



Charge-changing cross section measurement
of neutron-rich carbon isotopes at 50A MeV
and determination of their proton
distribution root-mean-square radii by using
Glauber model.

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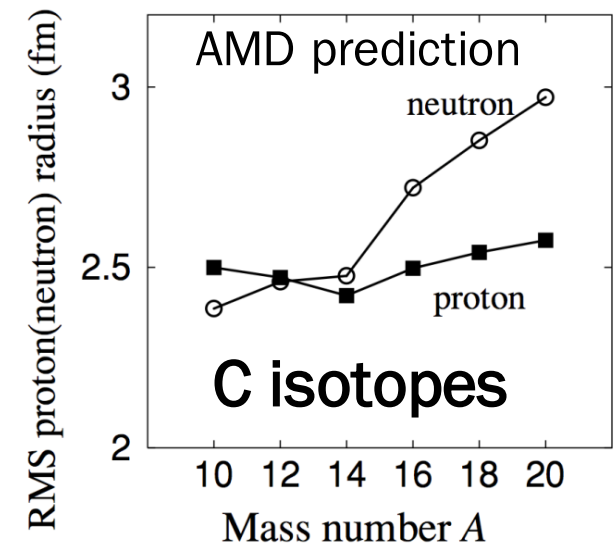
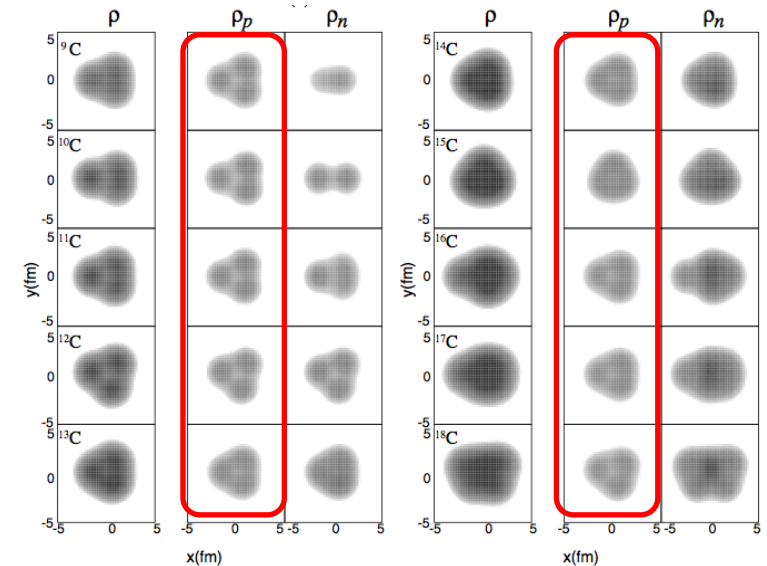
Motivation



- Nuclear radii is one of the most important gross property of nuclei.
- Proton and neutron distributions radii are good observables for testing nuclear structure model.
- The proton distribution radius together with matter radius are important in extracting the neutron skin thickness:

$$\Delta S_n = \langle R_n^2 \rangle^{\frac{1}{2}} - \langle R_p^2 \rangle^{\frac{1}{2}}, \quad N \langle R_n^2 \rangle = A \langle R_m^2 \rangle - Z \langle R_p^2 \rangle$$

- The AMD calculation predicted that proton densities (radii) of C isotopes are almost constant. We want to experimentally confirm this using CCCS measurement.





Nuclear Radii and Cross Section



- ❑ There is no model-independent method for determining the matter-density distribution of an unstable nucleus.
- ❑ For proton rms radii, electron scattering is only applicable for stable nuclei, isotope shift is challenging for p,p-sd shell nuclei.



- ❑ Geometrical model: $\sigma_R(P, T) = \pi(R_T + R_P)^2$

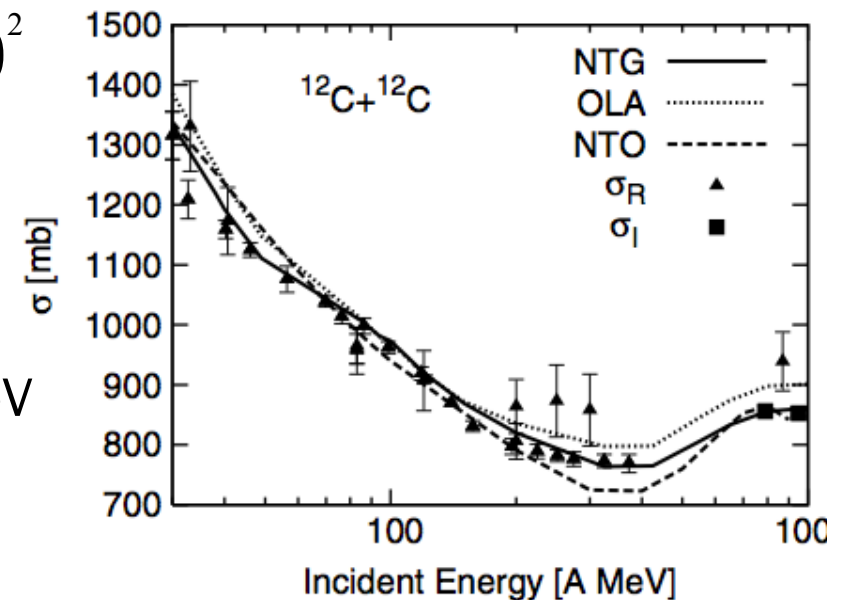
- ❑ Glauber model:

- ✓ It works very well for interaction or reaction cross section from 30A to 1000A MeV



$$\langle R^2 \rangle = \frac{\int r^2 [\rho_p(\mathbf{r}) + \rho_n(\mathbf{r})] d\mathbf{r}}{\int [\rho_p(\mathbf{r}) + \rho_n(\mathbf{r})] d\mathbf{r}}$$

W. Horiuchi *et al.*,
Phys.Rev.C75,044607 (200)





Extension of Glauber Model to CCCS



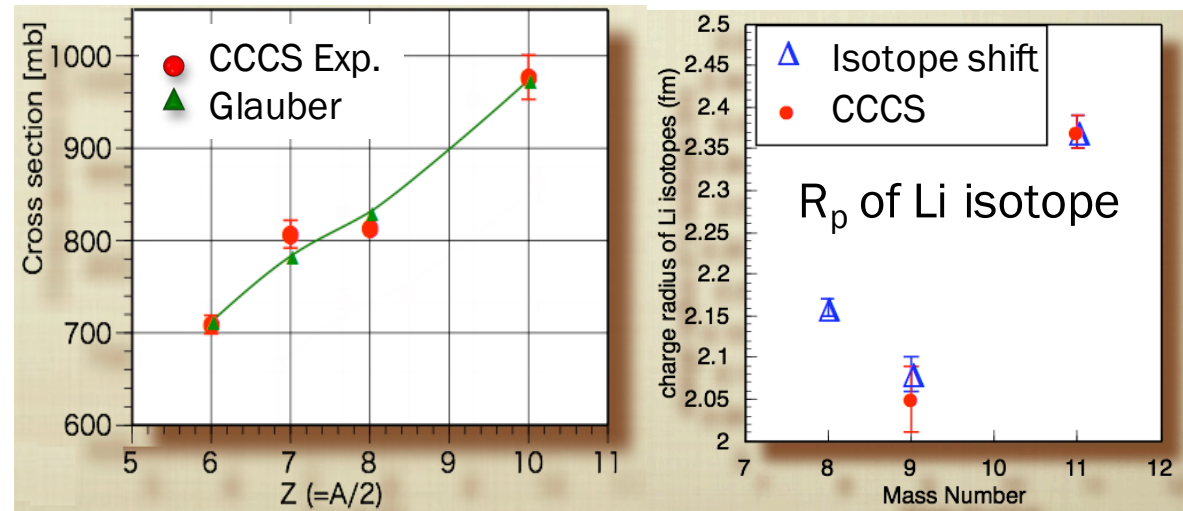
- ✓ **Charge-Changing Cross Section** σ_{cc} is the cross section for all (direct) processes which result in a **change of the atomic number(Z) of the projectile**

$$\sigma_{cc} = \int [1 - T_c(b)] d\mathbf{b}$$

$$T_c(\mathbf{b}) = \exp\left[-\sigma_{pp} \int \rho_{Pp}(\mathbf{r} - \mathbf{b}) \cdot \rho_{Tp}(\mathbf{r}) d\mathbf{r} - \sigma_{pn} \int \rho_{Pp}(\mathbf{r} - \mathbf{b}) \cdot \rho_{Tn}(\mathbf{r}) d\mathbf{r}\right]$$

- ✓ Tested by stable nuclei and Li isotopes at 900A MeV.

How about at energy around 50A MeV?



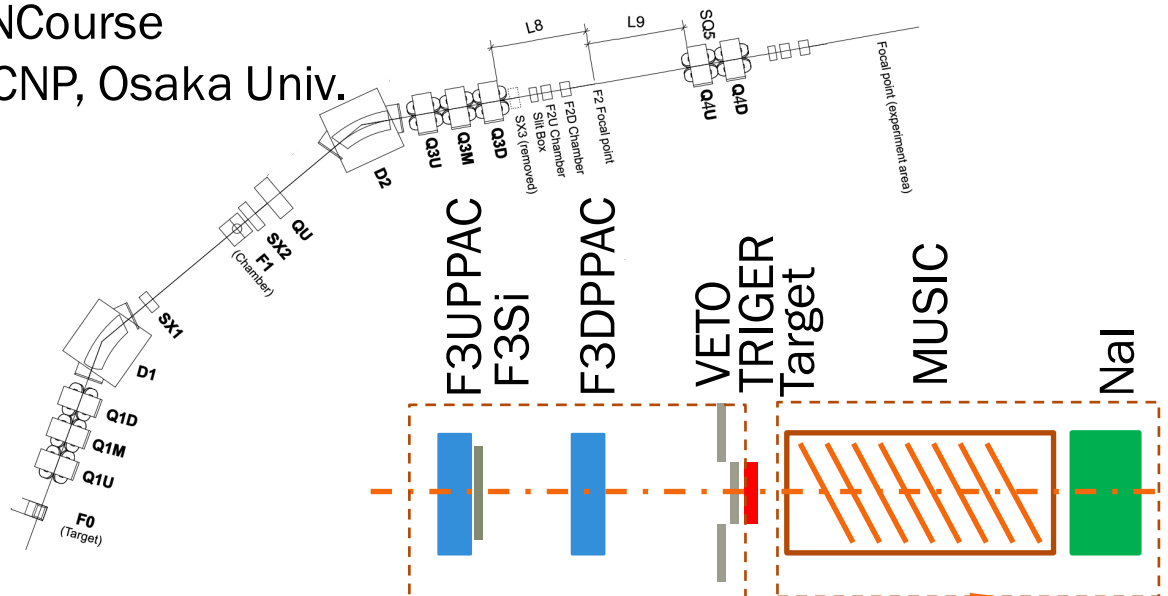
I. Tanihata *et al*, Prog.Part.Nucl.Phys. 68 (2013),215



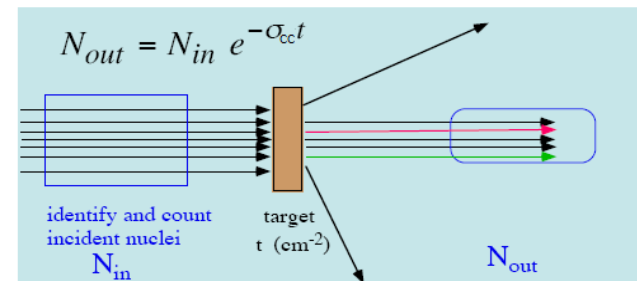
Experiment setup



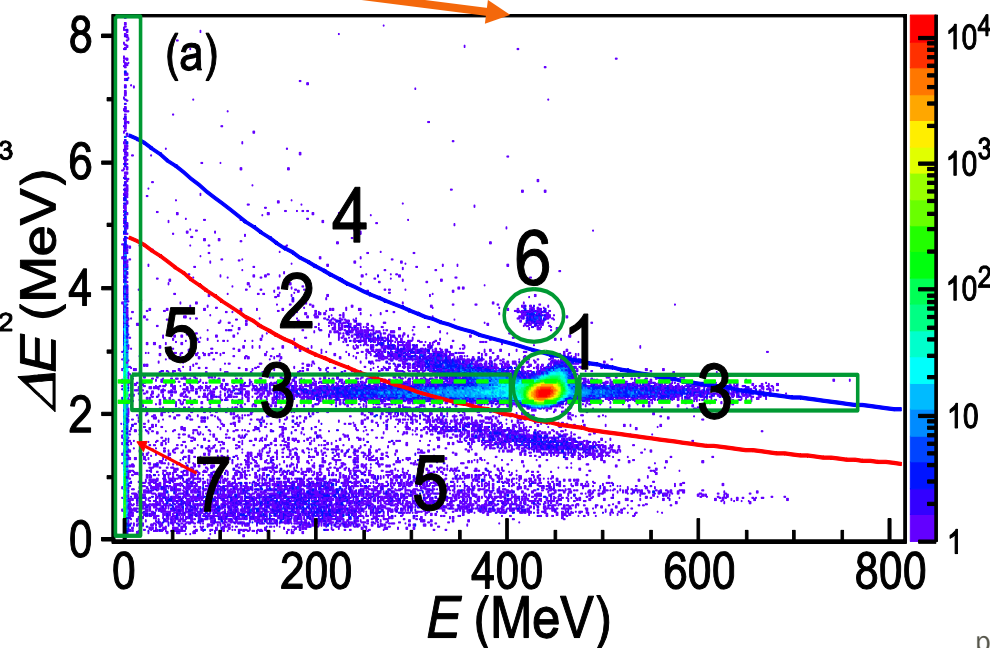
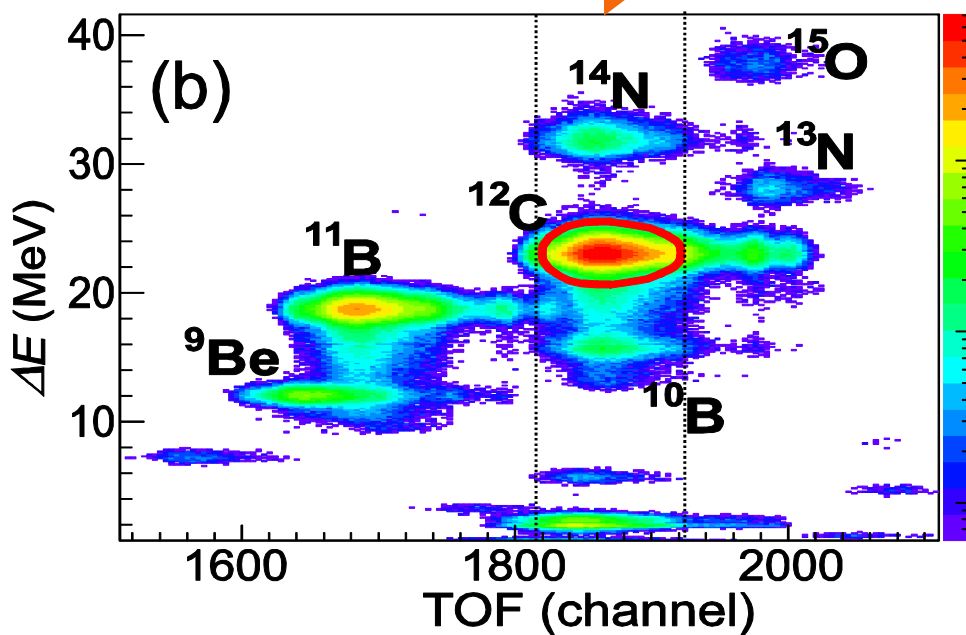
ENCourse
RCNP, Osaka Univ.



Transmission method.



Primary beam: ²²Ne, 80A MeV.
Primary target: Be
Reaction target: 450 mg/cm² natC

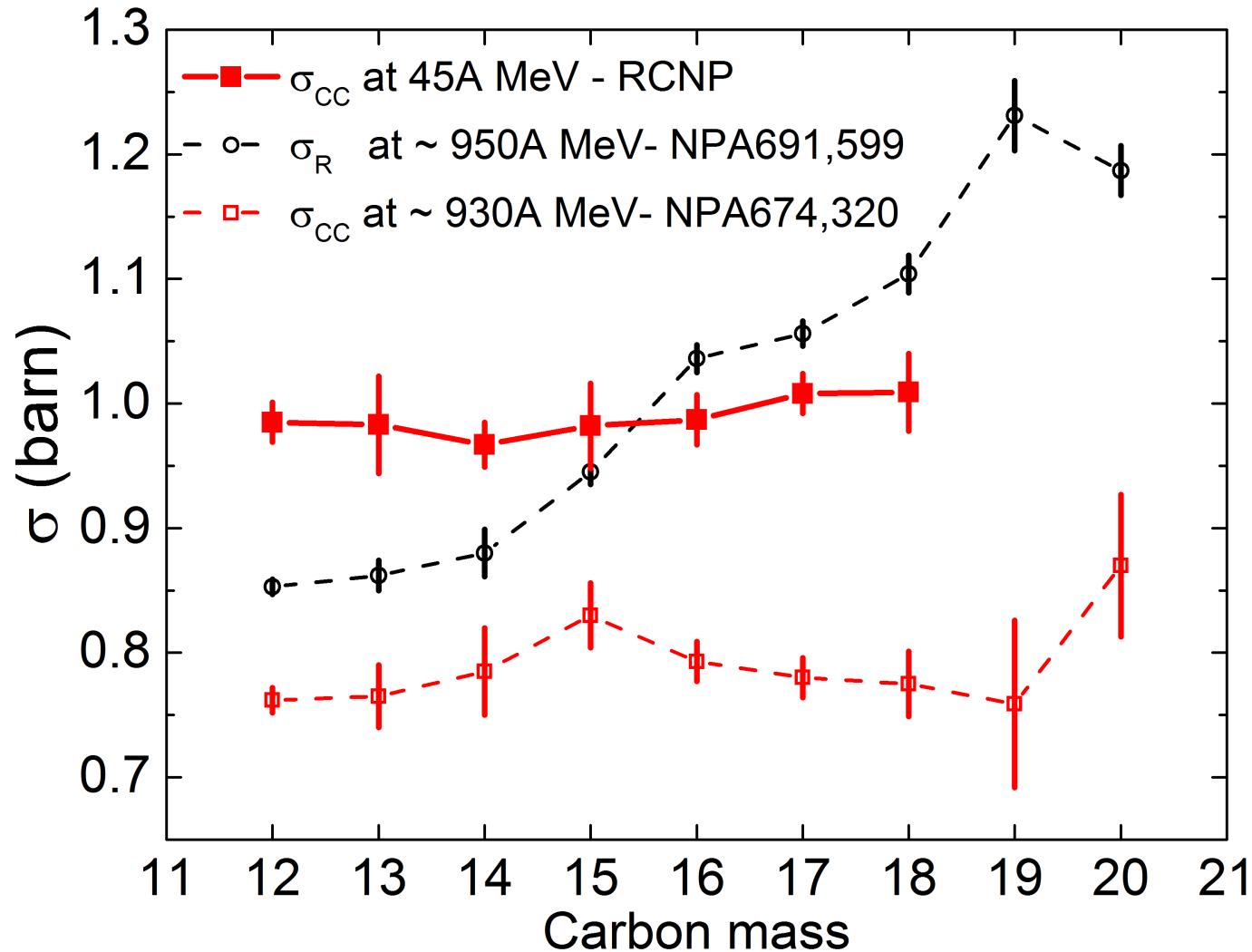




Charge-Changing Cross Section



- CCCSs of C isotopes increase slowly with neutron number for $^{12-18}\text{C}$.





