Pairing rotations in ground states of open-shell even-even deformed nuclei

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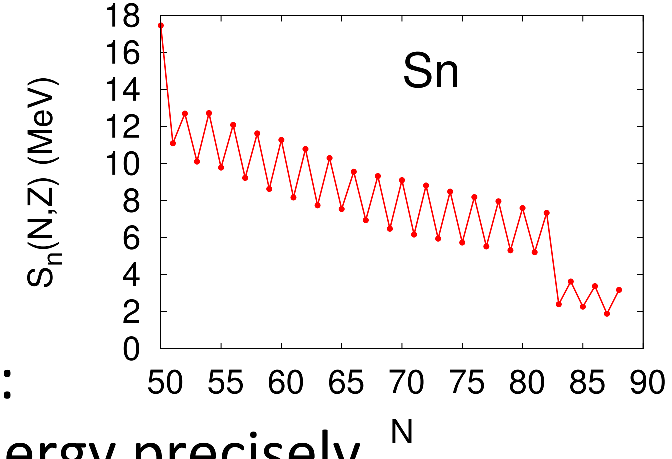
Pairing observables

nuclear pairing interaction: a nucleon pair gets additional binding

signature of pairing: odd-even mass staggering (OES)

$$\Delta^{(3)}(N) = \frac{(-1)^N}{2}[B(N-1) - 2B(N) + B(N+1)]$$

$$\Delta^{\mathrm{exp}}(N,Z) = \frac{1}{2}[\Delta^{(3)}(N-1) + \Delta^{(3)}(N+1)]$$



Energy density functional (mean-field theory):

difficult to compute odd-mass binding energy precisely

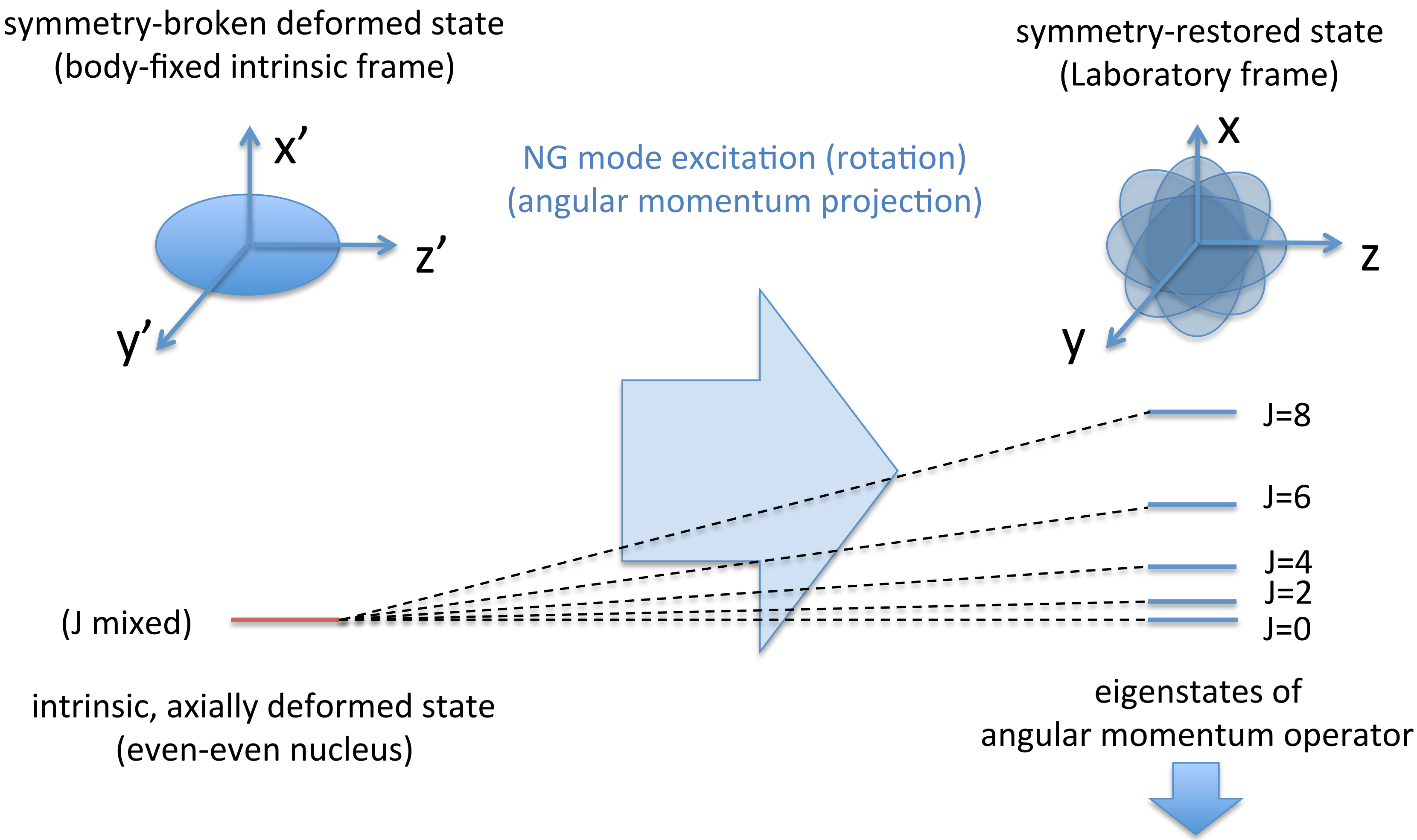
experimental OES ⟷ theoretical pairing gap

