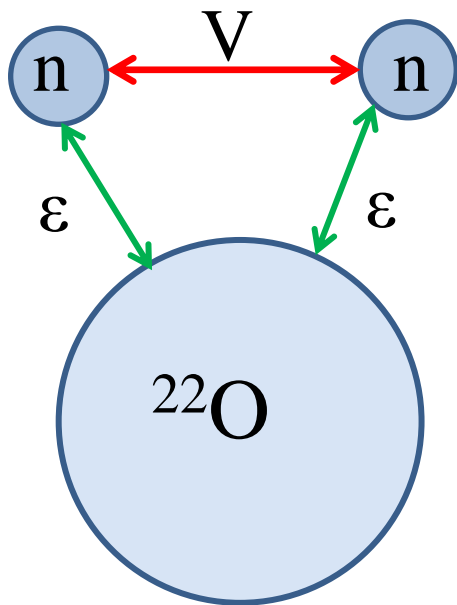
[G.A. Miller, M.K. Nefkens, I. Slaus, Phys. Rep. 194 \(1990\) 1;](#)

[C.R. Howell, et al., Phys. Lett. B 444 \(1998\) 252;](#)

[D.E. Gonzalez Trotter, et al., Phys. Rev. Lett. 83 \(1999\) 3788.](#)

$${}^{24}\text{O} = {}^{22}\text{O} + n + n$$

$${}^{23}\text{O} = {}^{22}\text{O} + n : \text{bound} \quad \varepsilon < 0$$



$$S_n ({}^{23}\text{O})_{\text{exp}} = 2.73 \text{ MeV} \rightarrow V_{\text{WS}} \text{ is determined}$$

$$+V_{\text{nn}} \rightarrow S_{2n} ({}^{24}\text{O}) = 6.94 \text{ MeV}$$

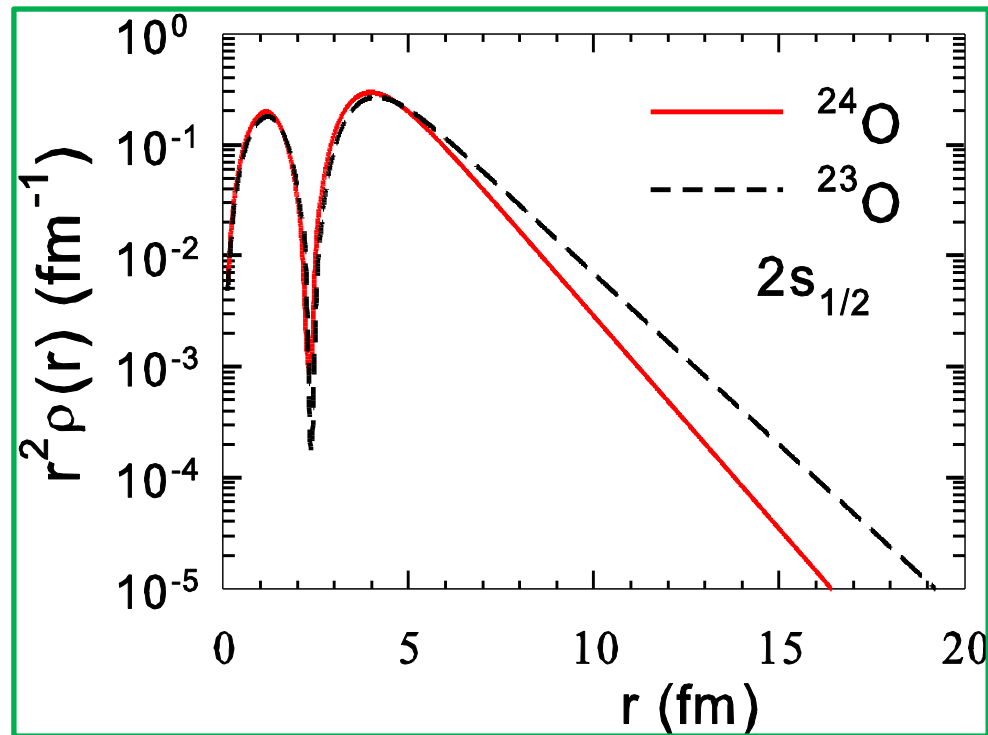
$$\text{cf. } S_{2n} ({}^{24}\text{O})_{\text{exp}} = 6.92 \text{ MeV}$$