Glauber model calculation



Formalism

$$\sigma_R = \int d\mathbf{b} \left(1 - \left| e^{i\chi(\mathbf{b})} \right|^2 \right)$$

$$e^{i\chi(\mathbf{b})} = \langle \Psi_0 \Theta_0 | \prod_{i \in P} \prod_{j \in T} \left[1 - \Gamma_{NN}(\mathbf{s}_i - \mathbf{t}_j + \mathbf{b}) \right] | \Psi_0 \Theta_0 \rangle$$

$$\Gamma_{NN}(\mathbf{b}) = \frac{1 - i\alpha_{NN}}{4\pi\beta_{NN}} \sigma_{NN} \exp\left[-\frac{b^2}{2\beta_{NN}} \right]$$

NTG (Nucleon-Target Glauber) $e^{i\chi_{NTG}(\mathbf{b})} = \exp\left[-\int d\mathbf{r} \rho_P(\mathbf{r}) \left\{ 1 - \exp\left[-\int d\mathbf{r}' \rho_T(\mathbf{r}') \Gamma_{NN}(\mathbf{s} - \mathbf{t} + \mathbf{b}) \right] \right\} \right]$

OLA $e^{i\chi_{OLA}(\mathbf{b})} = \exp\left[-\iint d\mathbf{r} d\mathbf{r}' \rho_P(\mathbf{r}) \rho_T(\mathbf{r}') \Gamma_{NN}(\mathbf{s} - \mathbf{t} + \mathbf{b}) \right]$

Zero range $e^{i\chi_{ZR}(\mathbf{b})} = \exp\left[-\int d\mathbf{r} \rho_P(\mathbf{r}) \rho_T(\mathbf{r} - \mathbf{b}) \sigma_{NN} \right]$



Glauber model calculation



Formalism

$$\sigma_R = \int d\mathbf{b} \left(1 - \left| e^{i\chi(\mathbf{b})} \right|^2 \right)$$

$$e^{i\chi(\mathbf{b})} = \langle \Psi_0 \Theta_0 | \prod_{i \in P} \prod_{j \in T} [1 - \Gamma_{NN}(\mathbf{s}_i - \mathbf{t}_j + \mathbf{b})] | \Psi_0 \Theta_0 \rangle$$

$$\Gamma_{NN}(\mathbf{b}) = \frac{1 - i\alpha_{NN}}{4\pi\beta_{NN}} \sigma_{NN} \exp\left[-\frac{b^2}{2\beta_{NN}}\right]$$

NTG (Nucleon-Target Glauber) $e^{i\chi_{NTG}(\mathbf{b})} = \exp\left[-\int d\mathbf{r} \rho_P(\mathbf{r}) \left\{ 1 - \exp\left[-\int d\mathbf{r}' \rho_T(\mathbf{r}') \Gamma_{NN}(\mathbf{s} - \mathbf{t} + \mathbf{b})\right] \right\}\right]$

OLA $e^{i\chi_{OLA}(\mathbf{b})} = \exp\left[-\iint d\mathbf{r} d\mathbf{r}' \rho_P(\mathbf{r}) \rho_T(\mathbf{r}') \Gamma_{NN}(\mathbf{s} - \mathbf{t} + \mathbf{b})\right]$

Zero range $e^{i\chi_{ZR}(\mathbf{b})} = \exp\left[-\int d\mathbf{r} \rho_P(\mathbf{r}) \rho_T(\mathbf{r} - \mathbf{b}) \sigma_{NN}\right]$

Input:

- σ_{NN} : Total cross section for nucleon-nucleon collisions.
- β_{NN} : Slope parameter of the NN elastic differential cross section
- ρ_P, ρ_T : Projectile, target nucleon density



Our approach



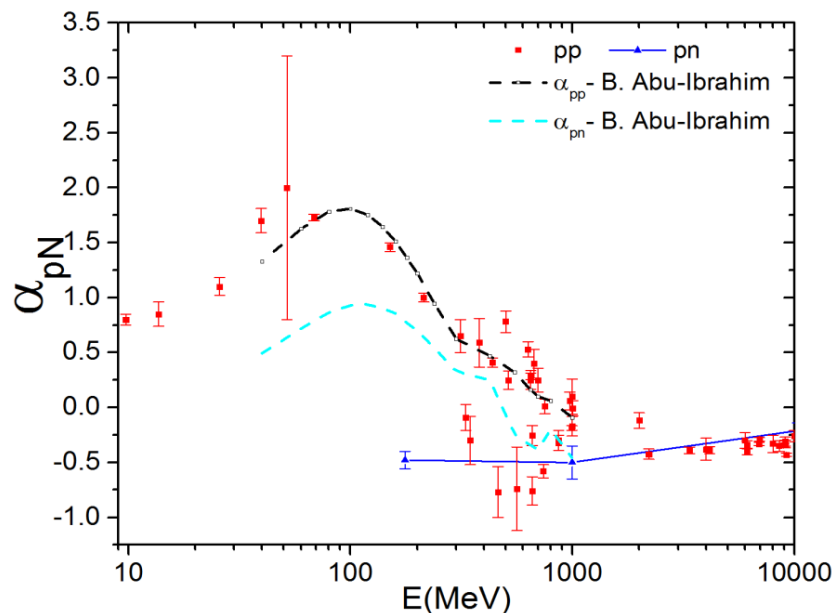
∞ The assumptions of this work:

◆ σ_{pN} : from Data Particle Group 2014.

◆ For β_{pN} :
$$\beta_{pp(n)} = \frac{1 + \alpha_{pp(n)}^2}{16\pi\sigma_{pp(n)}^{el}} \left(\sigma_{pp(n)}^{tot} \right)^2$$

❖ β_{pp} : using α_{pp} and σ_{pp}^{el} from PDG 2014.

❖ β_{pn} : determinate so as to fit experimental data of ^{12}C on carbon target.

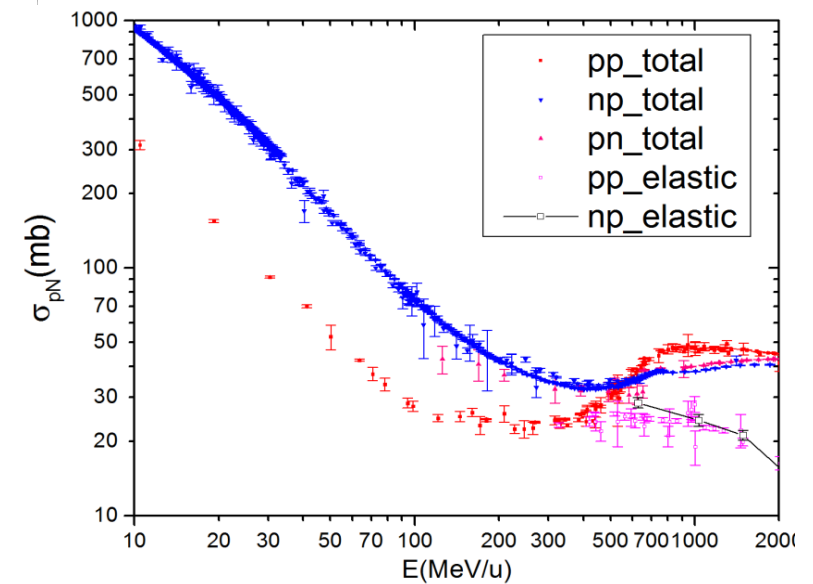


◆ For nucleon distribution:

❖ Target (ρ_T) - ^{12}C : proton distr. from electron scattering data; and neutron distr. are fitted harmonic oscillator (HO) distr. to reproduced σ_R and σ_{CC} at 950A MeV.

❖ Projectile (ρ_P) - ^AC : proton distr. is HO distr. fitted to σ_{CC} measured at RCNP; neutron distr. is also HO distr. fitted to σ_R at 950A MeV.

Exp. Points from Particle Data Group, 2014 and 2015 update





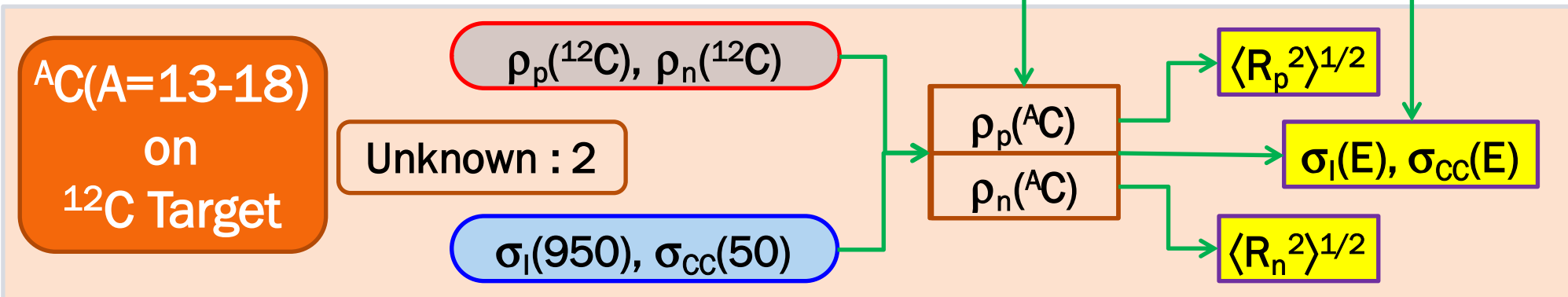
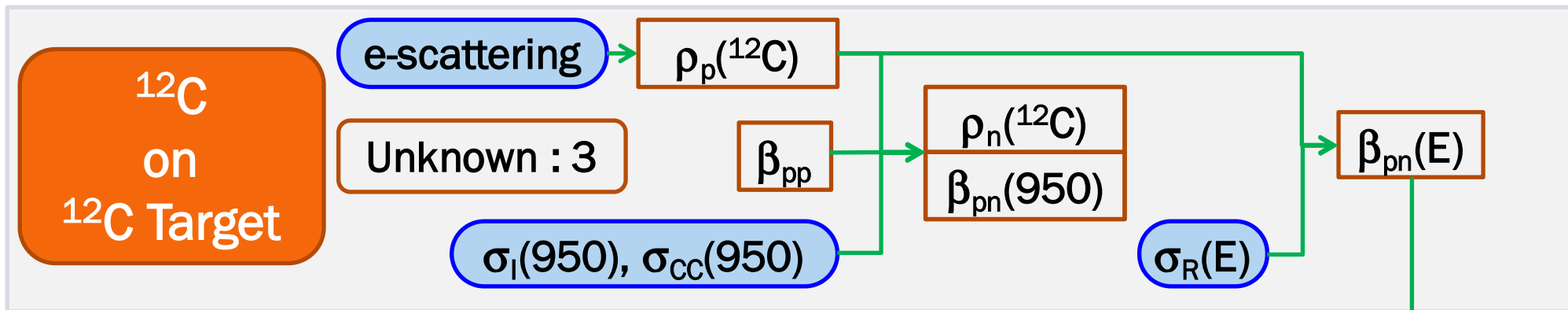
Extracting proton distribution radii



$$T(b) = \text{Exp} \left[- \int dz \sum_{i,k} \sigma_{ik} \iiint \rho_{Pi}(\vec{s}) \rho_{Tk}(\vec{t}) \Gamma(\vec{b} + \vec{s} - \vec{t}) d\vec{s} d\vec{t} \right], \text{ with } i, k : p, n$$

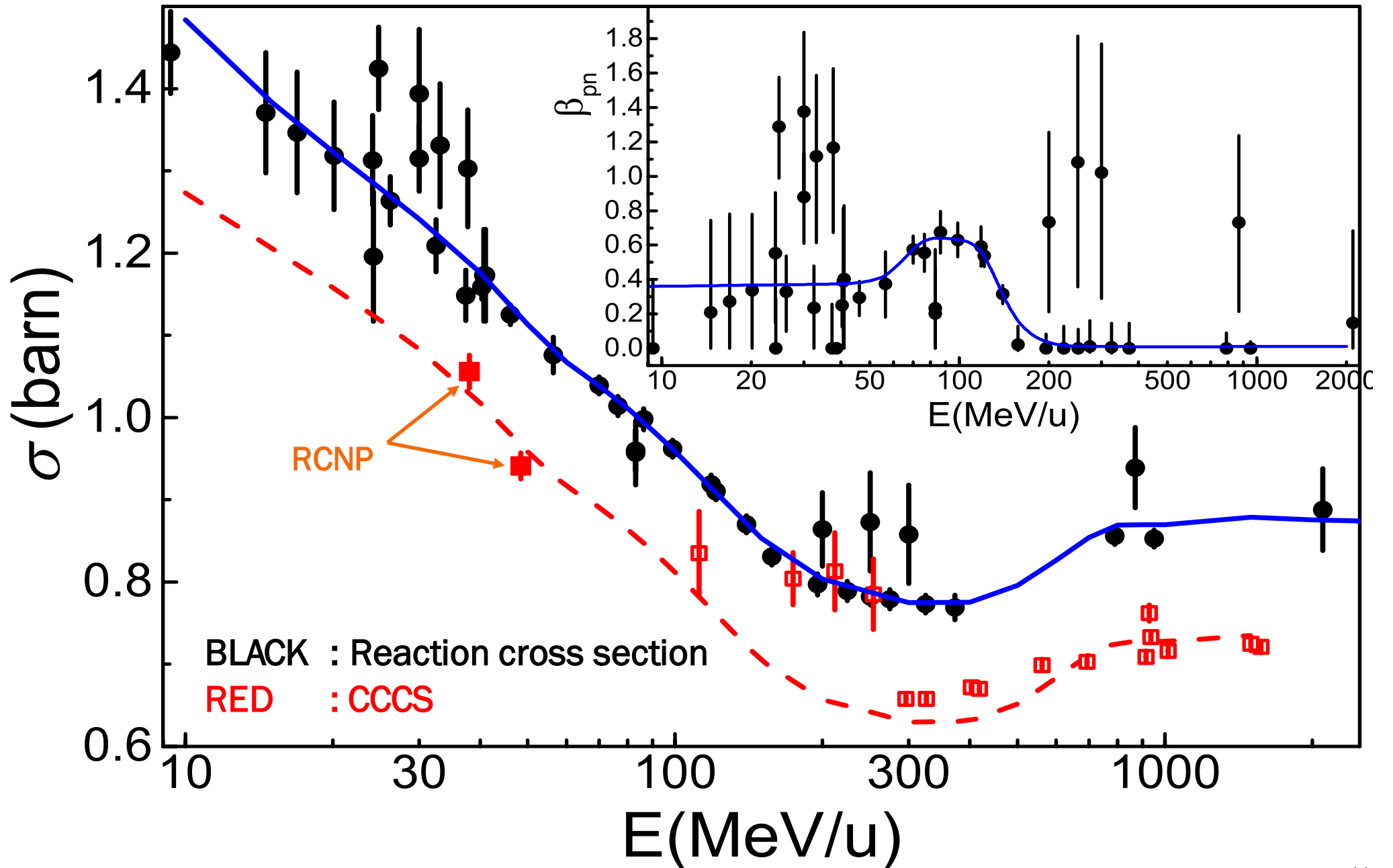
$$\Gamma(\vec{b}) = \frac{1}{4\pi\beta_{NN}^2} \text{Exp} \left[- \frac{b^2}{2\beta_{NN}^2} \right]$$