new (?) pairing observable: moment of inertia of pairing rotation

superconducting ground state (spontaneous breaking of gauge symmetry)
symmetry-restoring zero-energy Nambu-Goldstone mode -- pairing rotation

Brink and Broglia "Nuclear Superconductivity"
review: Broglia et al., Phys. Rep. **335**, 1(2000)

Rotational symmetry breaking (for comparison)



rotational energy $$E(J) = \frac{\hbar^2}{2\mathcal{J}_{\rm rot}} J(J+1)$$

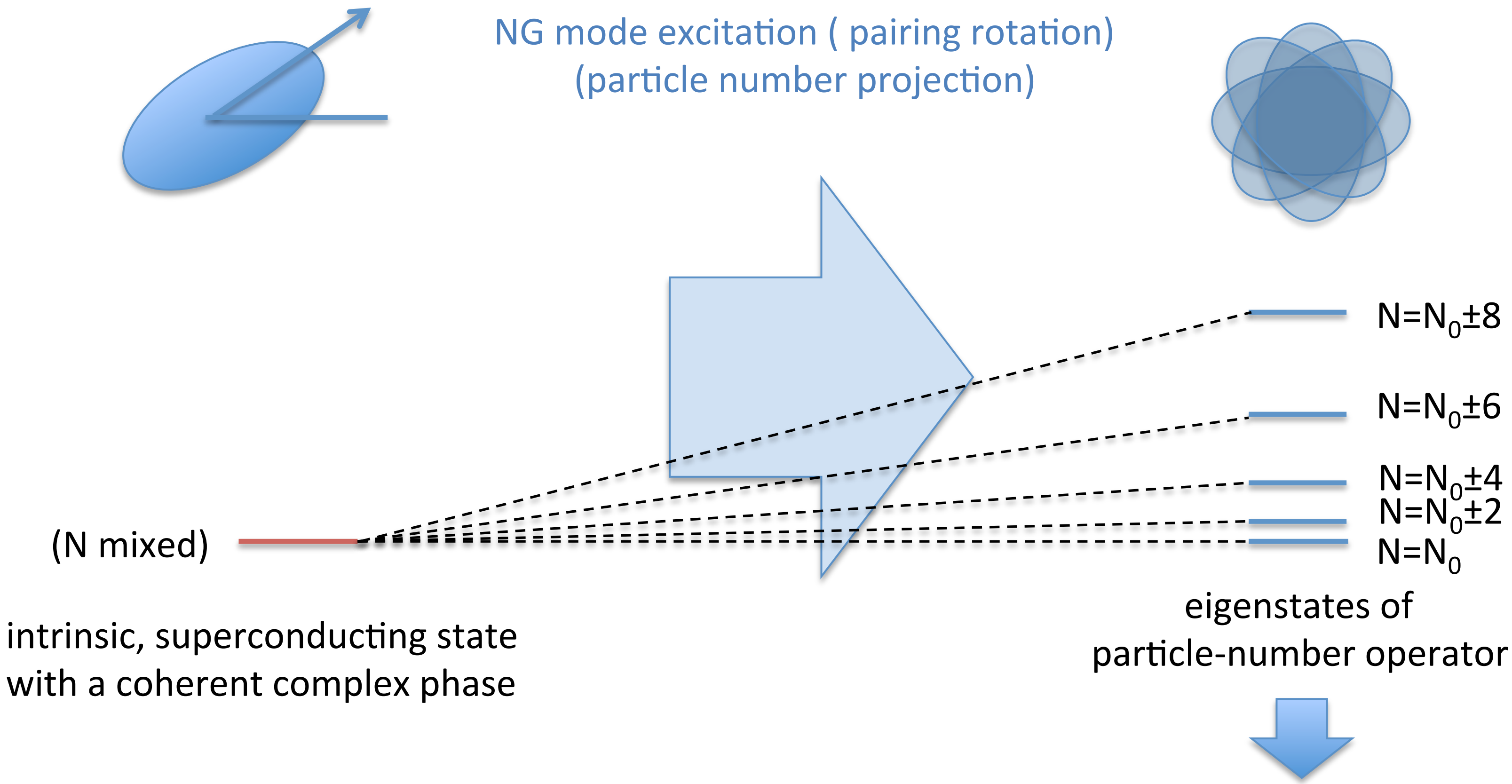
rotational band

moment of inertia ($E(2_1^+)$): magnitude of quadrupole collectivity

Gauge symmetry breaking

symmetry-broken superconducting state
(body-fixed intrinsic frame)