RMS matter radius (fm)

	present	Ozawa ^a	Kanungo ^b
${}^{22}\text{O}$:	2.85	2.88 ± 0.06	2.75 ± 0.15
${}^{23}\text{O}$:	2.97	3.20 ± 0.04	2.95 ± 0.23
${}^{24}\text{O}$:	3.03	3.19 ± 0.13	

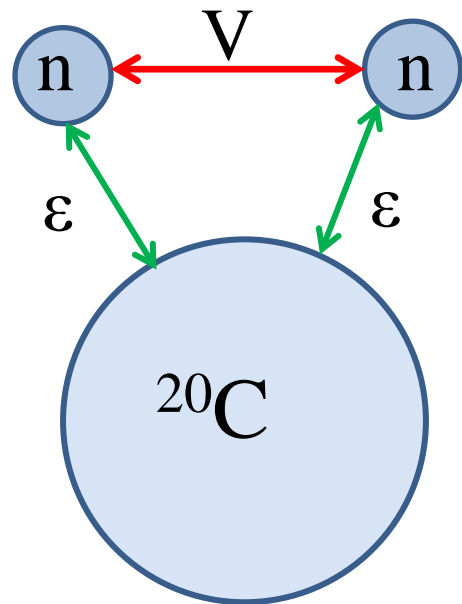
a) Ozawa et al., NP A691, 599 (2001)

b) Kanungo et al., PR C 84, 061304 (2011)



$$^{22}\text{C} = ^{20}\text{C} + \text{n} + \text{n}:$$

$$^{20}\text{C} = \text{closed-core} = 1\text{p}^{10} \text{v}1\text{d}_{5/2}^6 (0^+)$$



$$w(\vec{r}) = \langle \phi(\vec{r}') | v_{nn}(\vec{r} - \vec{r}') | \phi(\vec{r}') \rangle$$

