

# Island of Inversion by microscopically derived shell- model Hamiltonian

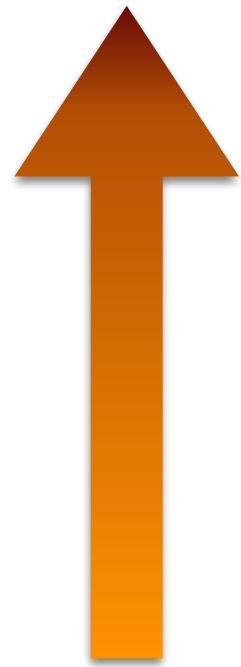
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2016/07/12, DREB2016, Halifax

# Neutron-rich nuclei~ island of inversion

large shell gap

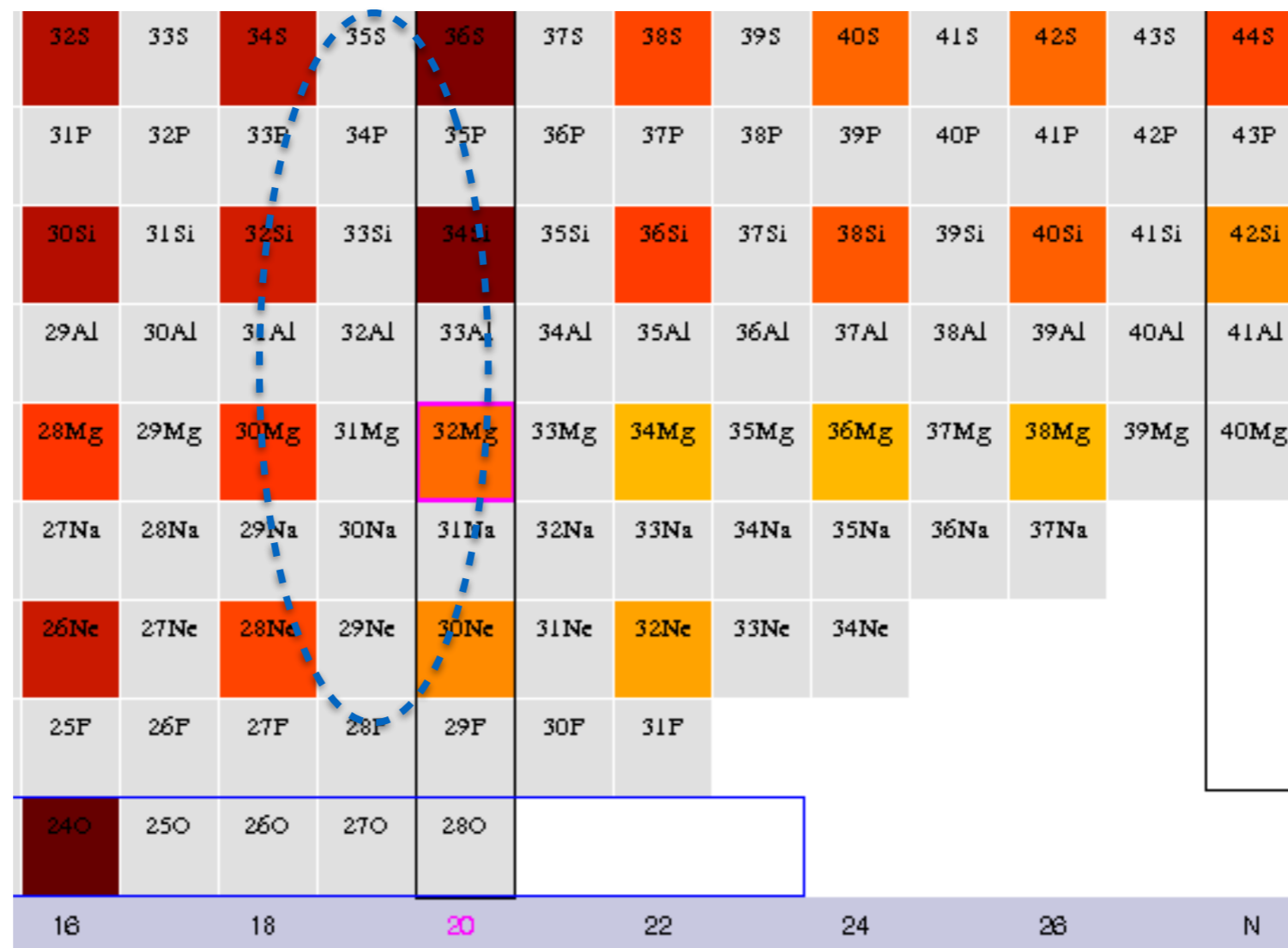


Si

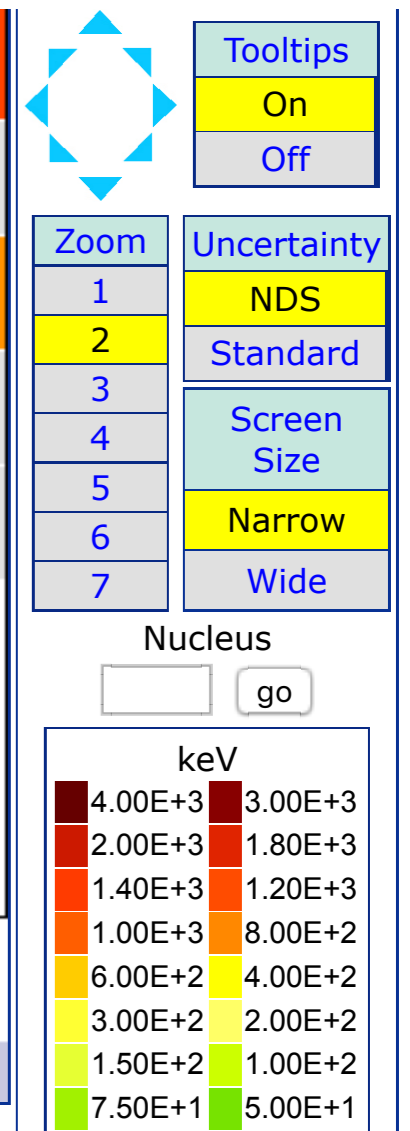
Mg

Ne

breaking of  
shell gap



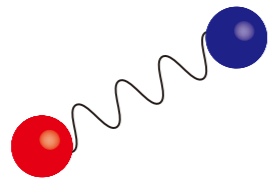
N=20



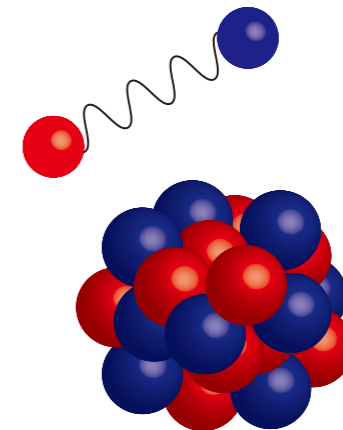
- $E(2+) \sim 1$  MeV on  $N=20$  indicate **breaking** of major shell gap
- Unified treatment beyond and below the  $N=20$  gap is necessary
- Two-major shell degrees of freedom is essential