symmetry-restored state
(Laboratory frame)

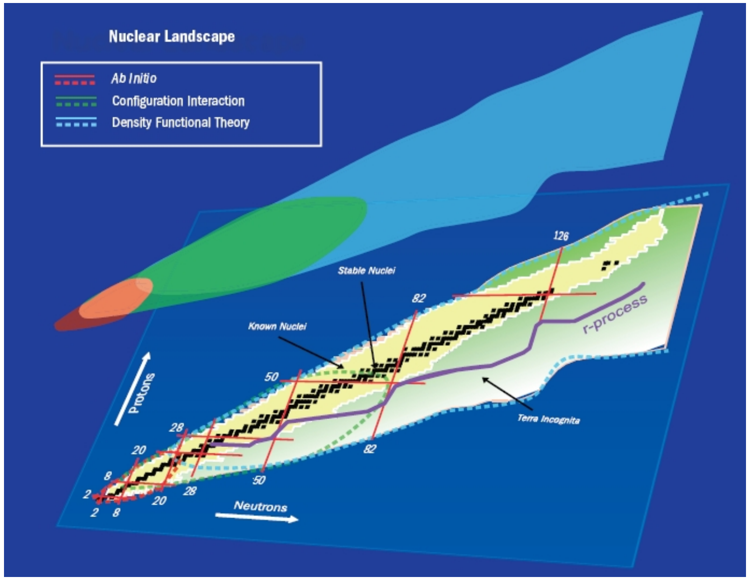


pairing rotational band

pairing rotational energy $E(N) = \frac{1}{2\mathcal{J}_{\mathrm{N}}}(N - N_0)^2$

pairing rotational moment of inertia: magnitude of pairing collectivity

Theoretical description of NG mode



Symmetry-broken state

Density Functional Theory

Skyrme HFB (HFBTHO, UNEDF1-HFB)

- ☐ harmonic oscillator basis
- ☐ axial deformation
- ☐ pairing superconductivity

NG mode excitation

Quasiparticle Random-Phase Approximation
(time-dependent density functional theory)

efficient solution based on linear response theory:

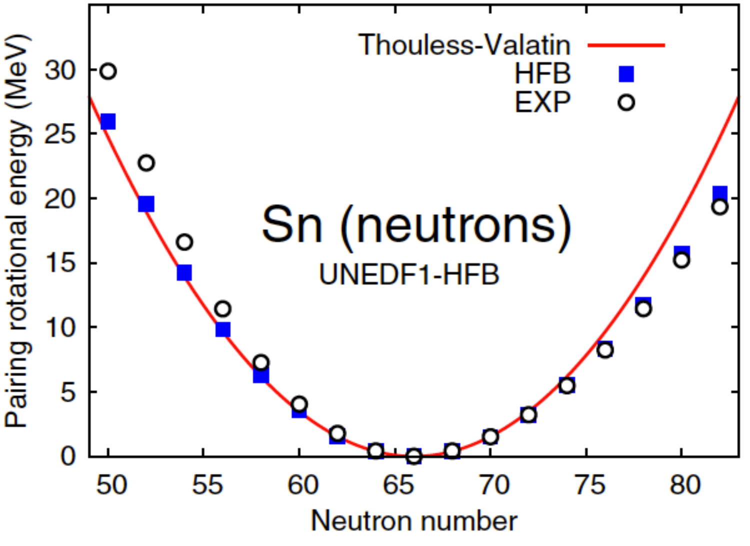
Finite-amplitude method (FAM)

Nakatsukasa et al., PRC**76**, 024318 (2007)

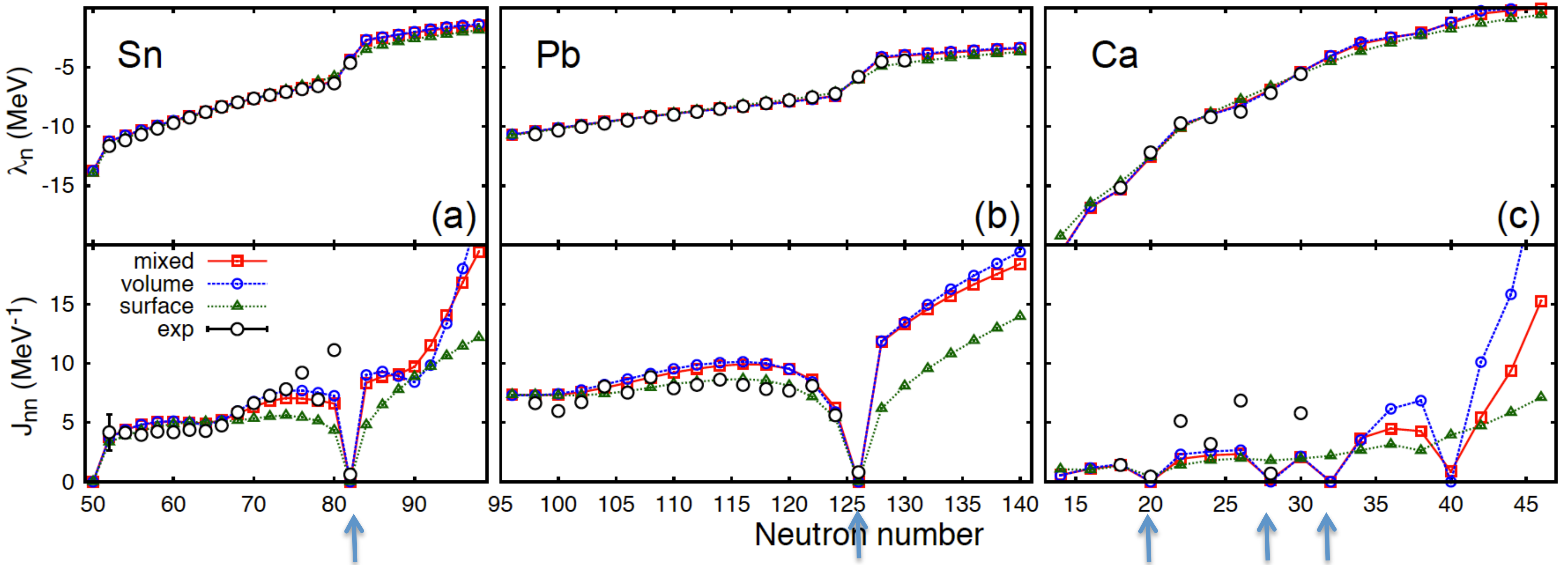
FAM formulation for zero-energy NG mode:
NH, PRC**92**, 034321 (2015)