$$V = \langle \phi(\vec{r}) | w(\vec{r}) | \phi(\vec{r}) \rangle$$

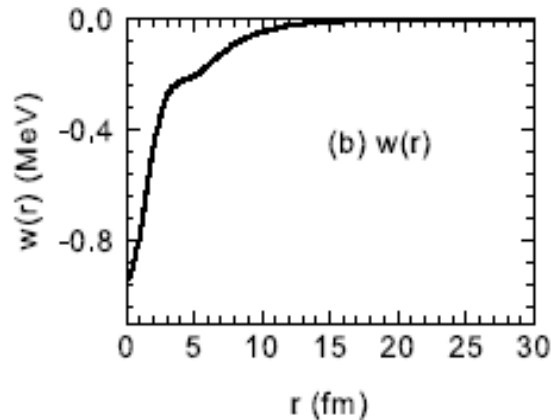
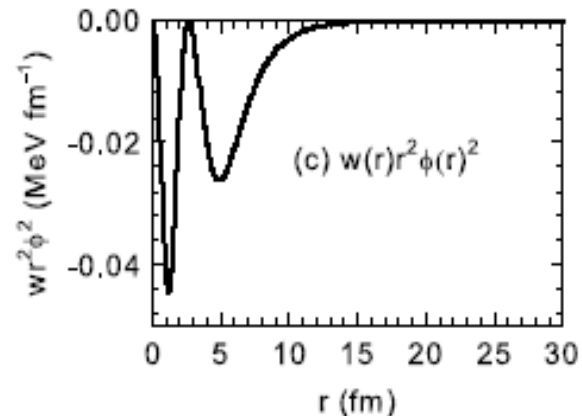
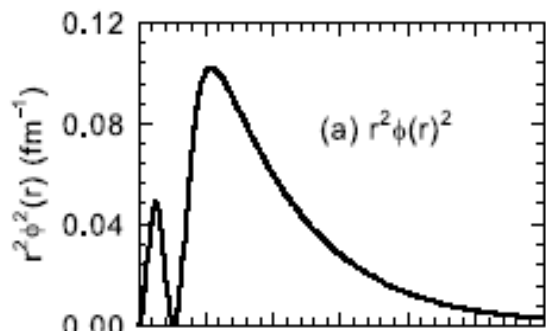
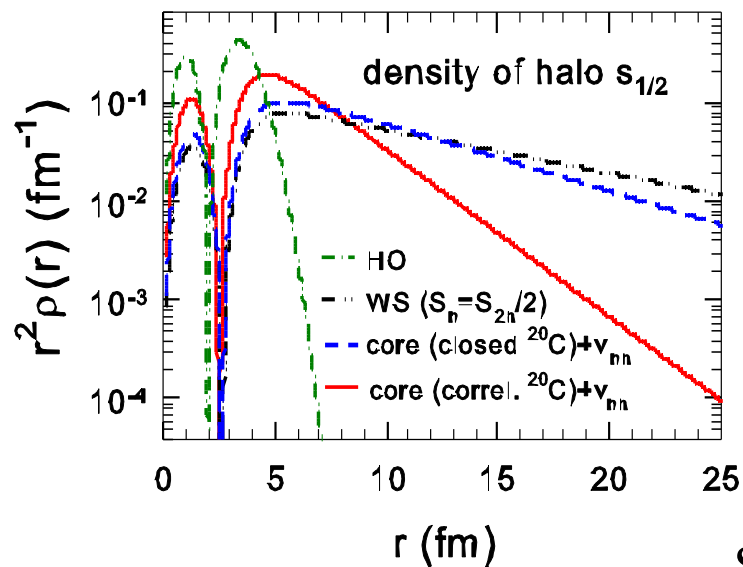


Fig. 3. (a) Radial wave function $\phi(r)$ of halo neutron, (b) potential $w(r)$ induced by the other halo neutron and (c) their product, as a function of the distance r .