Unknown parameters: 5





Results of Glauber model

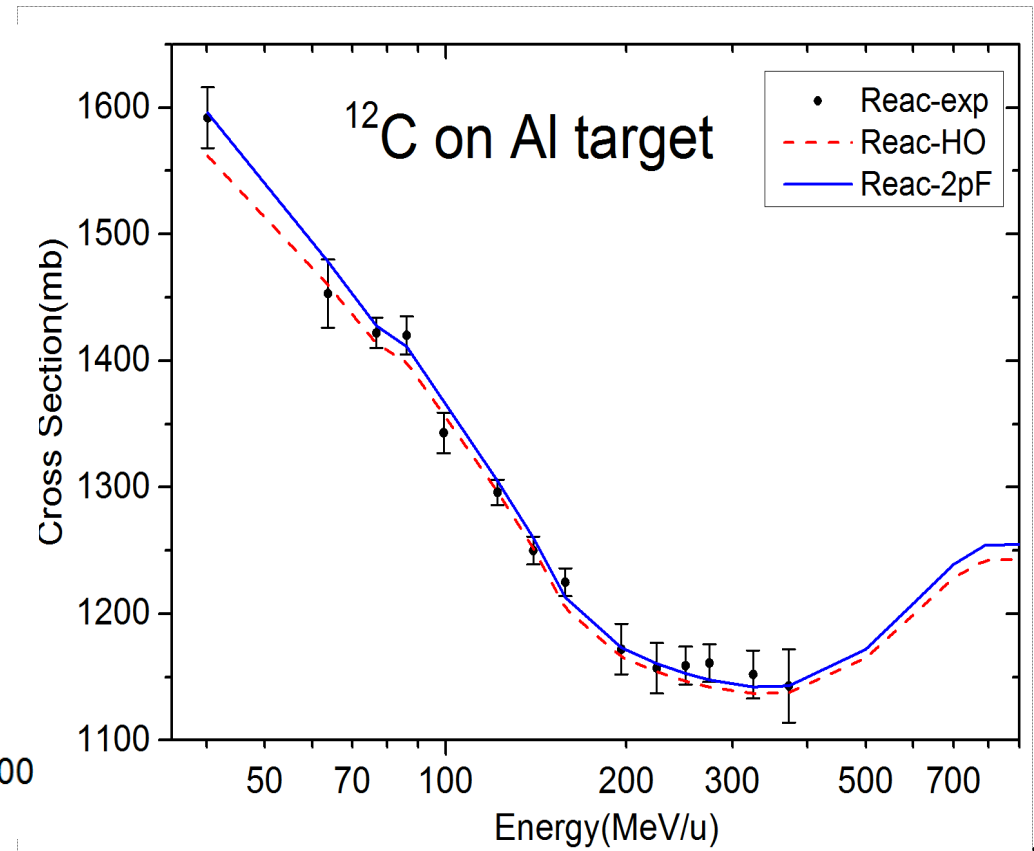
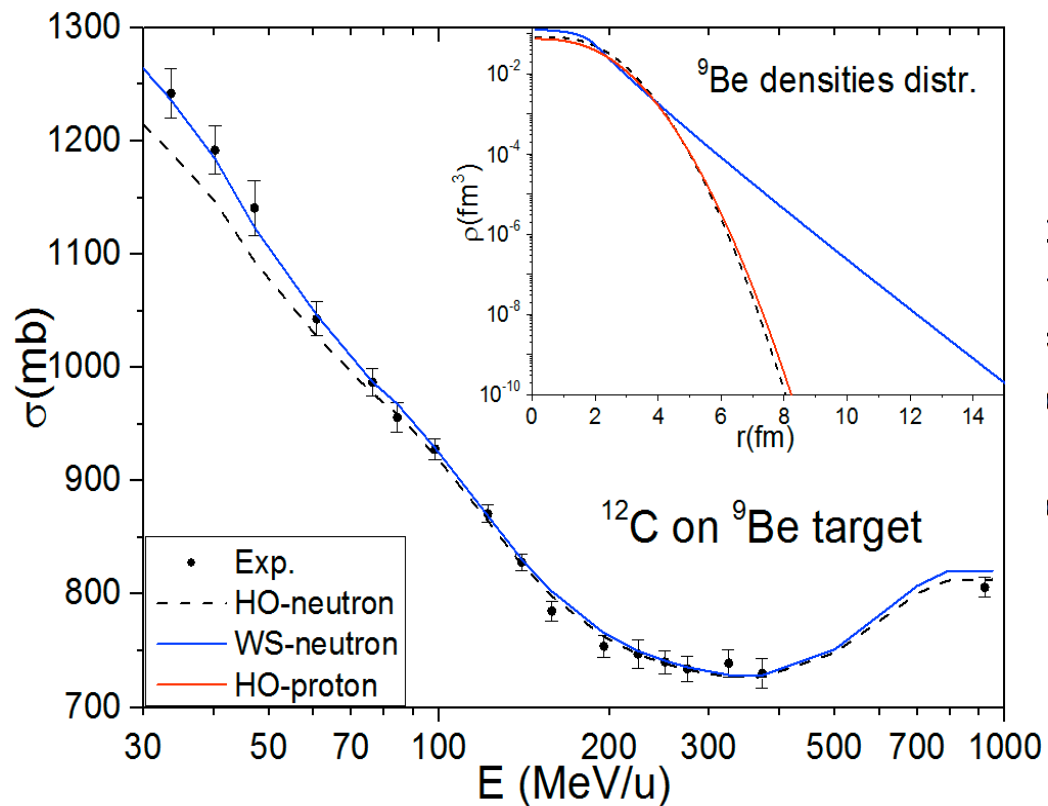




Results of Glauber model

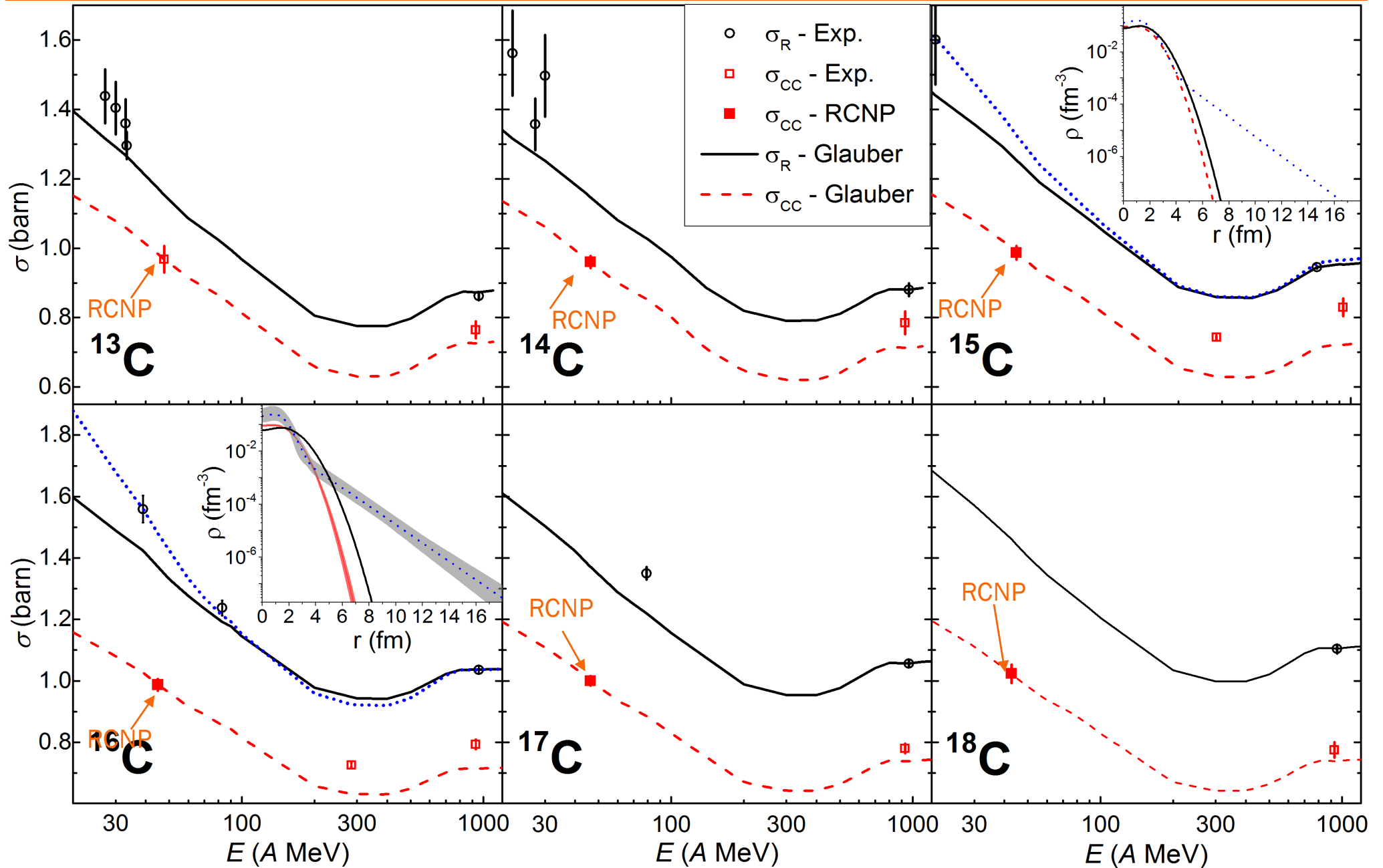


- ∞ HO: Hamonic osillator distribution.
- ∞ WS: Woods-Saxon (Two parameters Fermi) distribution.
- ∞ β_{pn} : Obtained from Carbon data.



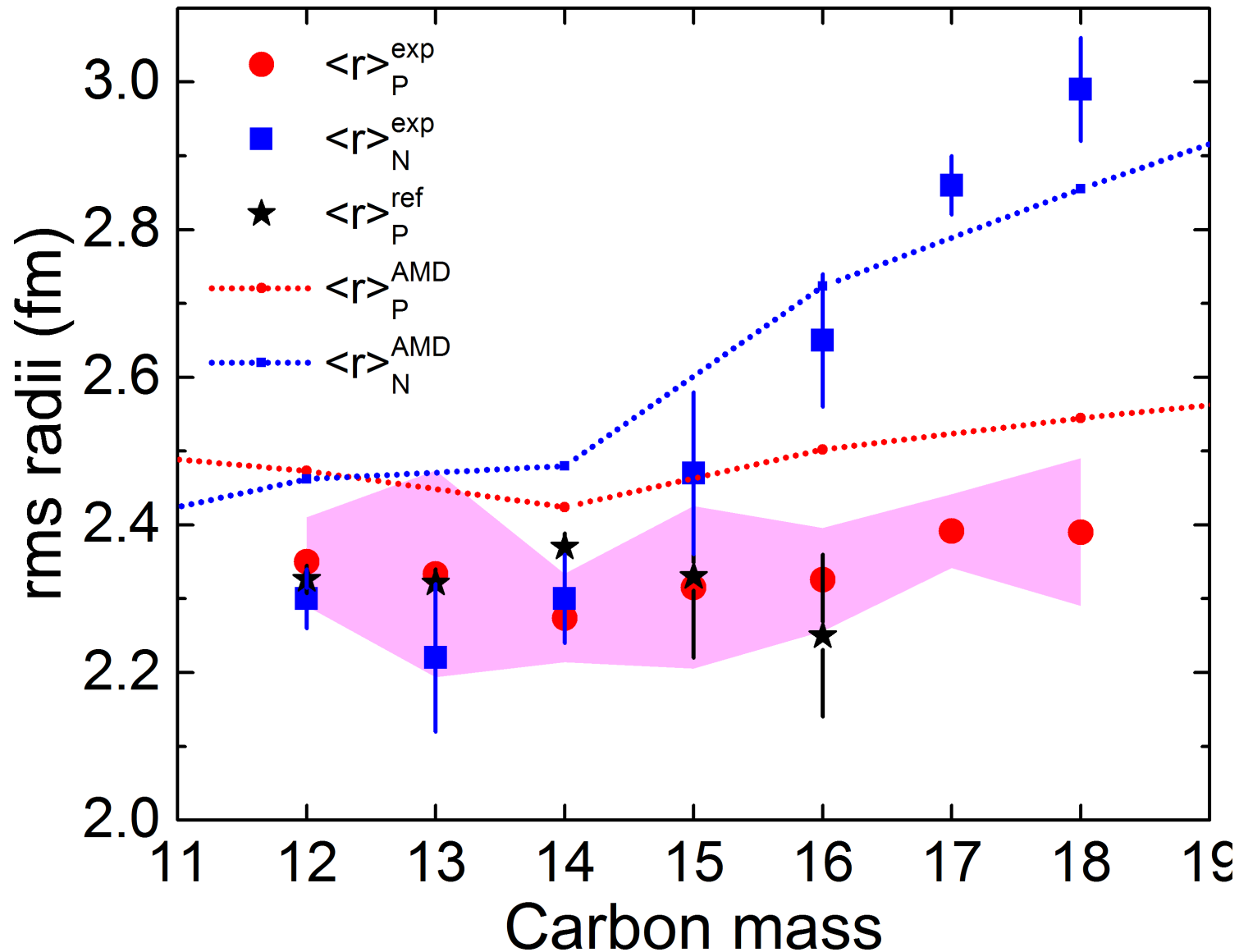


Consistency between low and high energy





Proton RMS radii



Ref:

I. Angeli, K.P. Marinova
AD&ND tabl 99,69(2013)

T. Yamaguchi et al.,
PRL107,032502(2011)

AMD:

Y. Kanada-En'yo,
PRC71,014310(2005)



Conclusion



- We have measured CCCSs of $^{10-18}\text{C}$ isotopes at 45A MeV using RI beam at EN Course, RCNP, Osaka University.
- The CCCSs at different beam energies are consistently understood by Glauber model with global parameters set.
- Rms proton radii of $^{12-18}\text{C}$ were determined by using Glauber model; they are almost constant. Which is consistent with electron scattering data and AMD prediction.
- The rms radii can be extracted from reaction/ CCCS measurement at low energy.



List of collaborators



D. T. Tran^{A,B}, T. T. Nguyen^{B,C}, I. Tanihata^{A,D}, M. Fukuda^E, H. J. Ong^A,
N. Aoi^A, Y. Ayyad^A, P.Y. Chan^A, T. H. Hoang^{A,B},
T. Hashimoto^F, E. Ideguchi^A, A. Inoue^A, T. Kawabata^G, H. K. Le^B,
K. Matsuta^E, M. Mihara^E, S. Momota^H, D. Nagae^I, A. Ozawa^I,
P.P. Ren^J, H. Sakaguchi^A, J. Tanaka^A, S. Terashima^D, R. Wada^J,
W.P. Liu^J, T. Yamamoto^A

^ARCNP, Osaka Univ.; ^BIOP Hanoi, Vietnam; ^CPham Ngoc Thach Univ. Vietnam;
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^GDept. of Phys. Kyoto Univ.; ^HKochi Univ. of Tech.; ^IIOP Univ. of Tsukuba;
^JIMP, China.

Thank you for your attention!



BACKUP





Charge Changing Cross Section (CCCS) Measurement of Neutron-rich Carbon Isotopes at 50A MeV.

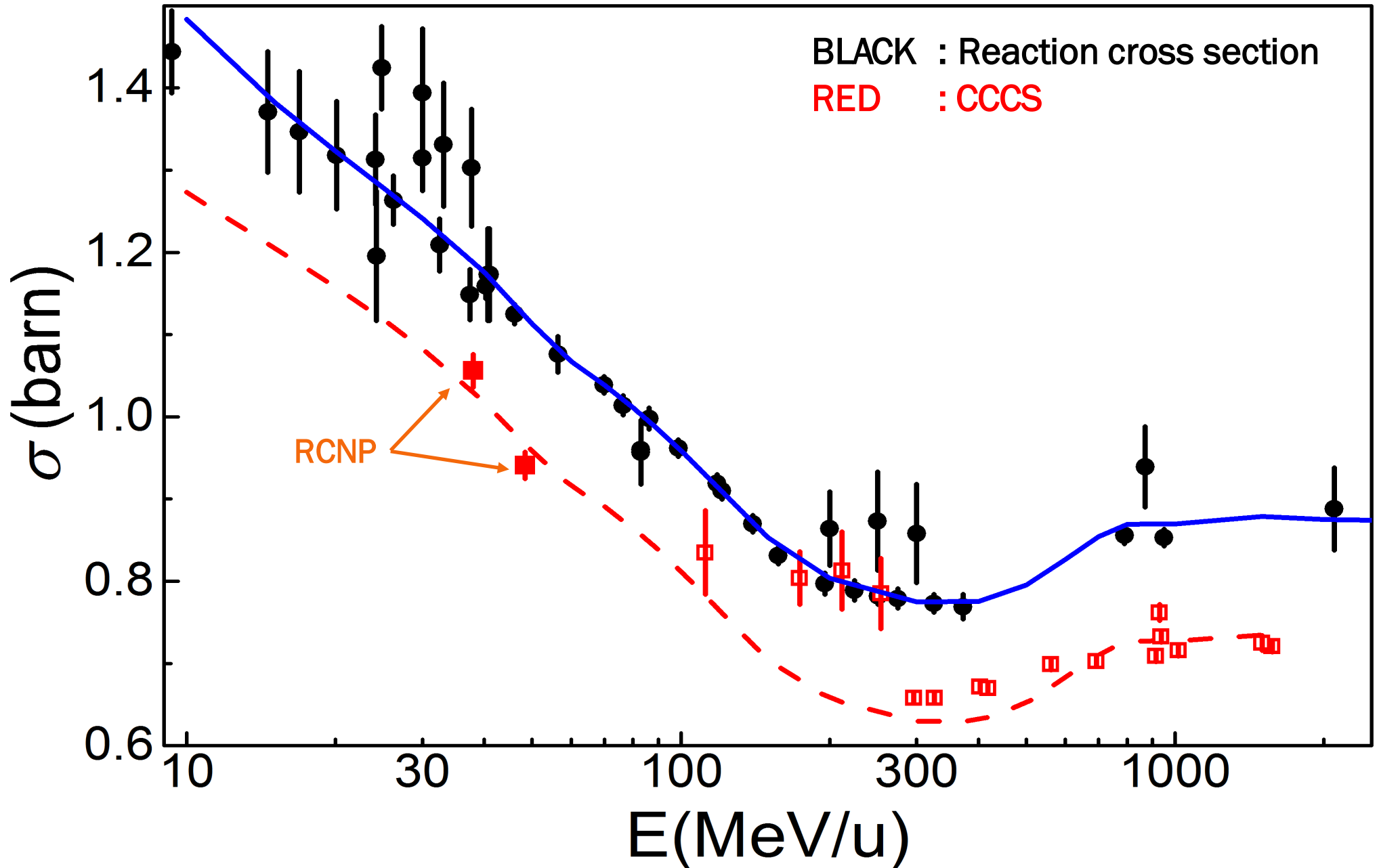
D. T. Tran^{A,B}, T. T. Nguyen^{B,C}, I. Tanihata^{A,D}, M. Fukuda^E, H. J. Ong^A, N. Aoi^A, Y. Ayyad^A,
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H. Sakaguchi^A, J. Tanaka^A, S. Terashima^D, R. Wada^J, W.P. Liu^J, T. Yamamoto^A

Presenter: Tran Dinh Trong

^ARCNP, Osaka Univ., ^BIOP Hanoi, Vietnam, ^CPham Ngoc Thach Univ. Vietnam,
^DBeihang Univ. China, ^EDept. of Phys. Osaka Univ., ^FIBS, Korea, ^GDept. of Phys. Kyoto Univ.,
^HKochi Univ. of Tech., ^IIOP Univ. of Tsukuba, ^JIMP, China.