QRPA moment of inertia: Thouless-Valatin

Thouless and Valatin, Nucl. Phys. **31** (1962)211



Pairing rotational MOI in single-closed shell nuclei



experimental pairing rotational moment of inertia

$$E(N+\Delta N) = E(N) + \lambda_n(N)\Delta N + \frac{(\Delta N)^2}{2\mathcal{J}_{nn}(N)} + O((\Delta N)^3)$$

$$E(N+2) = E(N) + 2\lambda_n(N) + \frac{4}{2\mathcal{J}_{nn}(N)}$$

$$E(N-2) = E(N) - 2\lambda_n(N) + \frac{4}{2\mathcal{J}_{nn}(N)}$$

$$\mathcal{J}_{nn}(N) = \frac{4}{E(N+2) - 2E(N) + E(N-2)}$$

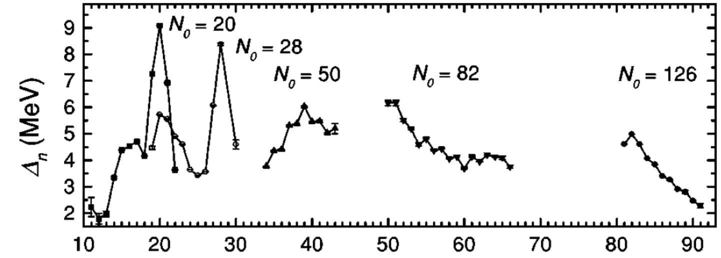
$$\lambda_{exp}(N_0) = \frac{1}{4}[E_{exp}(N_0-2) - E_{exp}(N_0+2)]$$

- experimental MOI: sensitive to the symmetry breaking
- pairing rotational assumption is not valid at next to magic numbers
- three kinds of pairing functional: sensitivity to pairing form in n-rich Ca

Shell gap and pairing rotational moment of inertia

Experimental pairing rotational moment of inertia

$$\mathcal{J}_{\rm nn}^{-1}(N) = \frac{1}{4}\delta_{\rm 2n}(N)$$



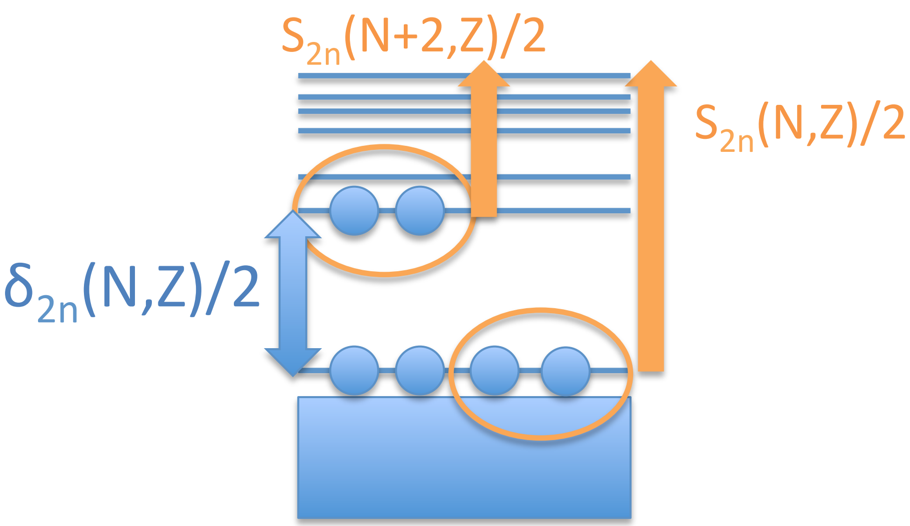
Lunney et al., Phys. Rep. **75**, 1021(2003)

empirical shell gap: difference of 2n separation energies

$$\delta_{\rm 2n}(N,Z) = E(N+2,Z) - 2E(N,Z) + E(N-2,Z) = S_{\rm 2n}(N,Z) - S_{\rm 2n}(N+2,Z)$$

weak pairing collectivity

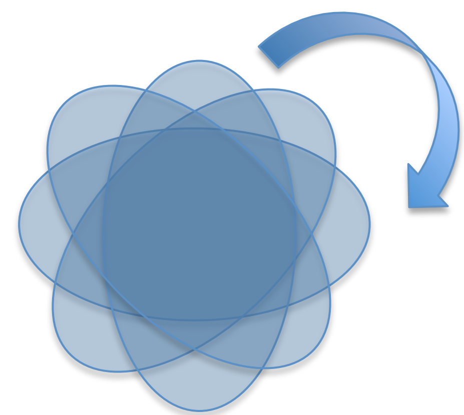
shell model picture
(before symmetry breaking)