$$S_{1n} = S_{2n}/2 - V/2 > S_{2n}/2$$

Halo radius

$$S_{2n} = -2(\varepsilon + V) + V = 2S_{1n} + V$$

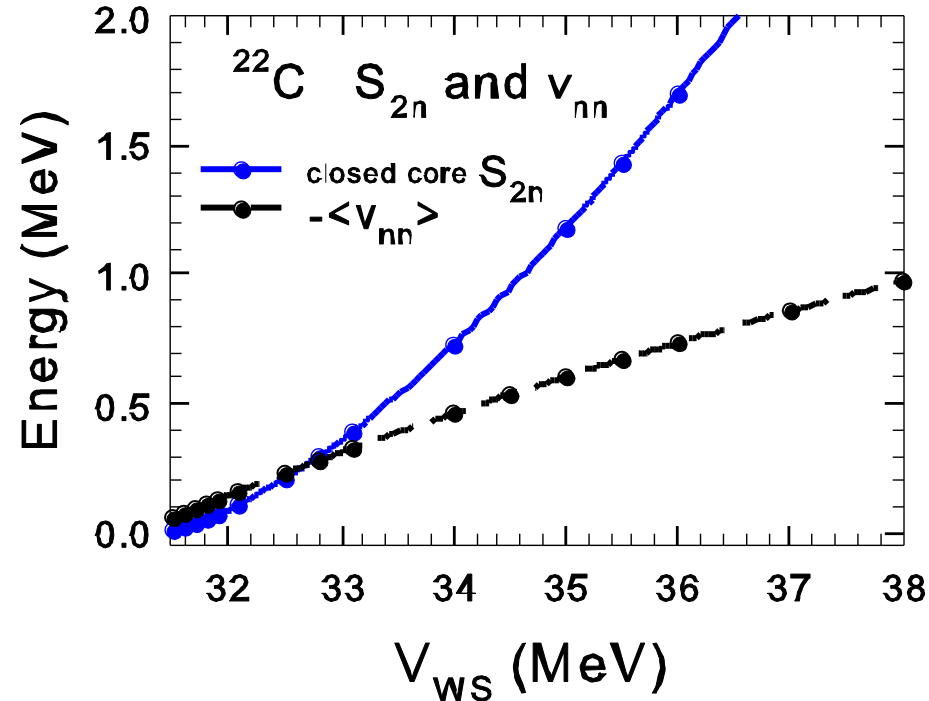
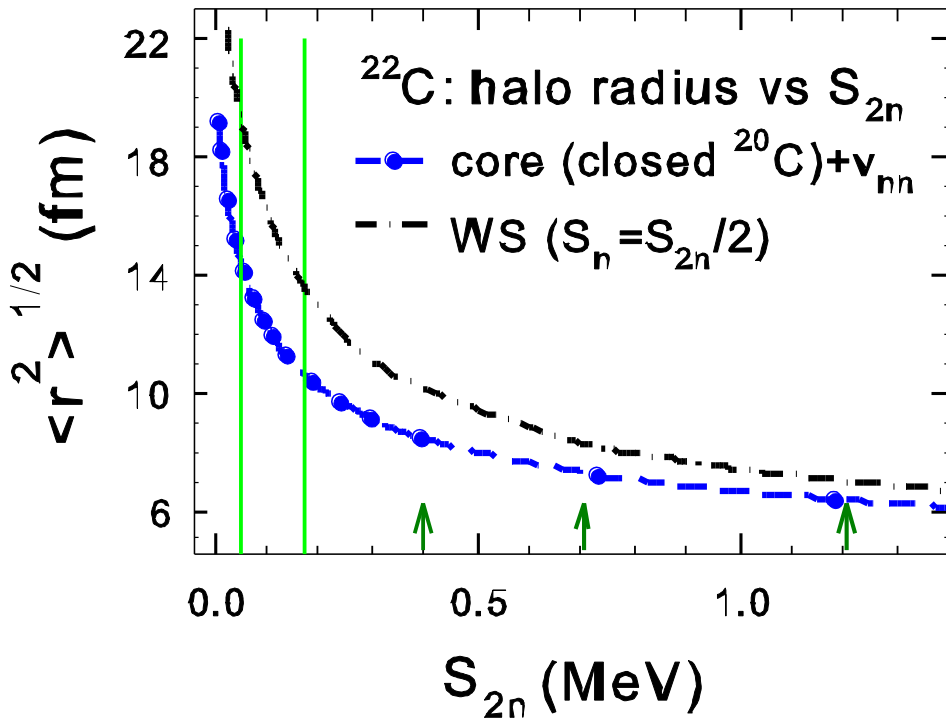
$$S_{1n} = (S_{2n} - V) / 2 > S_{2n} / 2$$

$$S_{2n} \text{ vs. } V = \langle v_{nn} \rangle$$

$$S_{2n} + V = -E_{nn} + V = -2\varepsilon$$

$$\varepsilon > 0 \Leftrightarrow S_{2n} < -V \Leftrightarrow {}^{21}\text{C: unbound}$$

$$\varepsilon < 0 \Leftrightarrow S_{2n} > -V \Leftrightarrow {}^{21}\text{C: bound}$$



Exp.