Results of Glauber model

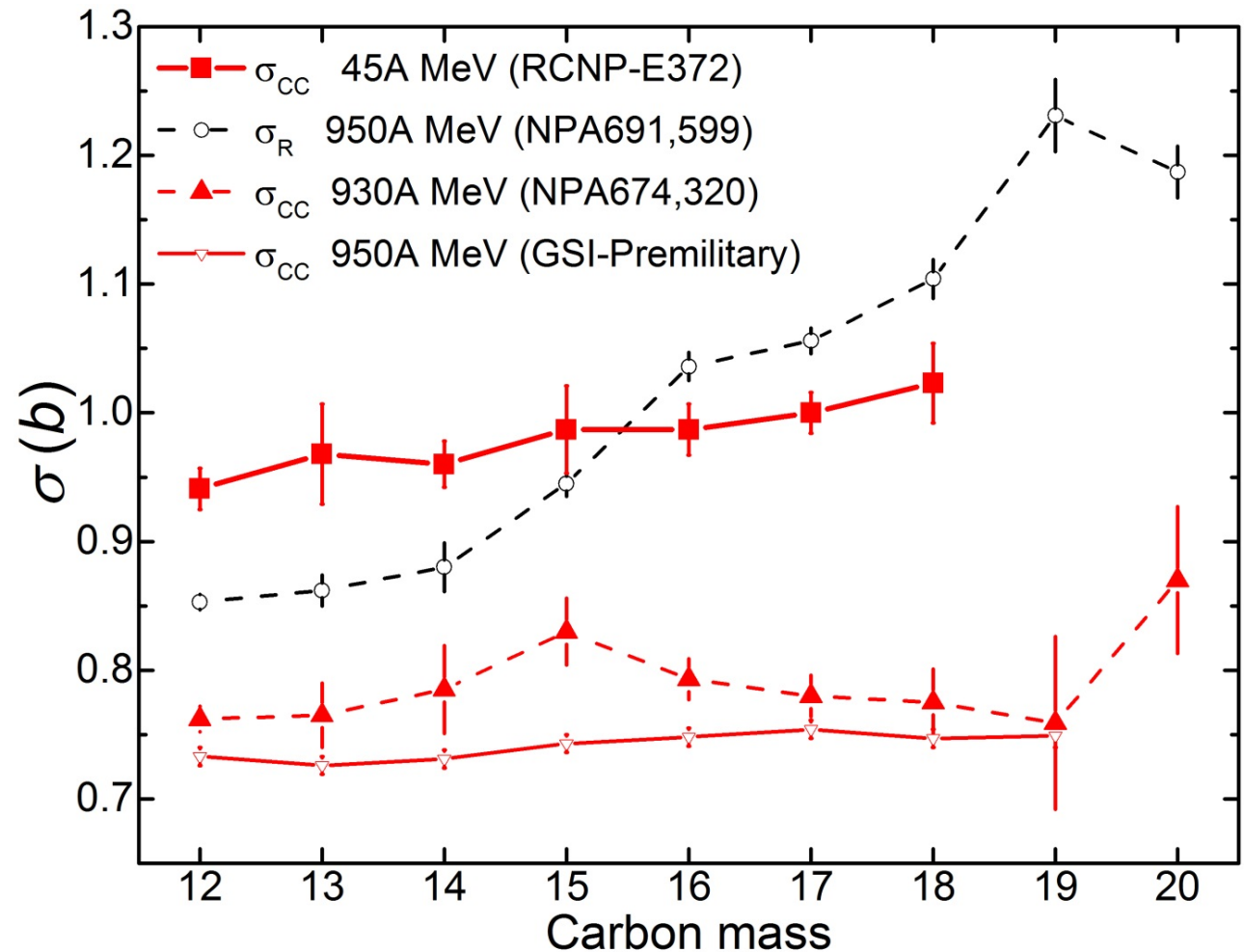




Charge-Changing Cross Section

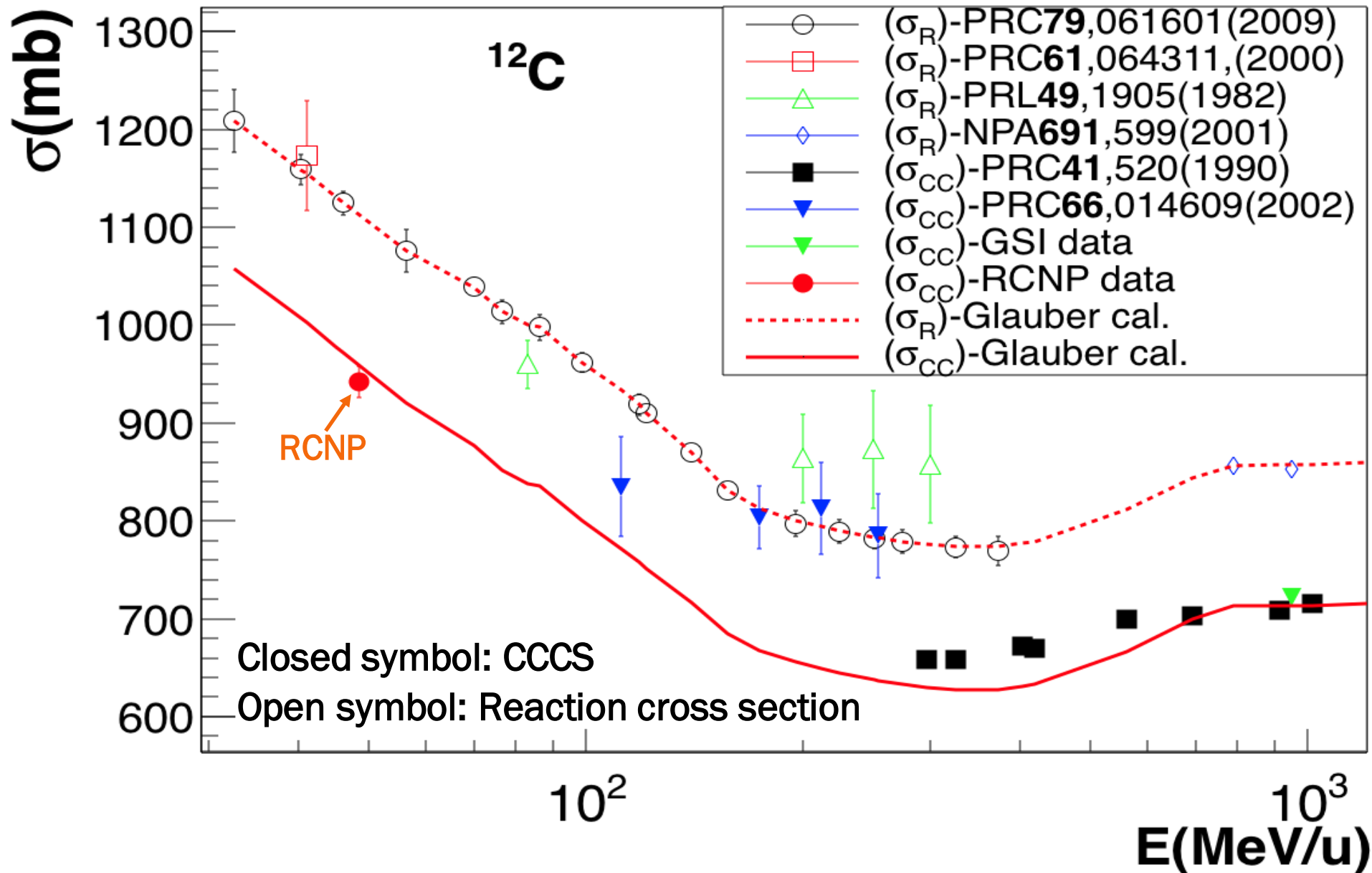


- CCCSs of C isotopes increase slowly with neutron number for $^{12-18}\text{C}$.
- This trend is similar with the high energy data.



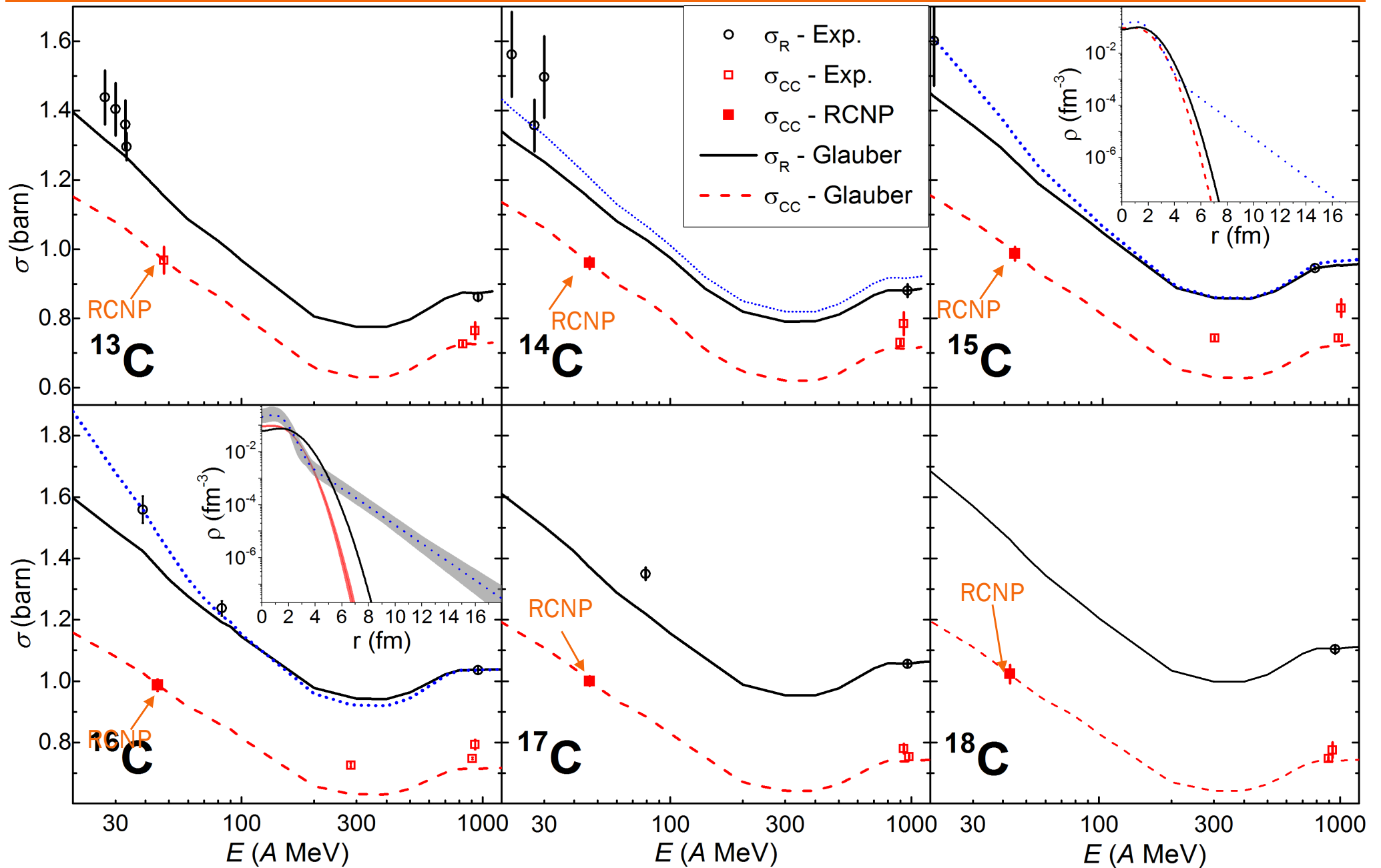


Results of Glauber model



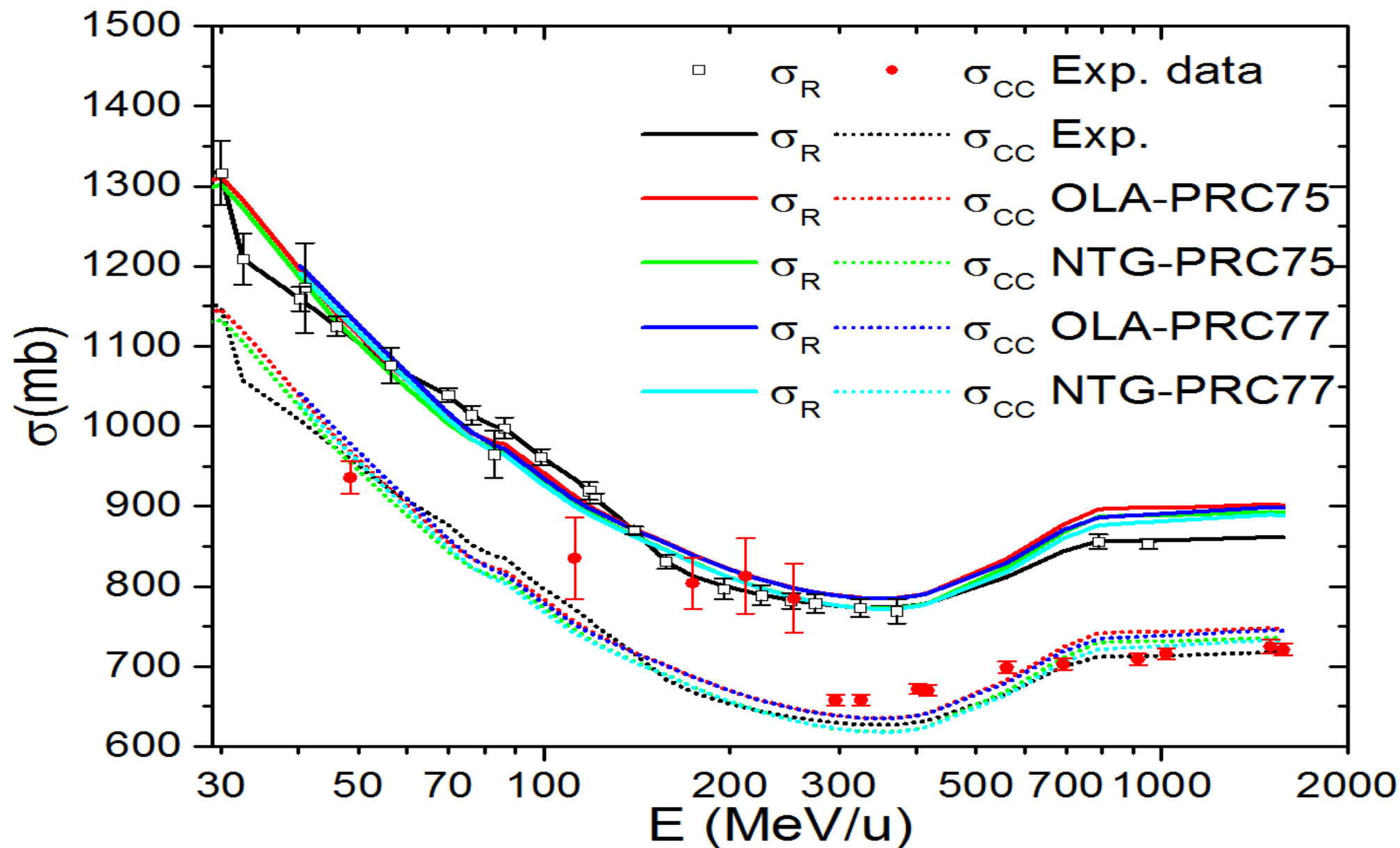


Consistency between low and high energy





Results of Glauber model





Content



- ☞ Motivation
- ☞ Experiment setup
- ☞ Data analysis
- ☞ Glauber calculation
- ☞ Results and discussion
- ☞ Conclusion

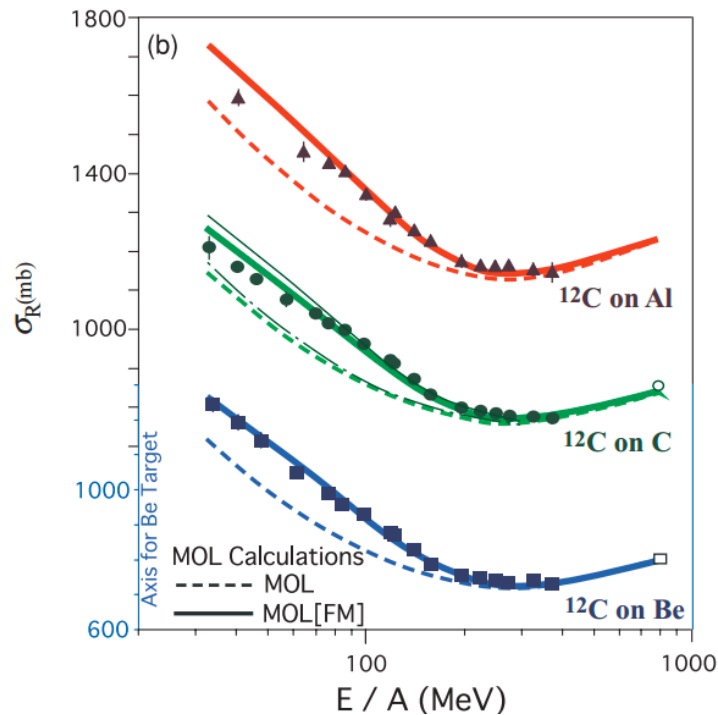
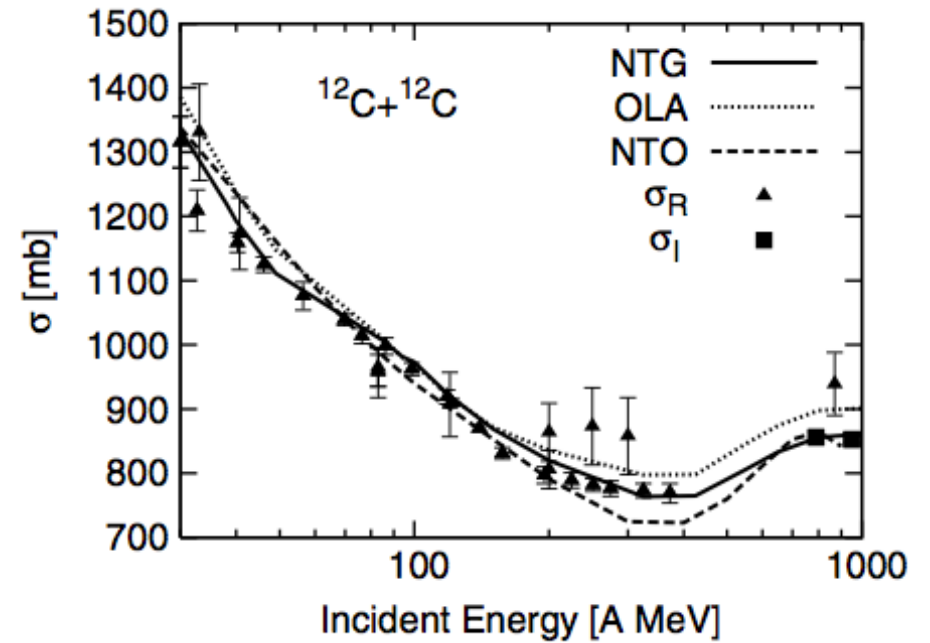