size of magic shell gap

strong pairing collectivity

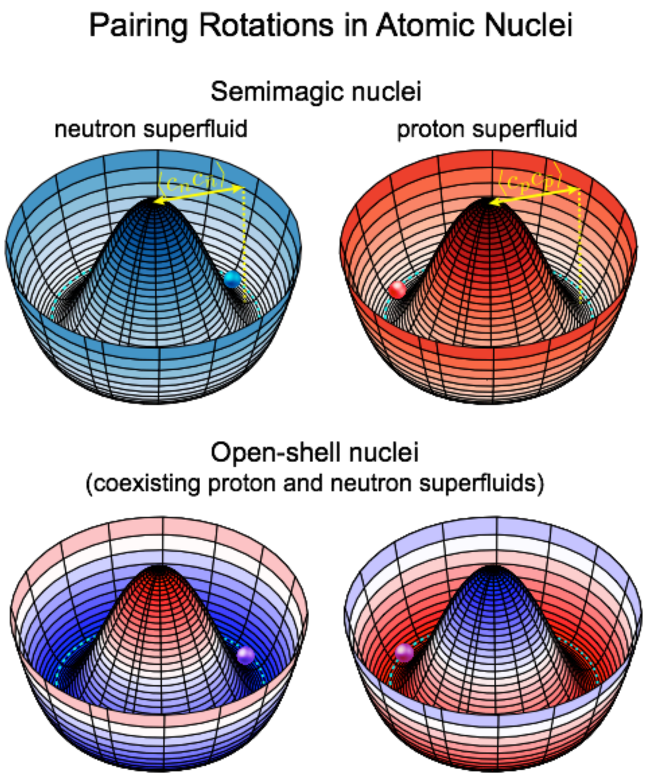
collective pairing picture
(after symmetry breaking)



$$E_{\rm pairrot}(N) = \frac{1}{2\mathcal{J}_{\rm nn}(N)}(\Delta N)^2$$

measure of gauge symmetry breaking

Pairing NG modes in doubly open-shell nuclei



neutron and proton gauge symmetry broken

(Δn≠0 and Δp≠0, $U(1)_n \times U(1)_p$)

$$[\hat{H}_{\mathrm{HFB}}, \hat{N}_{\mathrm{n}}] \neq 0 \qquad [\hat{H}_{\mathrm{HFB}}, \hat{N}_{\mathrm{p}}] \neq 0$$

neutron pairing rotational energy

proton pairing rotational energy

$$E(N,Z) = E(N_0,Z_0) + \lambda_{\mathrm{n}}(N_0,Z_0)\Delta N + \lambda_{\mathrm{p}}(N_0,Z_0)\Delta Z + \frac{(\Delta N)^2}{2\mathcal{J}_{\mathrm{nn}}(N_0,Z_0)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{\mathrm{pp}}(N_0,Z_0)}$$

NG modes as QRPA eigenmodes are neutron-proton mixed

formal theory: Marshalek, Nucl. Phys. A **275**,416 (1977)

first calculation: NH, Phys. Rev. C **92**,034321 (2015)

$$\hat{N}_1 = \hat{N}_{\mathrm{n}} \cos\theta + \alpha \hat{N}_{\mathrm{p}} \sin\theta$$
$$\hat{N}_2 = -\hat{N}_{\mathrm{n}} \sin\theta + \alpha \hat{N}_{\mathrm{p}} \cos\theta$$