S_{2n} : 110 ± 60 keV, NNDC **${}^{21}\text{C}$ unbound $\rightarrow S_{2n} \lesssim 0.3$ MeV \rightarrow RMS $\gtrsim 9$ fm**

0.4, 0.7, 1.2 MeV, Kobayashi et al., PR C86, 054604 (2012)

0.423 ± 1.140 MeV, Audi et al., NP A729, 337 (2003)

-0.140 ± 0.460 MeV, Gaudefroy et al., PRL 109, 202503 (2012)

RMS radius: $15.97 + 3.67 / - 3.97$ fm, ${}^{22}\text{C}$: Tanaka et al, PRL 104, 062701 (2010)

Correlated core of ^{20}C

Occupation number of neutron in $2s_{1/2}$ orbit ~ 1

Kobayashi et al., PR C86, 054604 (2013)

Shell-model calc. with YSOX: $1d_{5/2}^6 + 1d_{5/2}^4 2s_{1/2}^2$

Yuan, Suzuki, Otsuka, Xu, Tsunoda, PR C85, 064324 (2012)

Ground state energy of ^{20}C is lowered by admixture of the $1d_{5/2}^4 2s_{1/2}^2$ configurations.

Model:

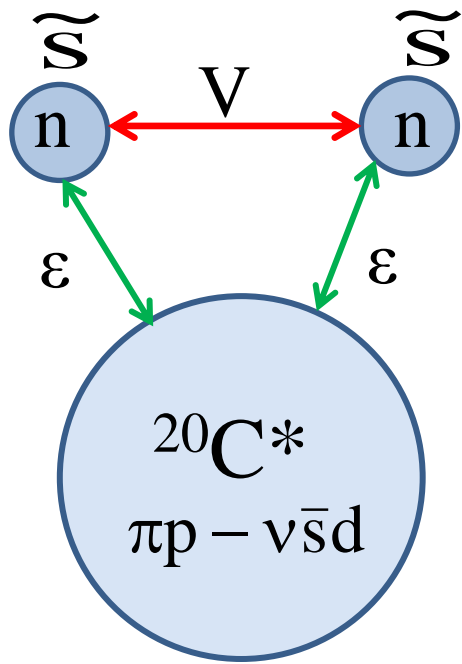
Halo s-orbit is occupied by 2 neutrons

Orthogonality condition between this halo s orbit and the s-orbit of the ^{20}C -core state is satisfied, that is, the core s state is made orthogonal to this halo s orbit by Gram-Schmidt method

→ Blocking effect on the core state

Energy of the ^{20}C core of the ^{22}C ground state is shifted with respect to the energy of the ^{20}C ground state : energy shift $=\Delta > 0$

$$S_{2n} = -E_{nn} - \Delta$$



$$|\tilde{s}\rangle = |s_{1/2}(\text{halo})\rangle = \alpha |2s_{1/2}(\text{H.O.})\rangle + \beta |\text{far-s}\rangle$$

$$|\bar{s}\rangle = |s_{1/2}(\text{core})\rangle = \beta |2s_{1/2}(\text{H.O.})\rangle - \alpha |\text{far-s}\rangle$$

$$\langle \bar{s} | \tilde{s} \rangle = 0$$

$$V_{\text{WS}} \rightarrow \infty \Rightarrow \beta \rightarrow 0, \alpha \rightarrow 1$$

\Rightarrow less $2s_{1/2}$ - components in $|\bar{s}\rangle$

\Rightarrow g.s. energy of ^{20}C is pushed up: $\Delta > 0$

$|s_{1/2}(\text{core})\rangle$ gets halo components.

Two-body m.e.'s of V_{YSOX} are modified.

Single-particle energy of $2s_{1/2}$ outside ^4He -core is also modified.

Shell-model calculation:

protons in p-shell, neutrons in sd-shell

\rightarrow g.s. energy of ^{20}C

\rightarrow energy shift $\Delta \quad \Delta \sim 1 \text{ MeV}$

$\rightarrow S_{2n} = -E_{\text{nn}} - \Delta$

S_{2n} vs. $V = \langle v_{nn} \rangle$

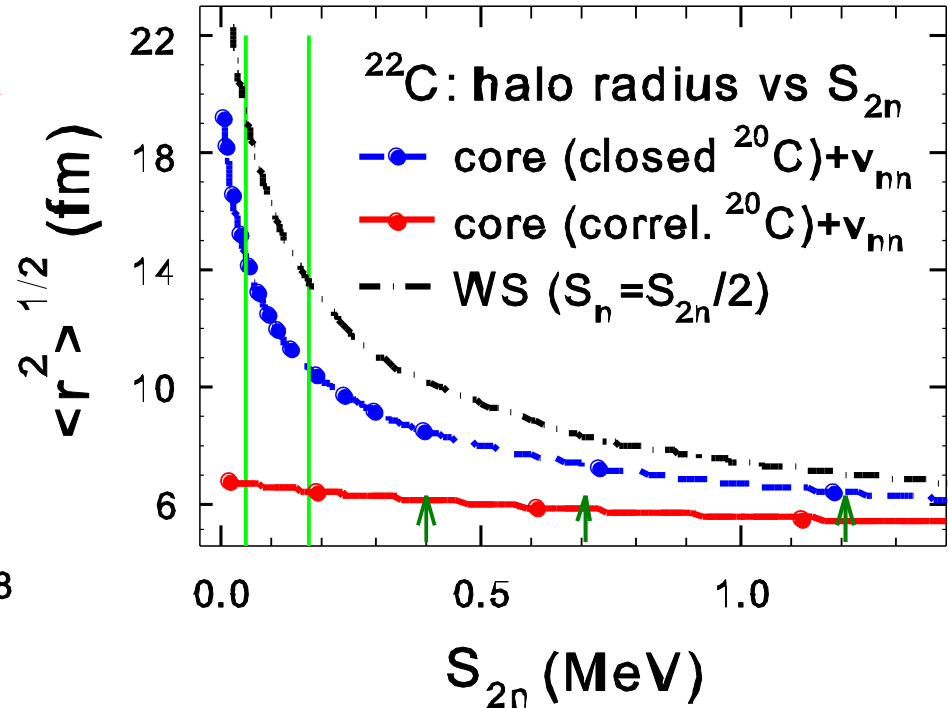
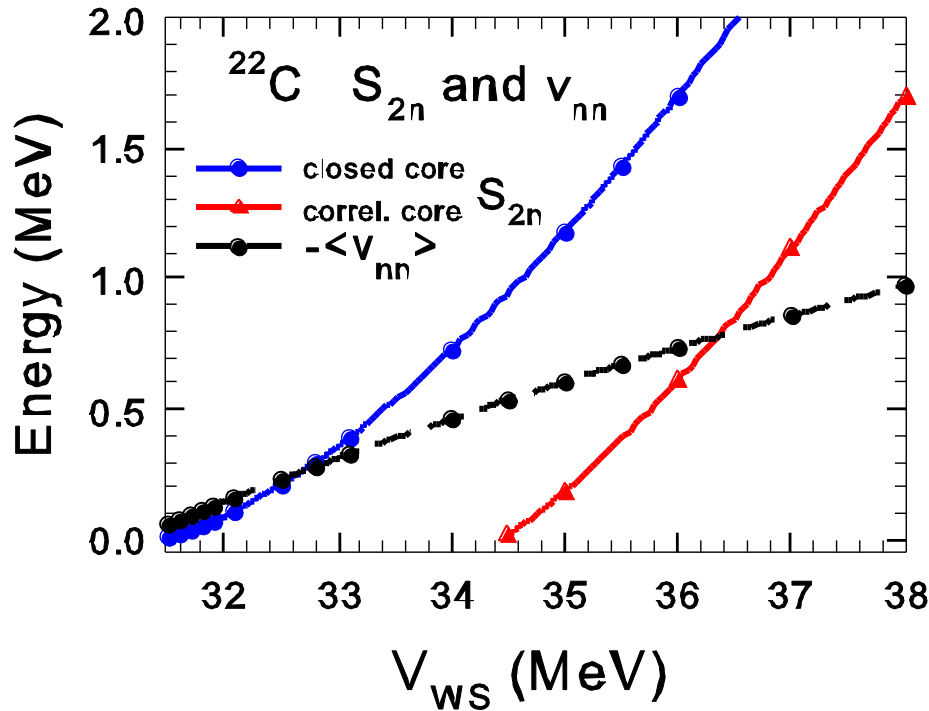
$$S_{2n} + V = -E_{nn} + V = -2\varepsilon$$