# Microscopically derived Hamiltonian

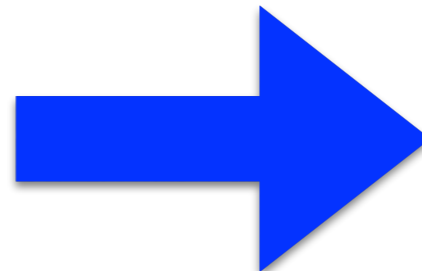
Nuclear force  
in free space



Effective nuclear force  
for model space



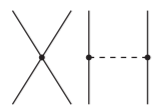
derive



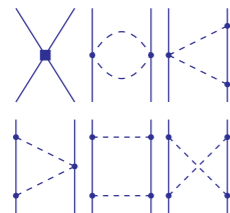
Many body  
perturbation theory

$\chi$  N3LO int. by  $\chi$  EFT

LO  
 $(Q/\Lambda_\chi)^0$



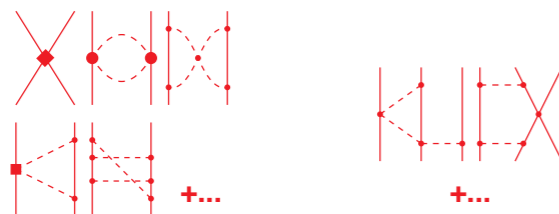
NLO  
 $(Q/\Lambda_\chi)^2$



NNLO  
 $(Q/\Lambda_\chi)^3$



N<sup>3</sup>LO  
 $(Q/\Lambda_\chi)^4$



Shell model Hamiltonian

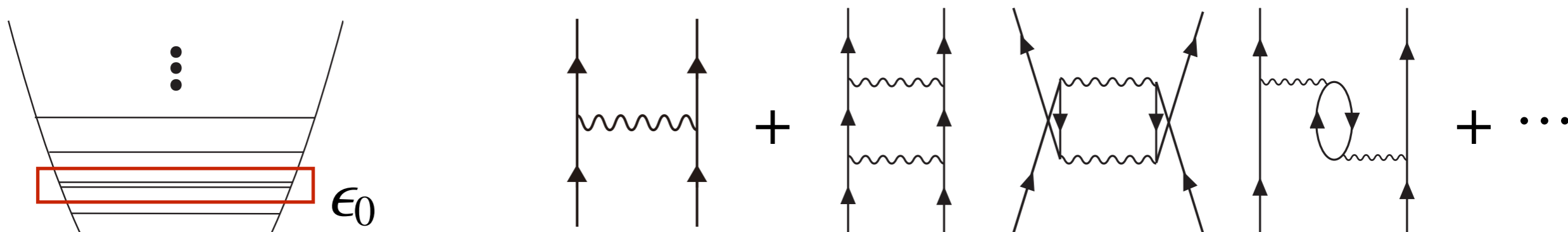
$$H = \sum_i \epsilon_i a_i^\dagger a_i + \sum_{ijkl} V_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k.$$

- Single particle energies
- Two-body matrix elements
- in many cases empirically constructed