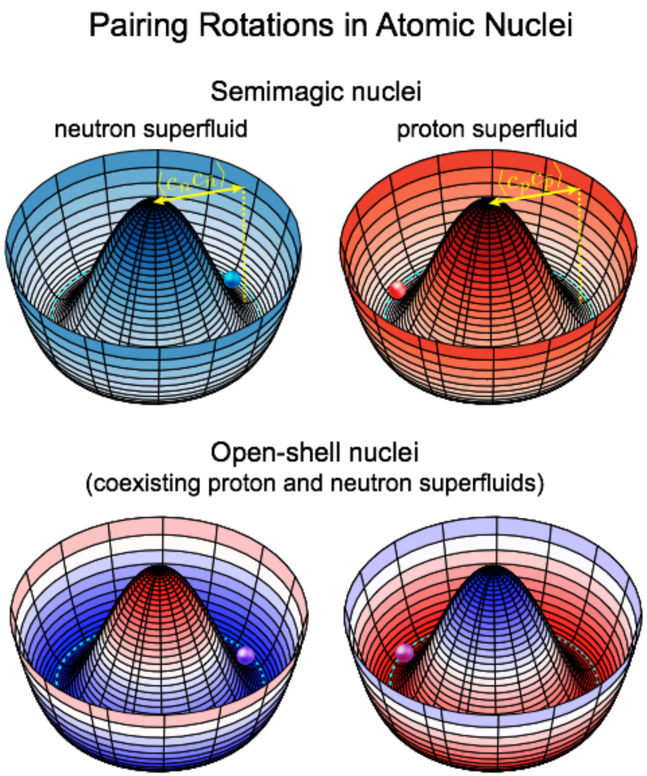
$$[\hat{H}_{\mathrm{HFB}}, \hat{N}_1] \neq 0 \qquad [\hat{H}_{\mathrm{HFB}}, \hat{N}_2] \neq 0$$

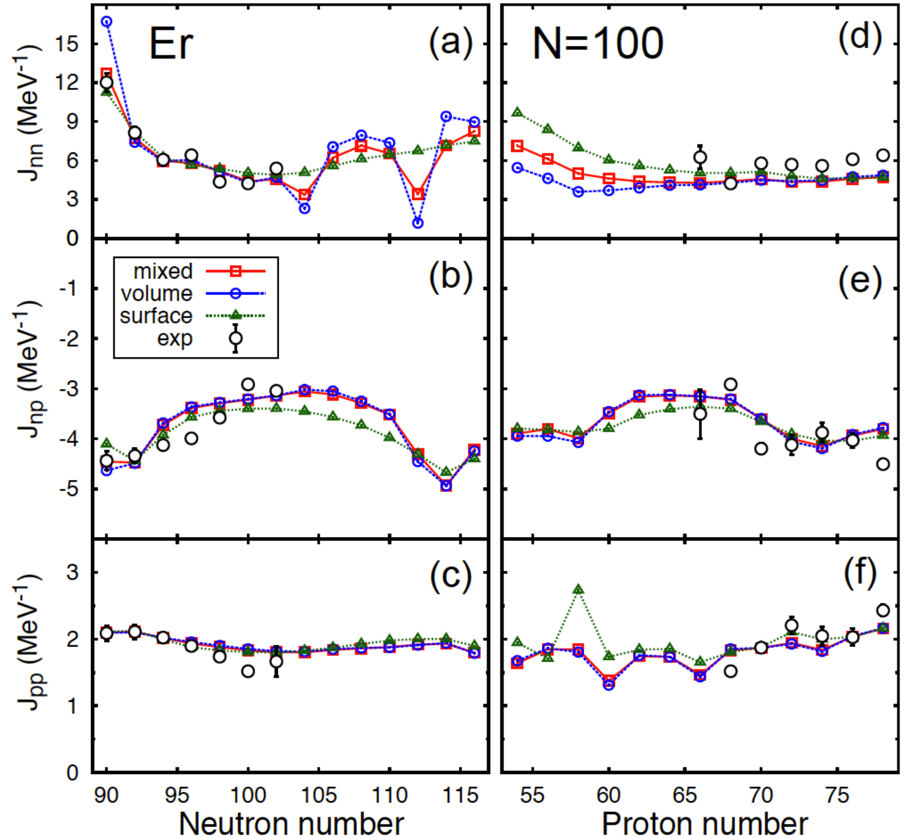
$$E(N,Z) = E(N_0,Z_0) + \lambda_1(N_0,Z_0)\Delta N_1 + \lambda_2(N_0,Z_0)\Delta N_2 + \frac{(\Delta N_1)^2}{2\mathcal{J}_1(N_0,Z_0)} + \frac{(\Delta N_2)^2}{2\mathcal{J}_2(N_0,Z_0)}$$

$$E_{\mathrm{pairrot}}(N,Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{\mathrm{nn}}(N_0,Z_0)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{\mathrm{np}}(N_0,Z_0)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{\mathrm{pp}}(N_0,Z_0)}$$

note: we don't have neutron-proton pairing

Pairing rotational MOI in doubly open-shell nuclei





neutron and proton modes mix in the NG modes in doubly-open shell nuclei
-- off-diagonal term in the pairing rotational moment of inertia