Existing Glauber model calculation

Literatures

◆ W. Horiuchi *et al.*, PRC75, 044607

$$\sigma_{pN}^{el} = \frac{1 + \alpha_{pN}^2}{16\pi\beta_{pN}} (\sigma_{pN}^{tot})^2.$$



◆ M. Takechi *et al.*, PRC79, 061601

$$\sigma_{NN}^{eff} = \int_{-\infty}^{+\infty} dp_{rel} \sigma_{NN}(p_{rel}) D(p_{rel}).$$

$$D(p_{rel}) = \frac{1}{\sqrt{2\pi(\langle p^2 \rangle^P + \langle p^2 \rangle^T)}} \times \exp[-(p_{rel} - p_{proj})^2 / 2(\langle p^2 \rangle^P + \langle p^2 \rangle^T)].$$

$$\sqrt{\langle p^2 \rangle} = 90 \text{ MeV}/c$$



Comparing Glauber Model



Calculations 1

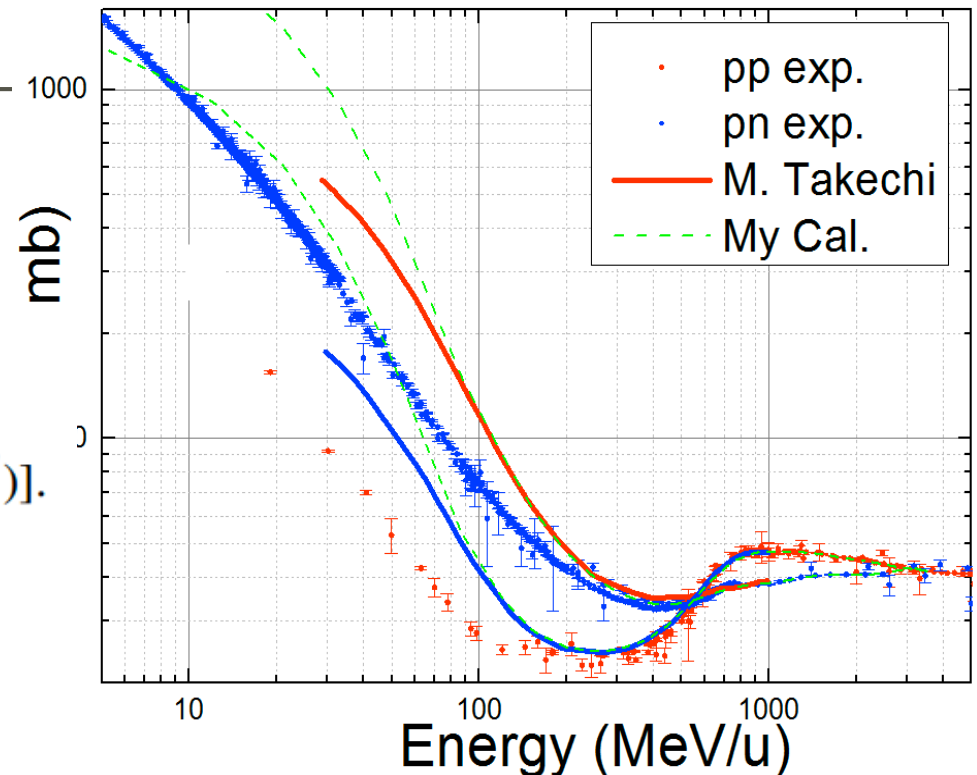
☞ M. Takechi at el., PRC79, 061601

$$\sigma_{NN}^{\text{eff}} = \int_{-\infty}^{+\infty} dp_{\text{rel}} \sigma_{NN}(p_{\text{rel}}) D(p_{\text{rel}}).$$

$$D(p_{\text{rel}}) = \frac{1}{\sqrt{2\pi(\langle p^2 \rangle^P + \langle p^2 \rangle^T)}} \times \exp[-(p_{\text{rel}} - p_{\text{proj}})^2 / 2(\langle p^2 \rangle^P + \langle p^2 \rangle^T)].$$

$$\sqrt{\langle p^2 \rangle} = 90 \text{ MeV}/c$$

I cannot reproduce her effective total cross section at low energy.



Are there any additional assumption?



Comparing Glauber Model



Calculations 2

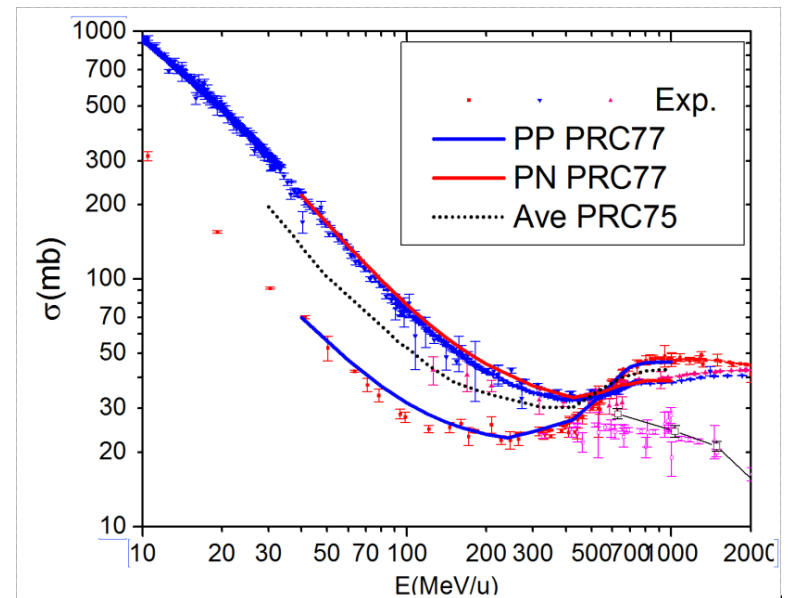
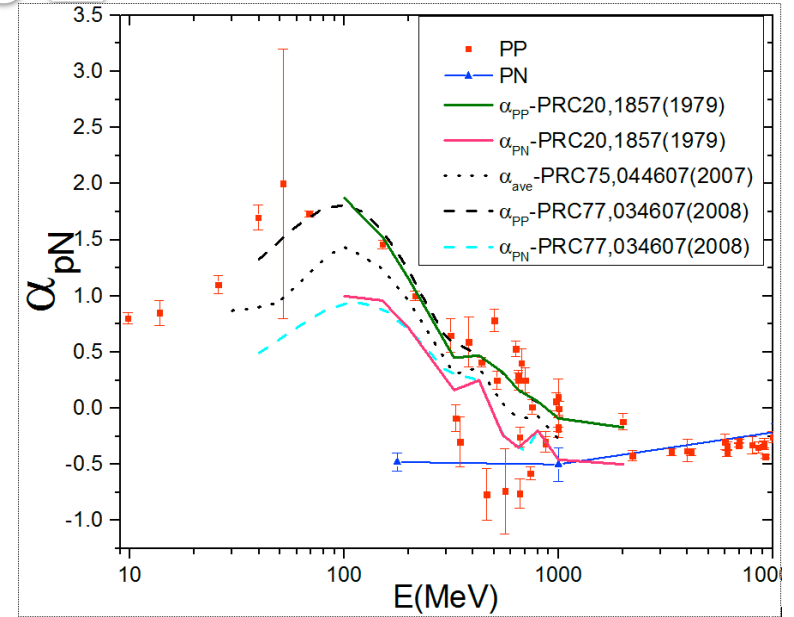
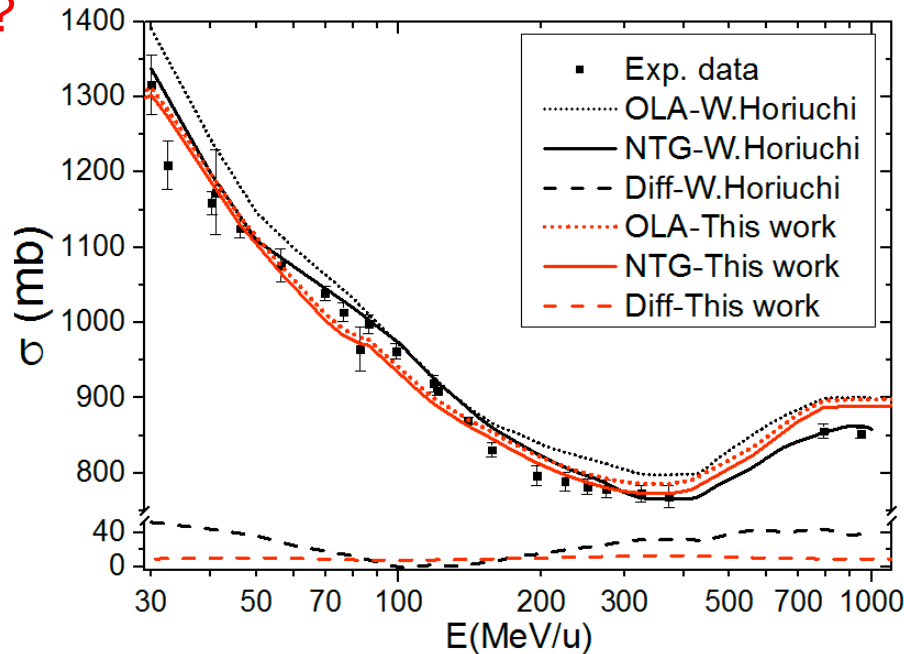
W. Horiuchi at el., PRC75, 044607

$$\sigma_{pN}^{el} = \frac{1 + \alpha_{pN}^2}{16\pi\beta_{pN}} (\sigma_{pN}^{tot})^2.$$

◆ No data at low energy for α_{pn} .

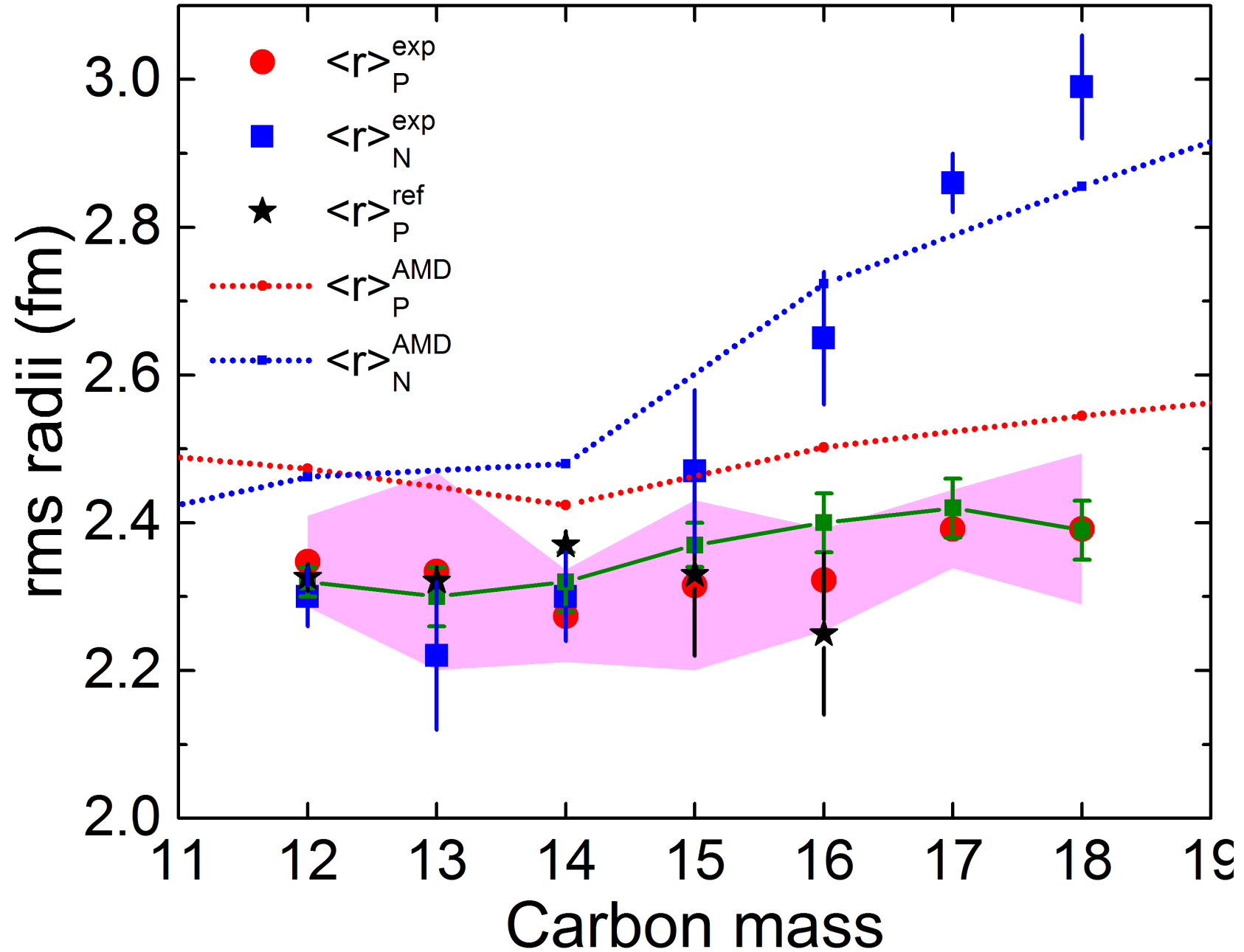
Extrapolation? How?

◆ Any assumption at low and high energy regions?



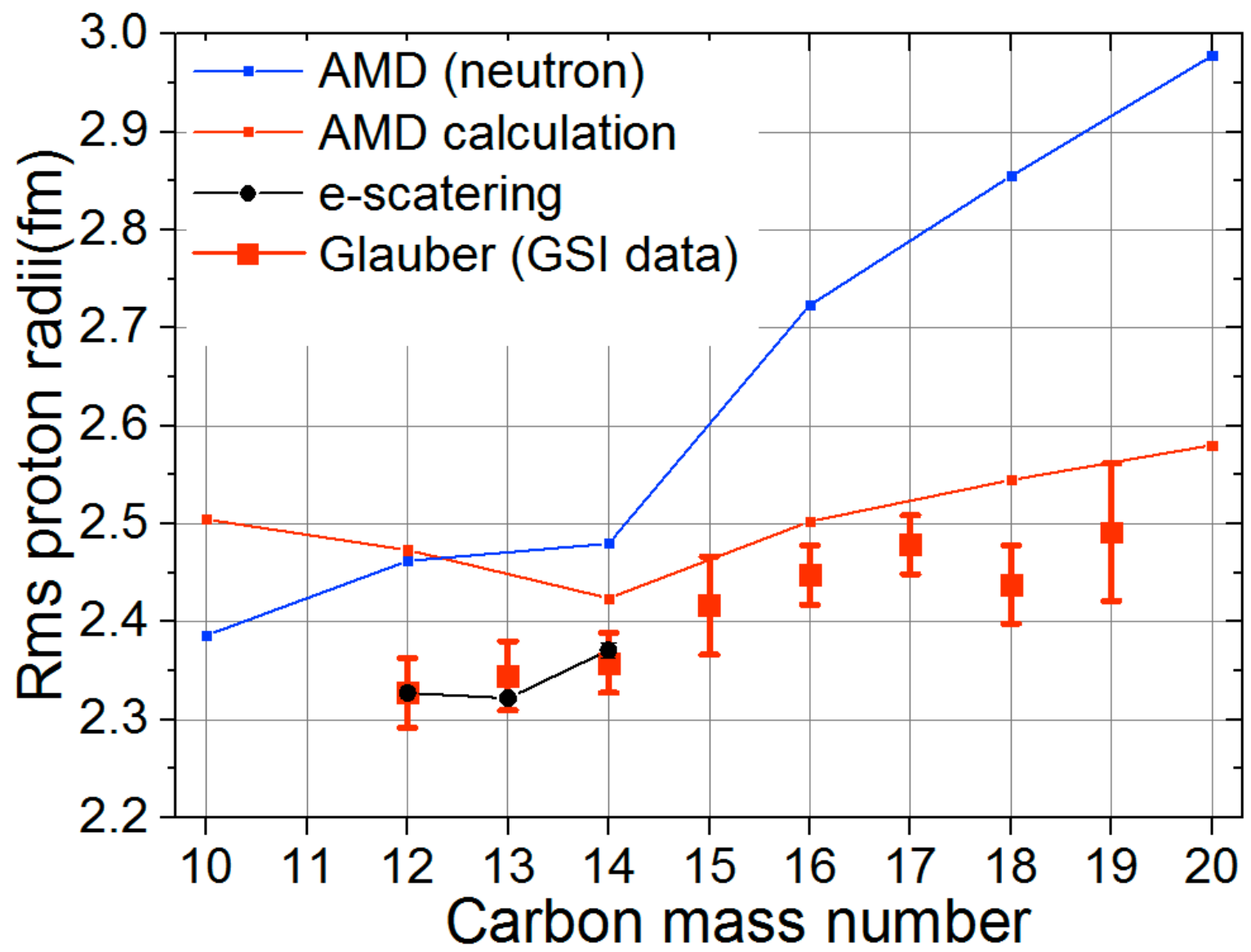


Results of Glauber model calculation





Results of Glauber model calculation



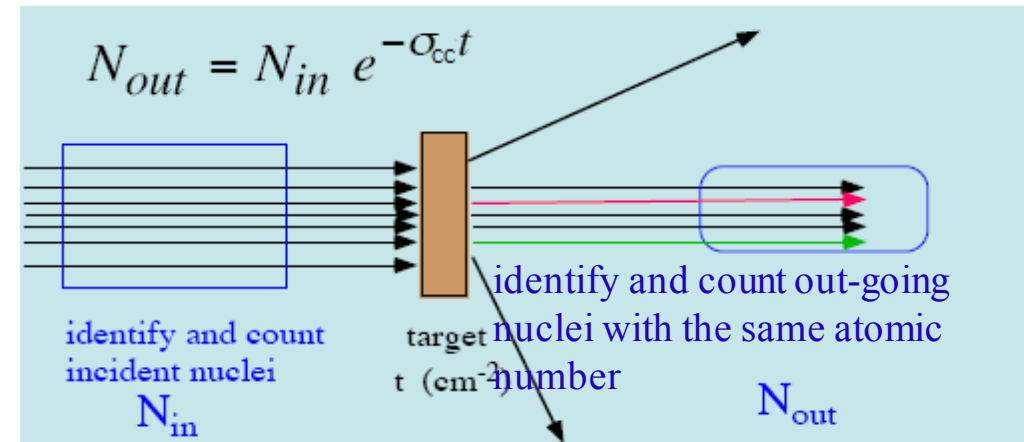


Experiment method



❖ Method: Transmission method

$$\sigma_{cc} = \frac{1}{t} \ln \left[\frac{\gamma_0 (1 - P_m)}{\gamma} \right]$$



➤ $\gamma = \mathbf{N_{out}(Z)/N_{in}(Z)}$

➤ γ, γ_0 : with C target and without C target

➤ $\mathbf{1 - P_m}$: Acceptance factor.