R. Machleidt and D. R. Entem, arXiv nucl-th, (2011).

# Many-body perturbation theory

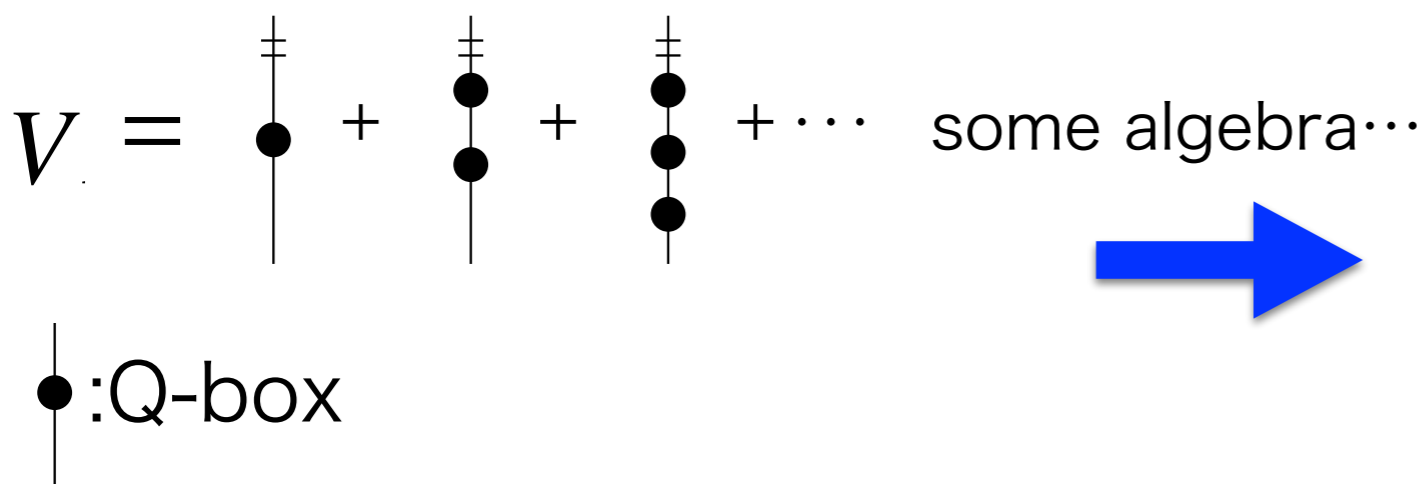
P: model space



$$\hat{Q}(E) = PVP + PVQ \frac{1}{E - QHQ} QVP$$

Q-box

As non-perturbative correction, we further include infinite repetition of Q-box



non-perturbative correction

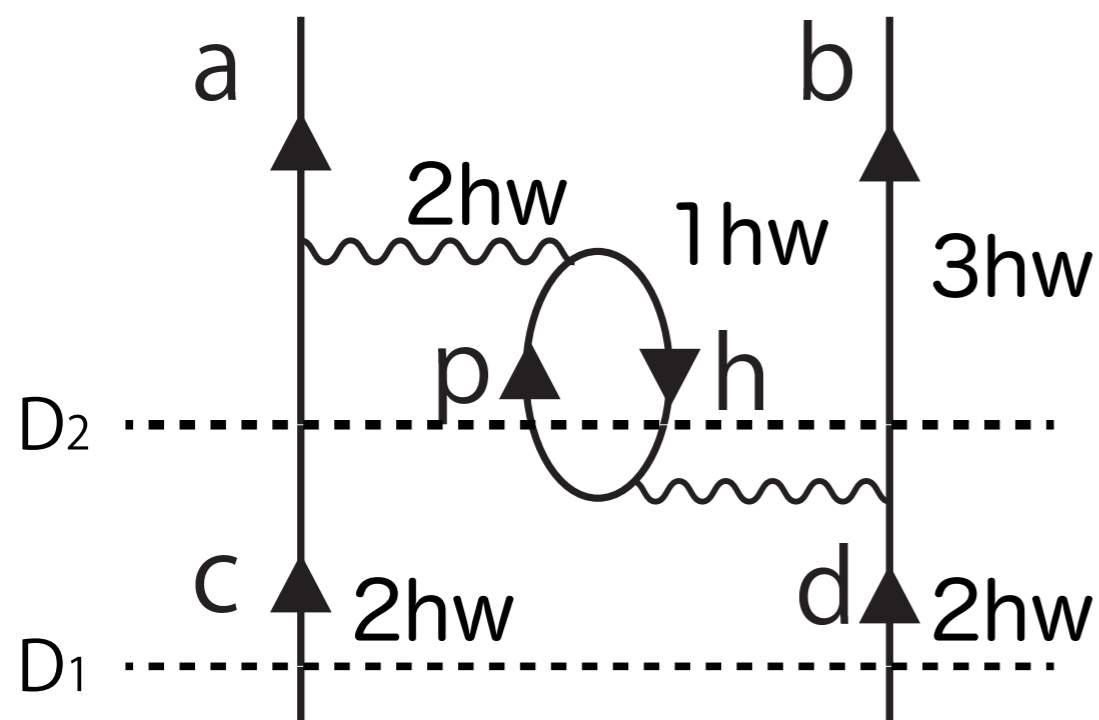
$$V_{\text{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0) \{V_{\text{eff}}^{(n-1)}\}^k$$