More pairing observables in open-shell systems!

$$\mathcal{J}_{\rm nn}(N,Z) = 4[E(N+2,Z) - 2E(N,Z) + E(N-2,Z)]^{-1}$$

$$\mathcal{J}_{\rm pp}(N,Z) = 4[E(N,Z+2) - 2E(N,Z) + E(N,Z-2)]^{-1}$$

empirical shell gaps

$$\mathcal{J}_{\rm np}(N,Z) = 4[E(N+2,Z+2) - E(N+2,Z) - E(N,Z+2) + E(N,Z)]^{-1}$$

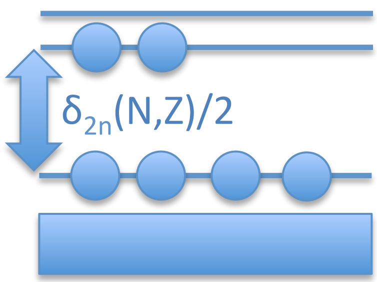
also known as δV_{pn}

Pairing picture of binding energy differences

pairing collectivity

empirical shell gap

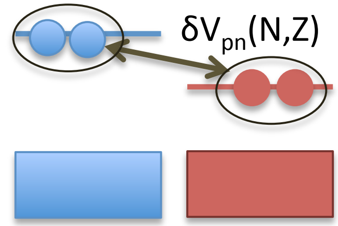
signature of magic number



δVpn

magic nuclei

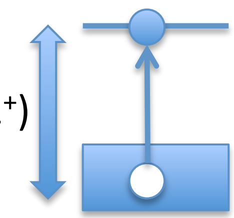
valence proton-neutron interaction energy



$E(2_1^+)$

doubly magic nuclei

^{208}Pb: $E(2^+)$=4.085 MeV
proton h9/2-h11/2
ph excitation



quadrupole collectivity

gauge symmetry breaking

superconducting nuclei

excitation of two NG modes

$$E_{\rm pairrot}(N,Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{\rm nn}(N_0,Z_0)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{\rm np}(N_0,Z_0)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{\rm pp}(N_0,Z_0)}$$

$$\mathcal{J}_{\rm nn}^{-1}(N) = \frac{1}{4}\delta_{2n}(N)$$

$$\mathcal{J}_{\rm pp}^{-1}(N,Z) = \frac{1}{4}\delta_{2p}(N,Z)$$

$$\delta V_{\rm pn}(N,Z) = -\mathcal{J}_{\rm np}^{-1}(N-2,Z-2)$$

magnitude of gauge symmetry breaking

rotational symmetry breaking

deformed nuclei

excitation of NG mode

$$E(2_1^+) \sim \frac{3}{\mathcal{J}_{\rm rot}}$$

magnitude of deformation

Summary

Summary

first systematic calculation of pairing rotational MOI in doubly open-shell nuclei
Evidence of neutron and proton mixed pairing Nambu-Goldstone modes

Pairing rotational moments of inertia:
new pairing observable from even-even systems only
New interpretation of double binding energy differences:
shell gap and proton-neutron interaction energy in terms of gauge symmetry breaking
New motivation for mass measurement experiment:
binding energy difference may determine unknown pairing property

Future extensions, impact to direct reactions

Pair transfer amplitudes: another good pairing observable for gauge symmetry breaking (cf. $B(E2:2_1^+ \rightarrow 0_1^+)$ in rotational case)
Pairing rotation is more universal than expected: even in deformed open-shell systems!
- Importance of pairing rotational picture in Sn isotopes [Potel et al, PRL **107**,092501 (2011)]
→ extension to open-shell nuclei
Extension with neutron-proton pairing (T=1)

Collaborator: Witek Nazarewicz (MSU)

References: NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)
NH, Phys. Rev. C **92**, 034321 (2015)