➤ t : target thickness.

❖ Primary beam: ^{22}Ne , 80A MeV.

❖ Secondary beam: C isotopes, $\sim 50\text{A}$ MeV.



How are proton rms radii measured?



- ☑ Electron scattering is the most successful method to investigate the charge distribution of nuclei.
- ☒ It could be applied only to stable nuclei.
- ☑ The isotope shift technique has been applied for unstable isotopes.
- ☒ Challenging for nuclei with $4 < Z < 11$ due to uncertainty in atomic physics calculation.
- ☒ Difficult to access to neutron drip-line due to limitation of beam intensity.

Electron Scattering Experiments

“It would be of great scientific interest if it were possible to have a supply of electrons ... of which the individual energy of motion is greater even than that of the alpha particle.”

[Ernest Rutherford, Royal Society, London, (as PRS) 30 Nov 1927]

1950s
Hoffstadter

First
observation
of finite proton size
using 2 MeV e beam

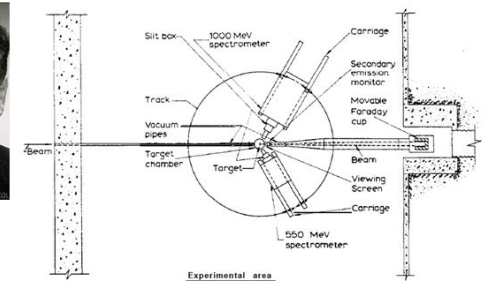
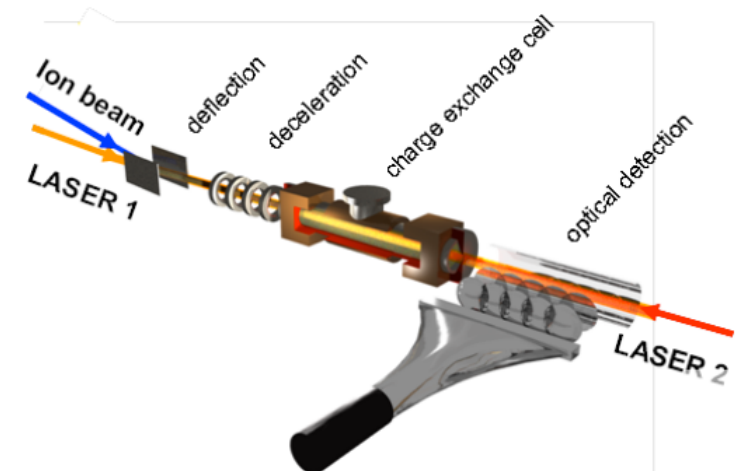
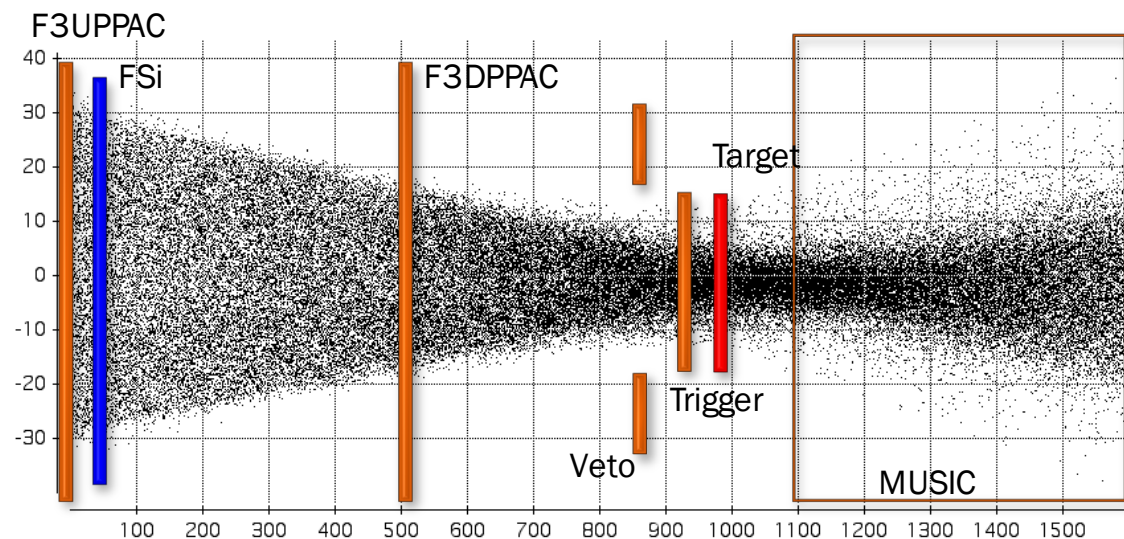
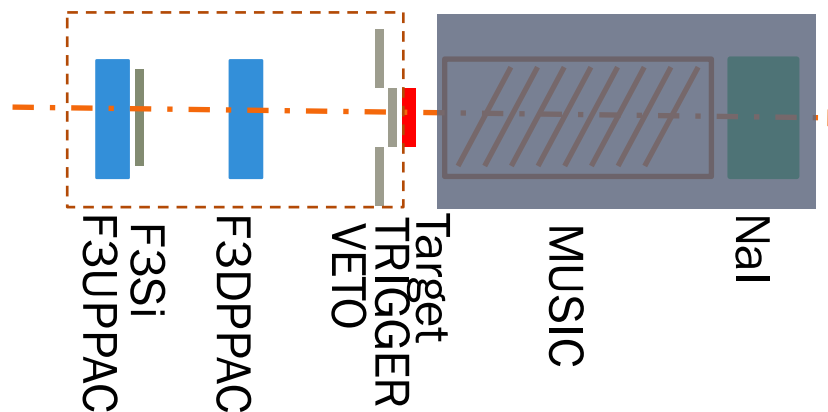
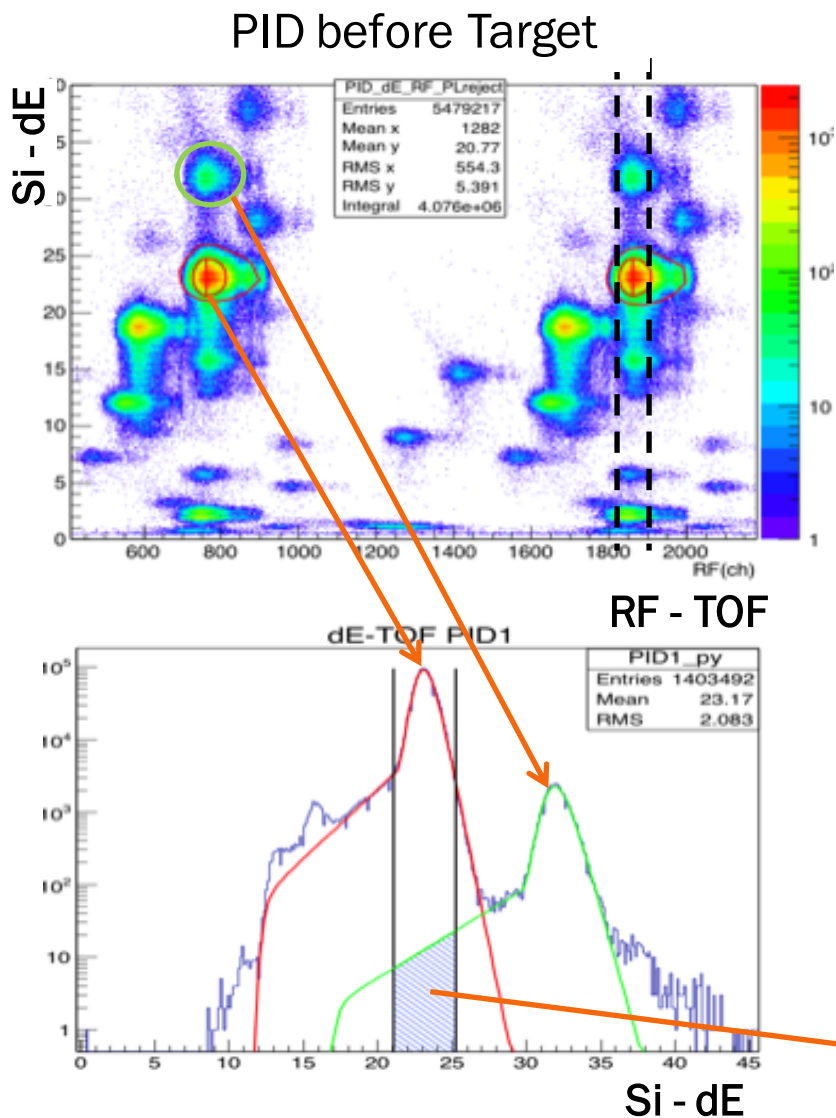


Fig. 2. This figure shows a schematic diagram of a modern electron-scattering experimental area. The track on which the spectrometers roll has an approximate radius of 13.5 feet.





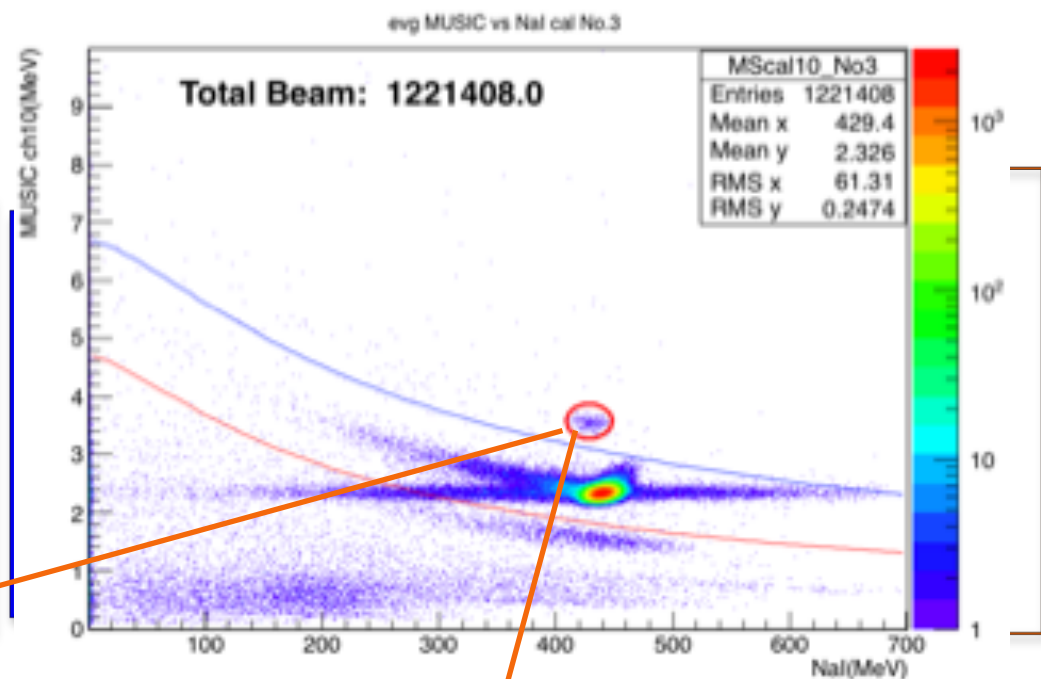
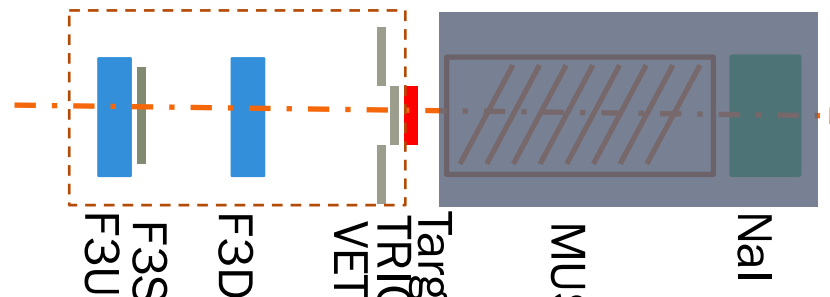
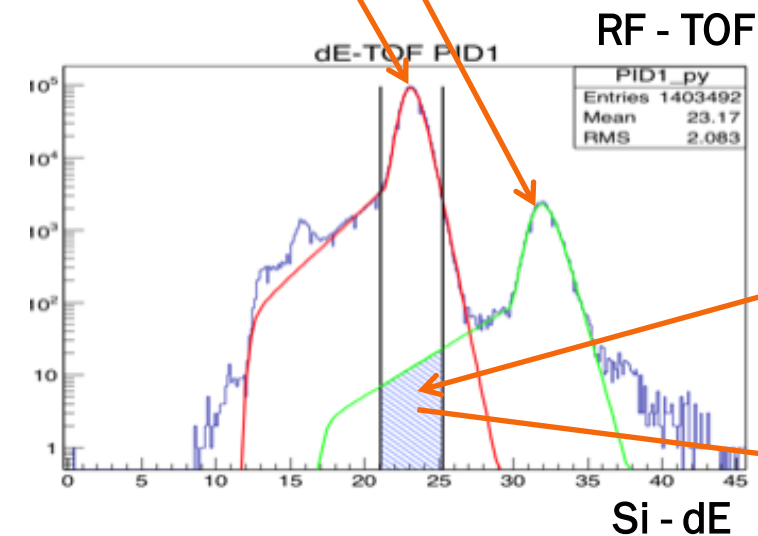
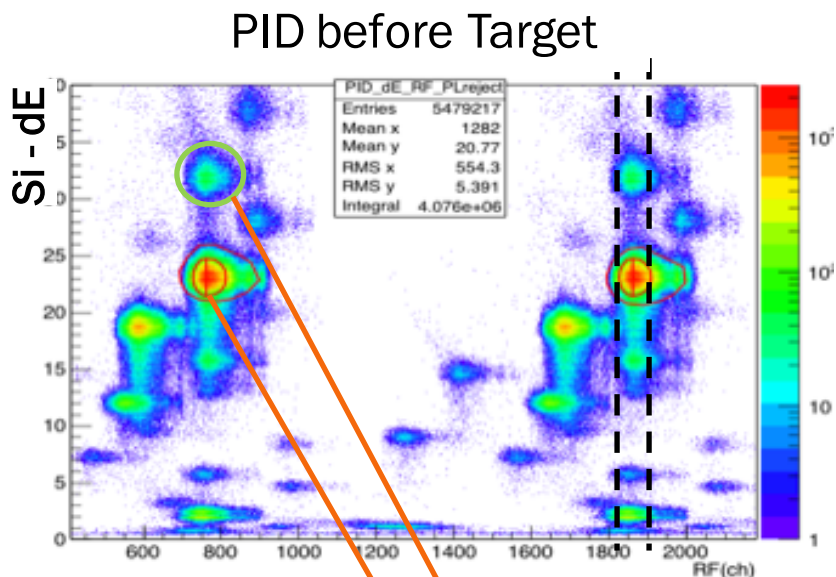
Tracking and Counting Beam - N_{in}



-Estimation: $\rightarrow 246/1221408=0.02014\%$



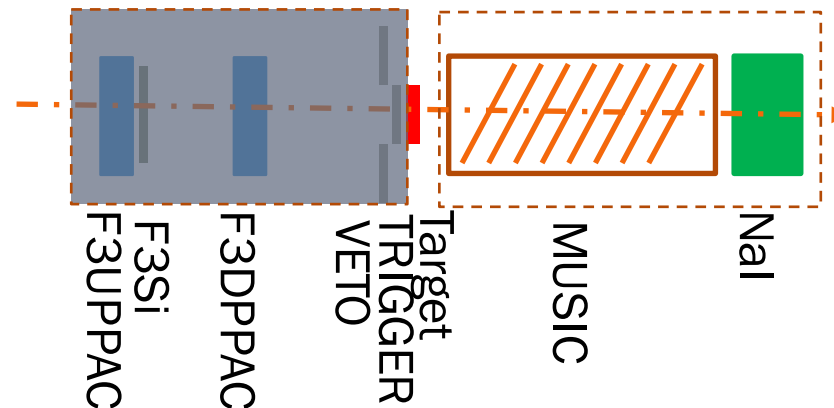
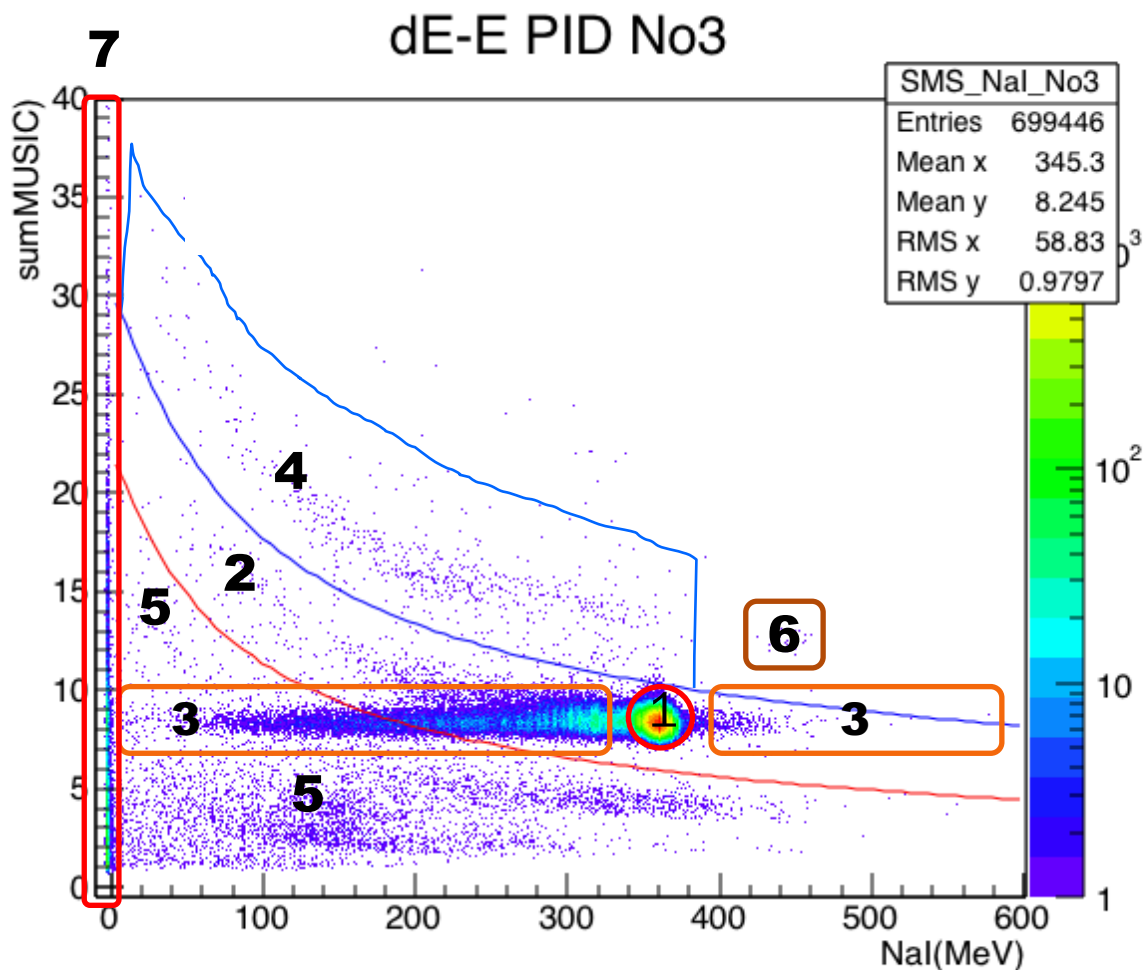
Tracking and Counting Beam - N_{in}