$\varepsilon > 0 \Leftrightarrow S_{2n} < -V \Leftrightarrow {}^{21}\text{C}$: unbound

$$S_{2n} < 0.8 \text{ MeV} \quad \alpha^2 \approx 50\text{-}60\%$$

Cf. $15.97 + 3.67 / -3.97 \text{ fm}$, ${}^{22}\text{C}$: Tanaka et al, PRL 104, 062701 (2010)

→ RMS radius of halo = 6~7 fm
Dependence on S_{2n} is small

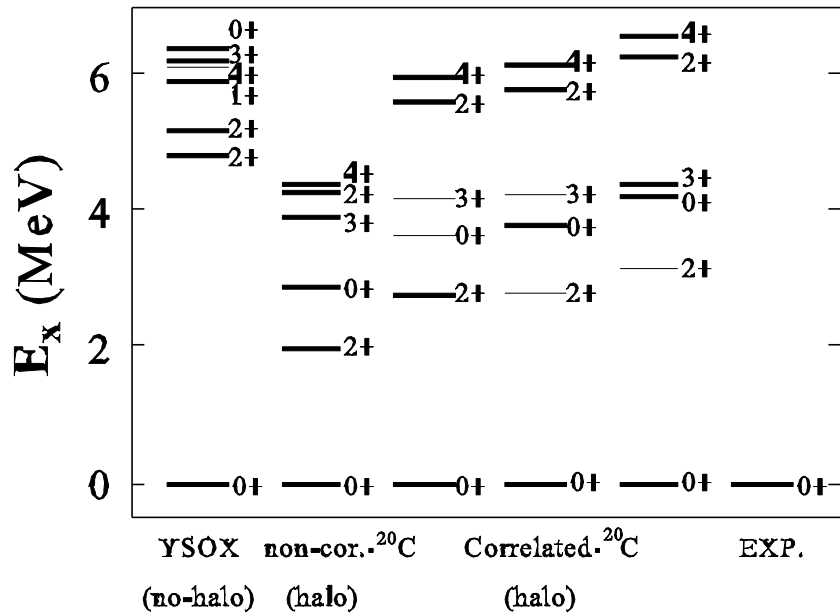


The upper bound on the radius of the halo contradicts the hypothesis of Efimov states, which implies the appearance of similar states at different scales near threshold. The ground state of ${}^{22}\text{C}$ is already close to this upper bound, and there are no excited bound states.

The state of two-neutron halo ${}^{22}\text{C}$ can be called a single Efimov state for the correlated core.

Energy levels of ^{22}C

^{22}C



E_x of 2^+ state depends sensitively on the models, closed or correlated ^{20}C .

Experimental value?

Where is 2^+ state?

V_{ws} : 34.5, 35.0, 36.0

$N(s_{1/2})$: 0.94, 0.28, 0.24, 0.18

Summary

- 3-body model with low-energy n-n interaction, which reproduces s-wave scattering length and effective range, is shown to be successful to make two valence neutrons bound in drip-line nuclei, ^{24}O and ^{22}C .
- S_{2n} and RMS radius of valence neutron in ^{24}O are well reproduced.
- Relation between S_{2n} and RMS radius of halo neutron in ^{22}C are presented for “closed-core” and “correlated-core” models for ^{20}C . For the “correlated-core” model, S_{2n} is constrained to be < 0.8 MeV and RMS radius of halo neutron in ^{22}C is obtained to be 6-7 fm for the condition that ^{21}C is unbound.
This suggests non-existence of multiple (excited) Efimov states.
cf. Acharya, Ji and Phillips, PL B723, 196 (2013)
- Spectrum of ^{22}C is shown to be sensitive to the models, “closed-” or “correlated-core”.