-At downstream det.: $284/1221408=0.02325\%$
 -Estimation: $246/1221408=0.02014\%$



Identify scattered particles



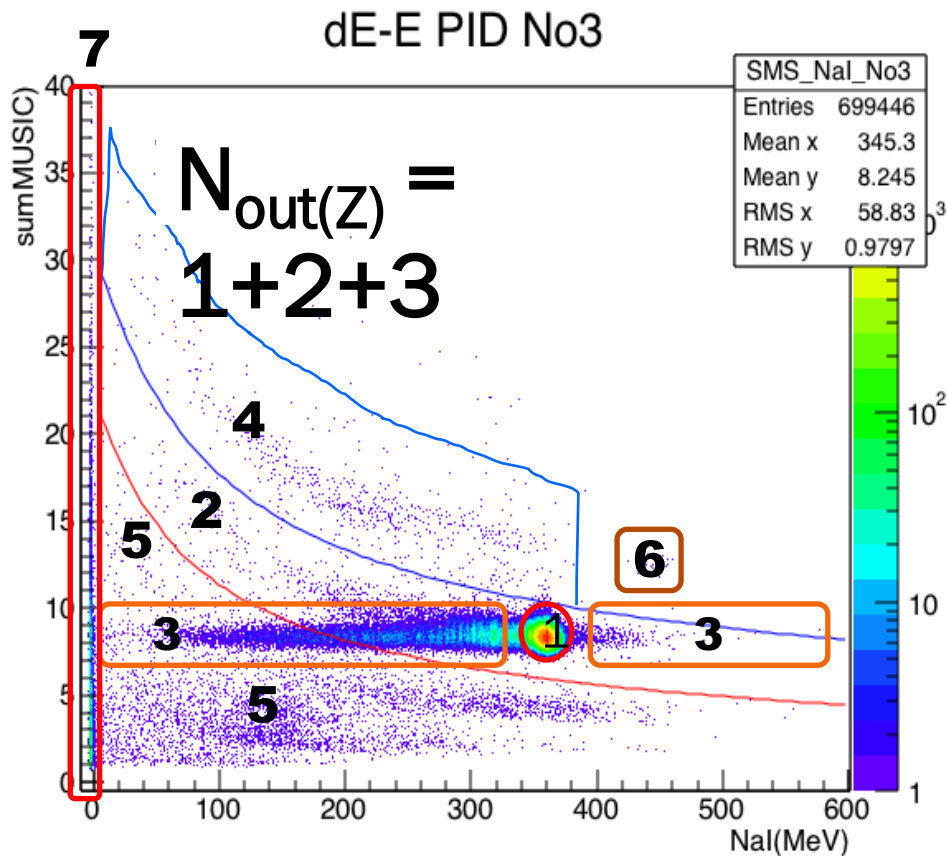
1. **Beam.**
2. **Elastic and inelastic.**
3. **Reaction in NaI detector.**
4. Proton pickup.
5. Proton removal.
6. Contaminant.
7. **Outside acceptance.**



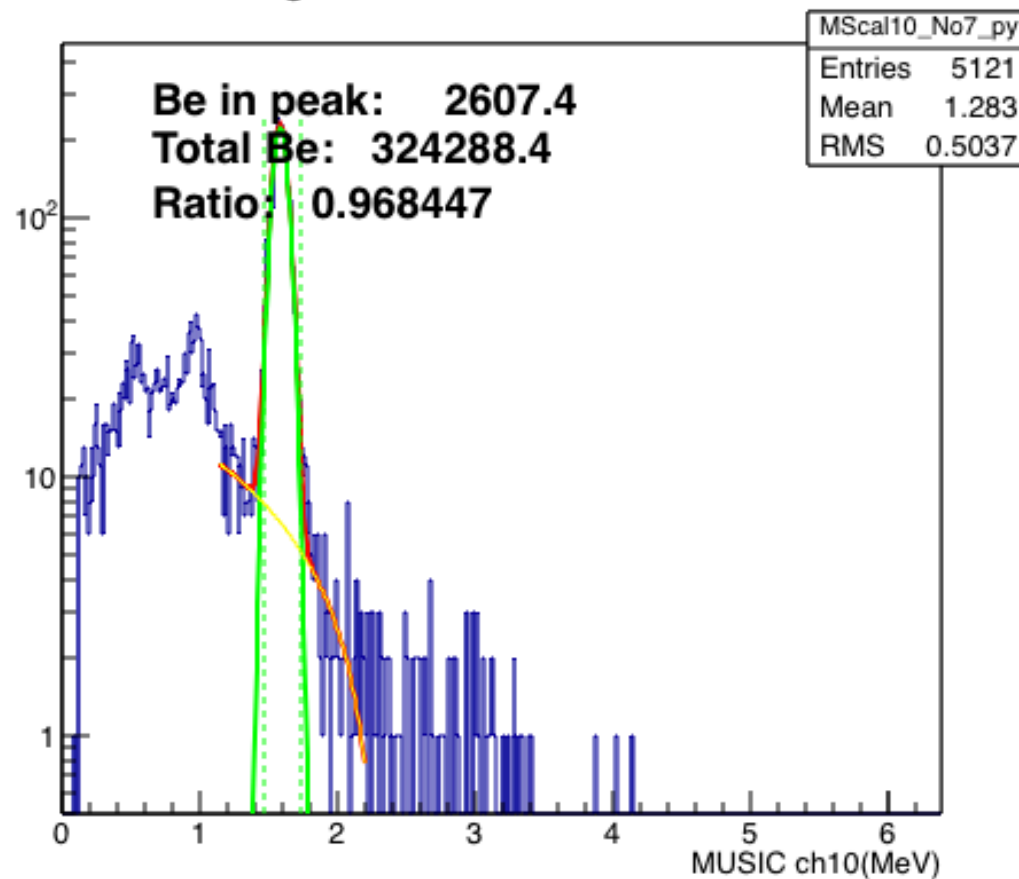
Count scattered particles – $N_{out}(Z)$



$$N_{out}(Z) = 1+2+3$$



evg MUSIC vs Nal cal No.7

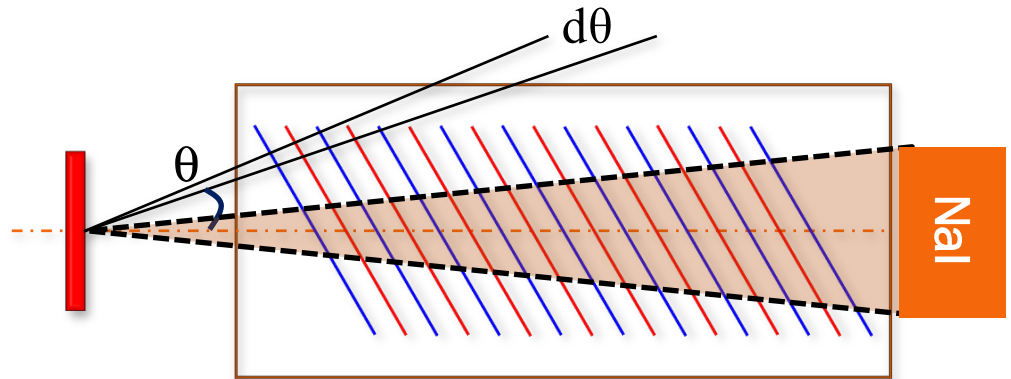




Acceptance $(1 - P_m)$



- Acceptance of each MUSIC layer was determined.
- Rutherford scattering was used to fit and calculate out-of-acceptance factor.
- Checked with GEANT4 simulation.



$$\sigma_{cc} = \frac{1}{t} \ln \left[\frac{\gamma_0 (1 - P_m)}{\gamma} \right]$$

e.g)

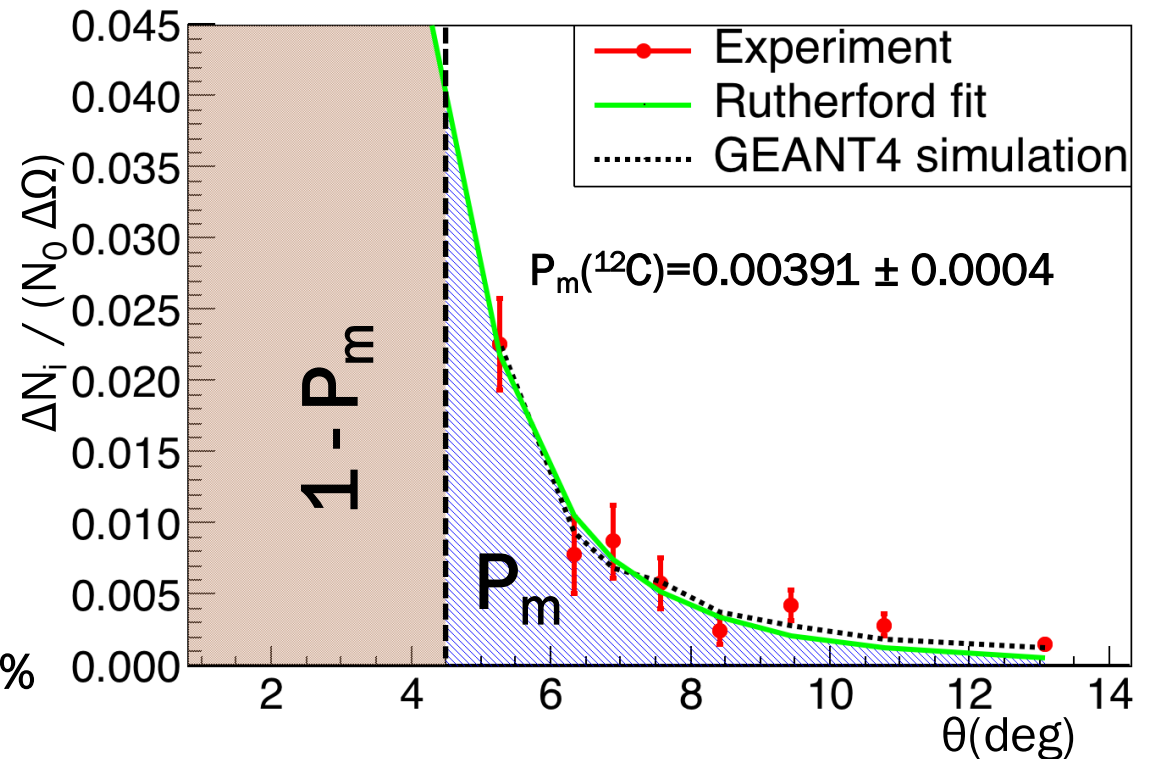
$P_m = 0$

$\sigma_{cc} = 1128 \text{ mb}$

$P_m = 0.00391$

$\sigma_{cc} = 942 \text{ mb}$

186 -> ~20%





Glauber Model



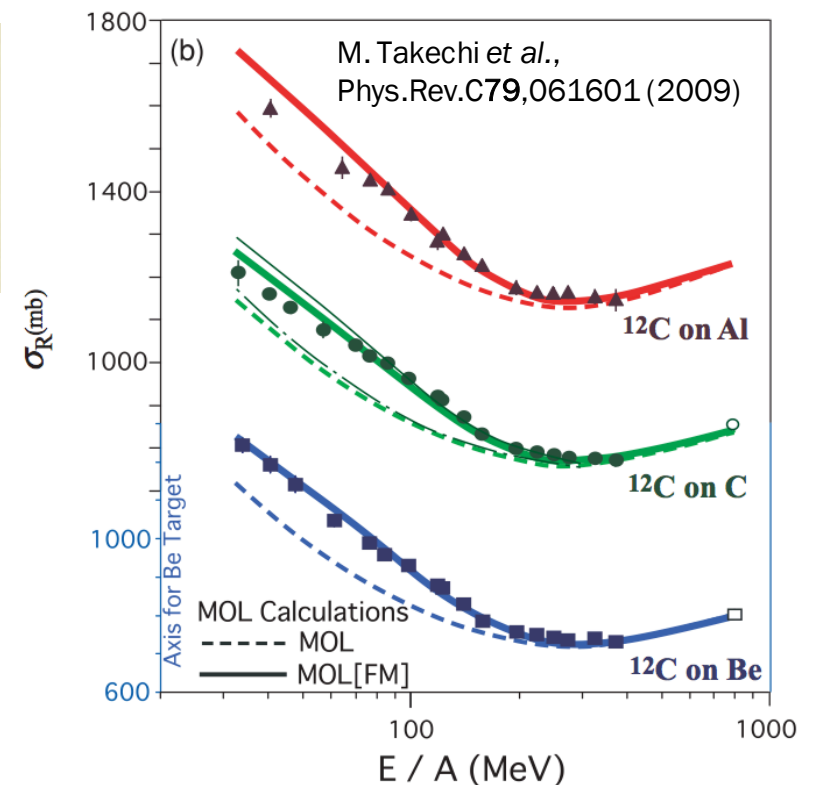
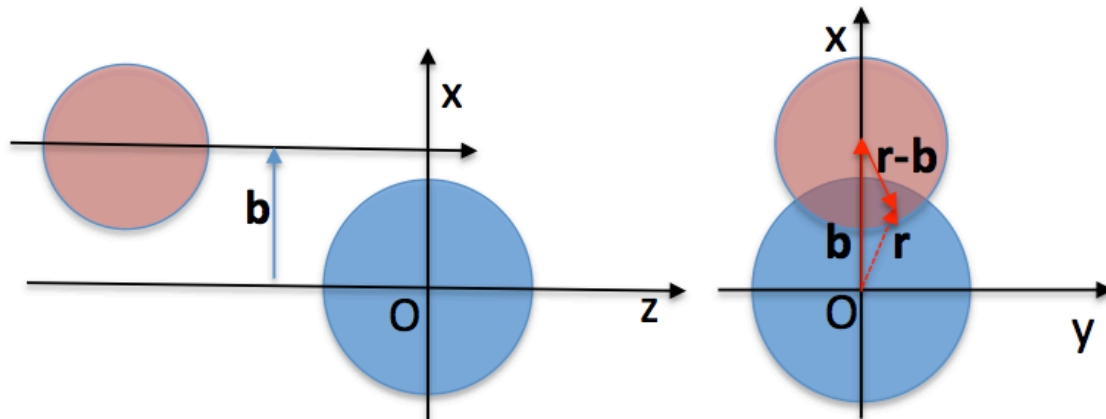
☞ Nuclear radii and interaction cross section:

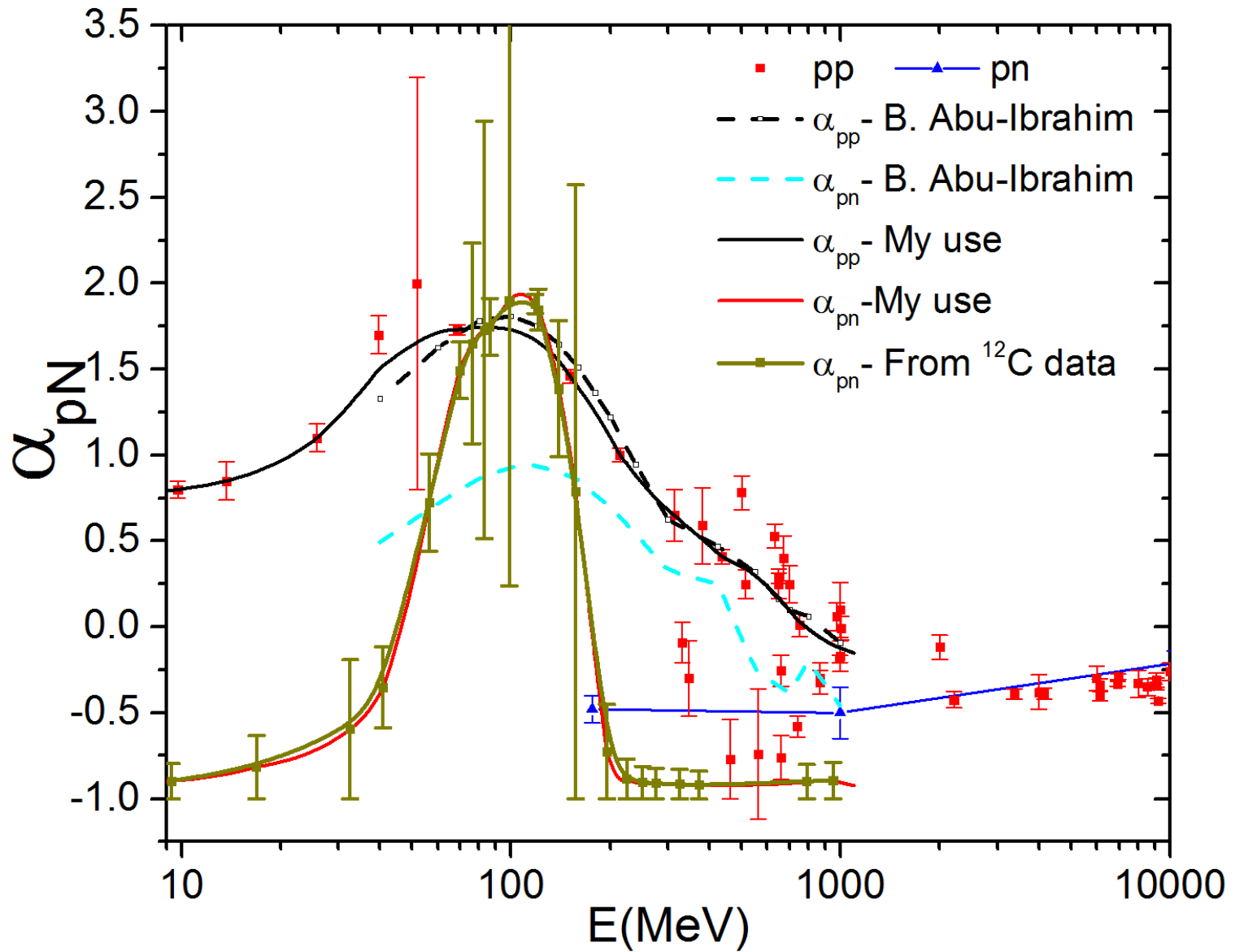
$$\sigma_I(P, T) = \pi (R_T + R_P)^2$$

☞ Glauber model for interaction cross section works very well from 30A to 1000A MeV

$$\sigma_I(P, T) = \int [1 - T(b)] d\mathbf{b}$$

$$T(\mathbf{b}) = \exp[-\sigma_{pp} \int \{\rho_{Pp}(\mathbf{r} - \mathbf{b}) \cdot \rho_{Tp}(\mathbf{r}) + \rho_{Pn}(\mathbf{r} - \mathbf{b}) \cdot \rho_{Tn}(\mathbf{r})\} d\mathbf{r} \\ - \sigma_{pn} \int \{\rho_{Pp}(\mathbf{r} - \mathbf{b}) \cdot \rho_{Tn}(\mathbf{r}) + \rho_{Pn}(\mathbf{r} - \mathbf{b}) \cdot \rho_{Tp}(\mathbf{r})\} d\mathbf{r}]$$







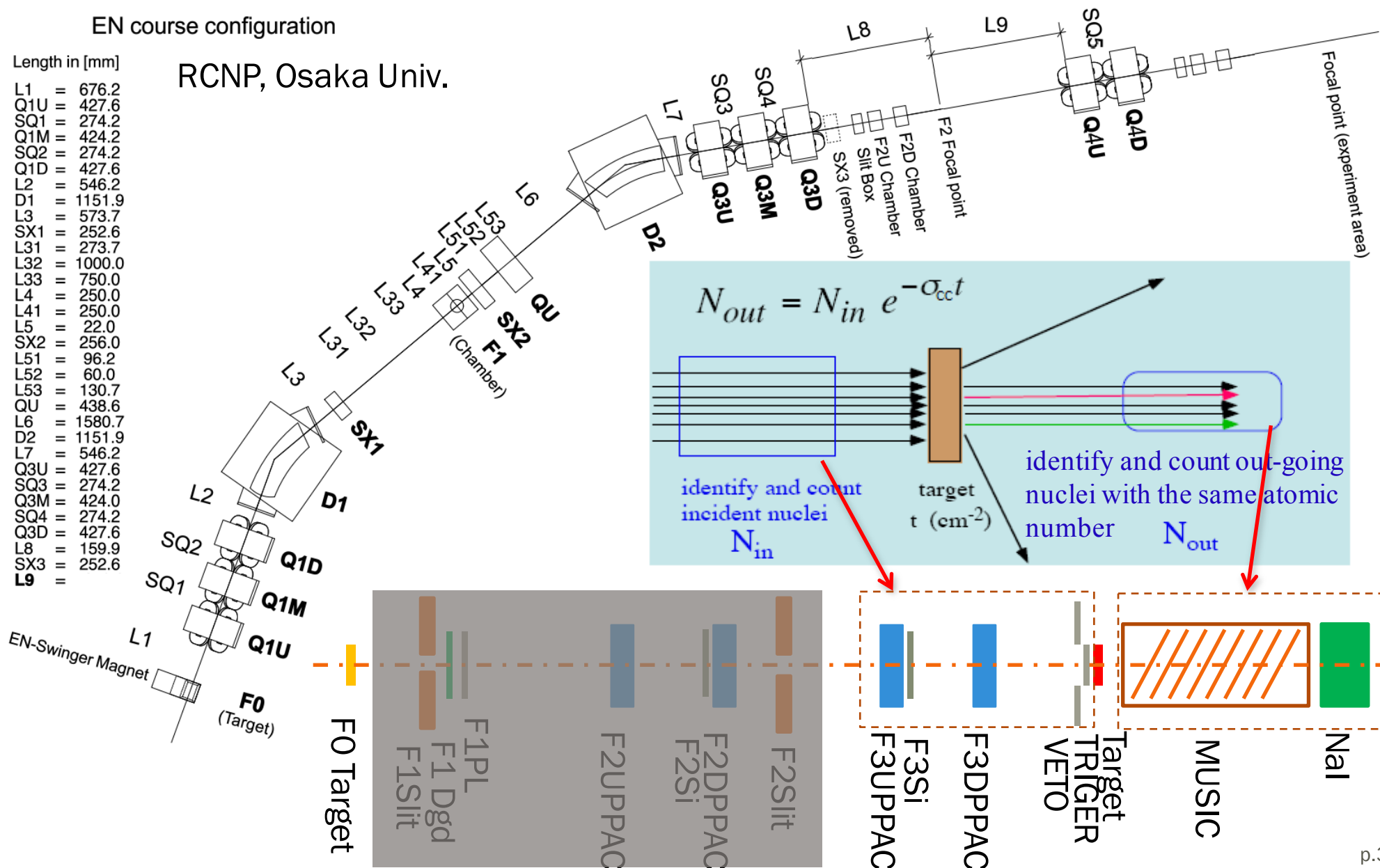
Experiment setup

EN course configuration

Length in [mm]

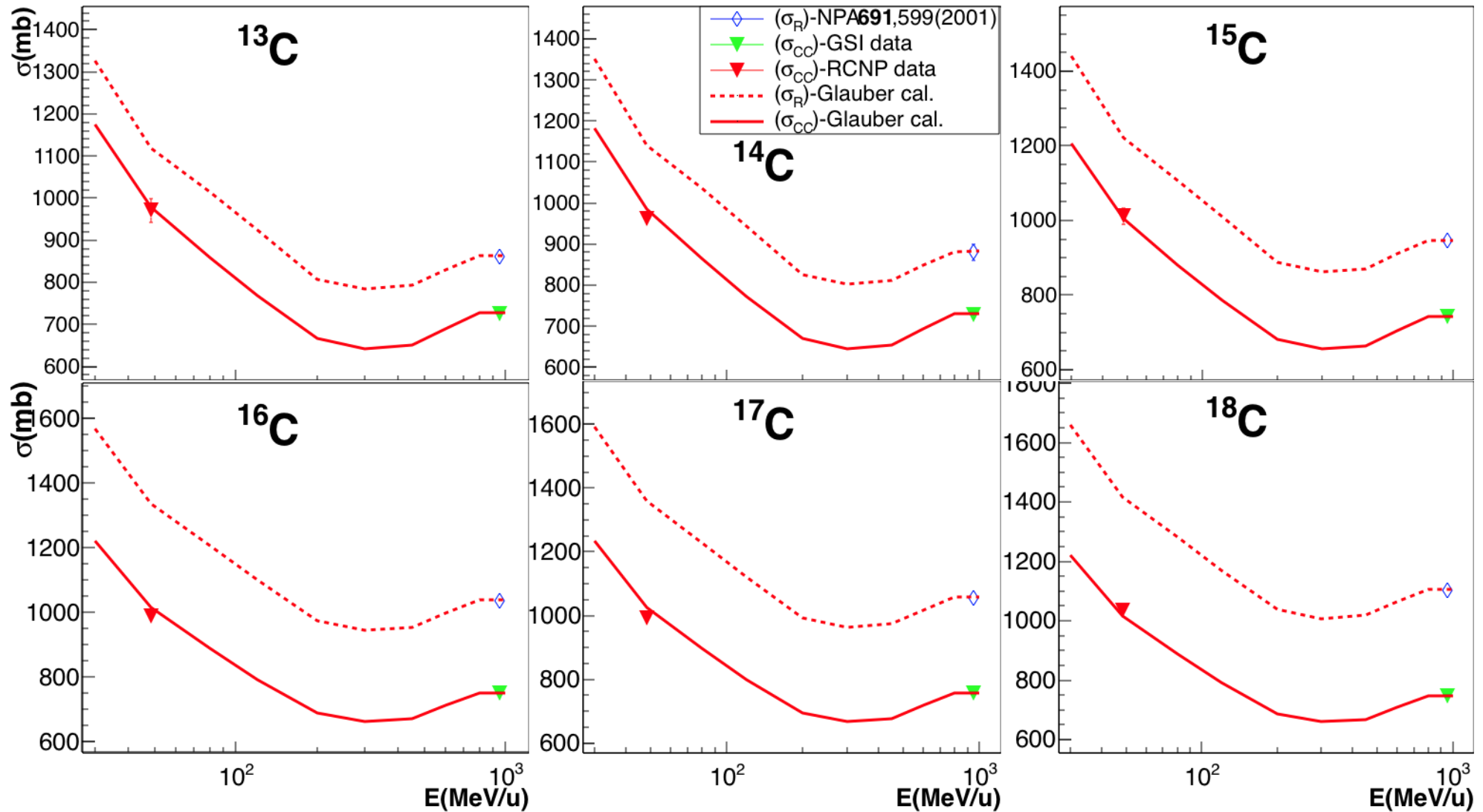
- L1 = 676.2
- Q1U = 427.6
- SQ1 = 274.2
- Q1M = 424.2
- SQ2 = 274.2
- Q1D = 427.6
- L2 = 546.2
- D1 = 1151.9
- L3 = 573.7
- SX1 = 252.6
- L31 = 273.7
- L32 = 1000.0
- L33 = 750.0
- L4 = 250.0
- L41 = 250.0
- L5 = 22.0
- SX2 = 256.0
- L51 = 96.2
- L52 = 60.0
- L53 = 130.7
- QU = 438.6
- L6 = 1580.7
- D2 = 1151.9
- L7 = 546.2
- Q3U = 427.6
- SQ3 = 274.2
- Q3M = 424.0
- SQ4 = 274.2
- Q3D = 427.6
- L8 = 159.9
- SX3 = 252.6
- L9 =

RCNP, Osaka Univ.





Glauber model calculation





Extracting proton distribution radii

$$T(b) = \text{Exp} \left[- \int dz \sum_{i,k} \sigma_{ik} \iiint \rho_{Pi}(\vec{r}) \rho_{Tk}(\vec{r}) \Gamma(\vec{b} + \vec{s} - \vec{t}) d\vec{s} d\vec{t} \right], \text{ with } i, k : p, n$$

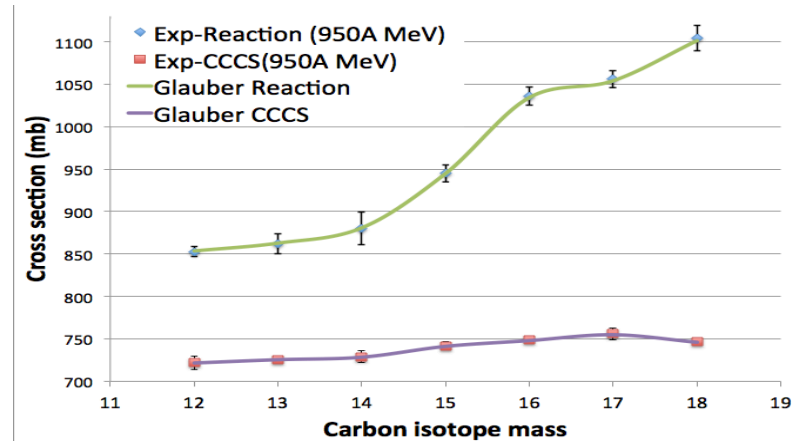
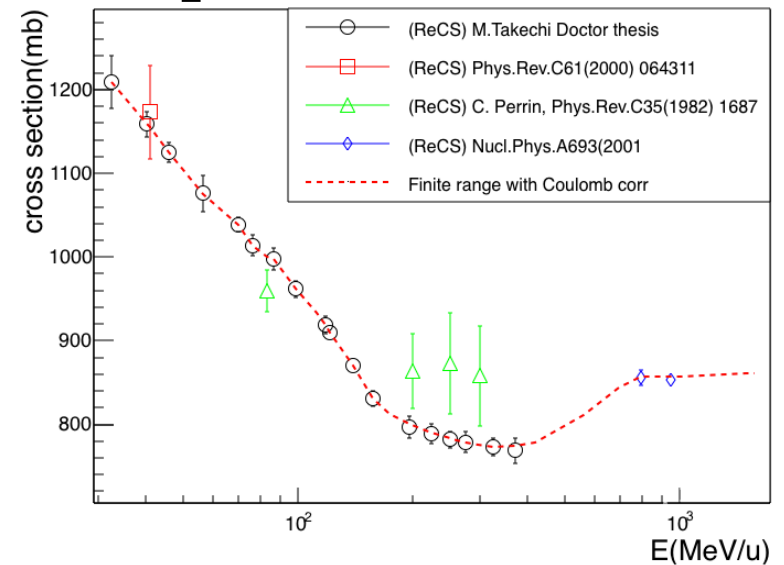
$$\Gamma(\vec{b}) = \frac{1}{4\pi\beta_{NN}^2} \text{Exp} \left[- \frac{b^2}{2\beta_{NN}^2} \right]$$

➤ For ¹²C case:

- ρ_p: from electron scattering data.
- ρ_n, β_{NN} using fit Glauber model with reaction cross section and CCCS.
- β_{NN}(E): using dependence reaction cross section on energy.

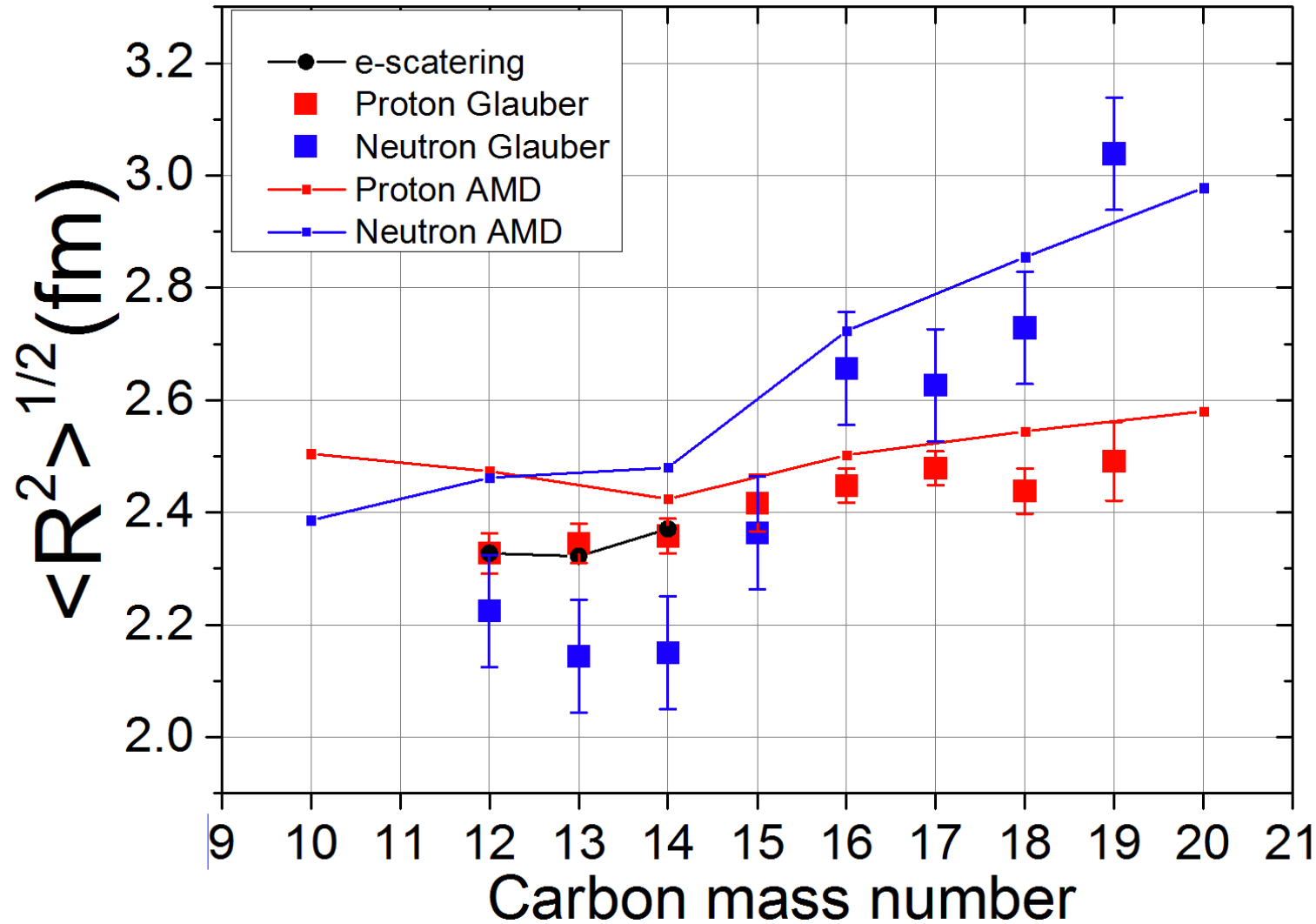
➤ For other isotopes:

Using β_{NN}(E) from ¹²C data, fit Glauber model with reaction cross section and CCCS to obtain ρ_p, ρ_n.





Results of Glauber model calculation



Intereaction cross section