$$\hat{Q}_k(E) = \frac{1}{k!} \frac{d^k \hat{Q}(E)}{dE^k}$$

**Kuo-Krenciglowa method**

# Divergent problem and EKK method

Ex: core-polarization diagram



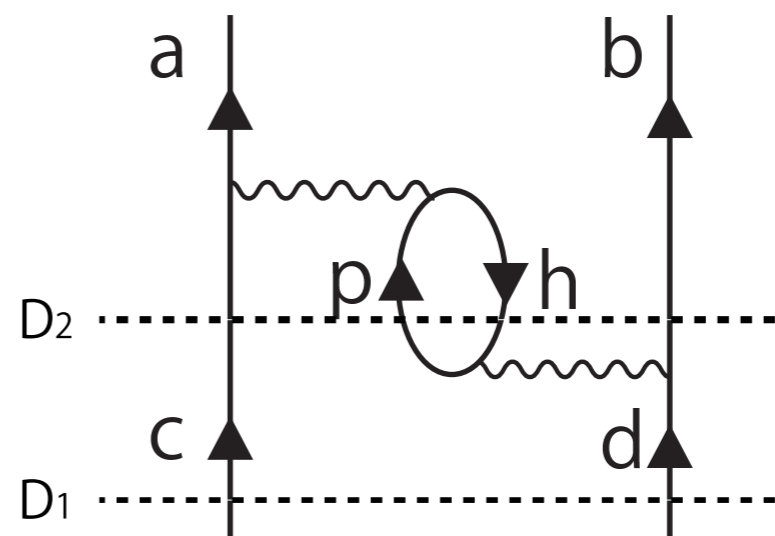
$$(D1-D2) = (2+2)hw - (2-1+3)hw = 0hw$$

-> diverges with naive perturbation theory

KK method

$$V_{\text{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0) \{V_{\text{eff}}^{(n-1)}\}^k.$$

then, we introduce EKK method to avoid unwanted divergence with re-summation



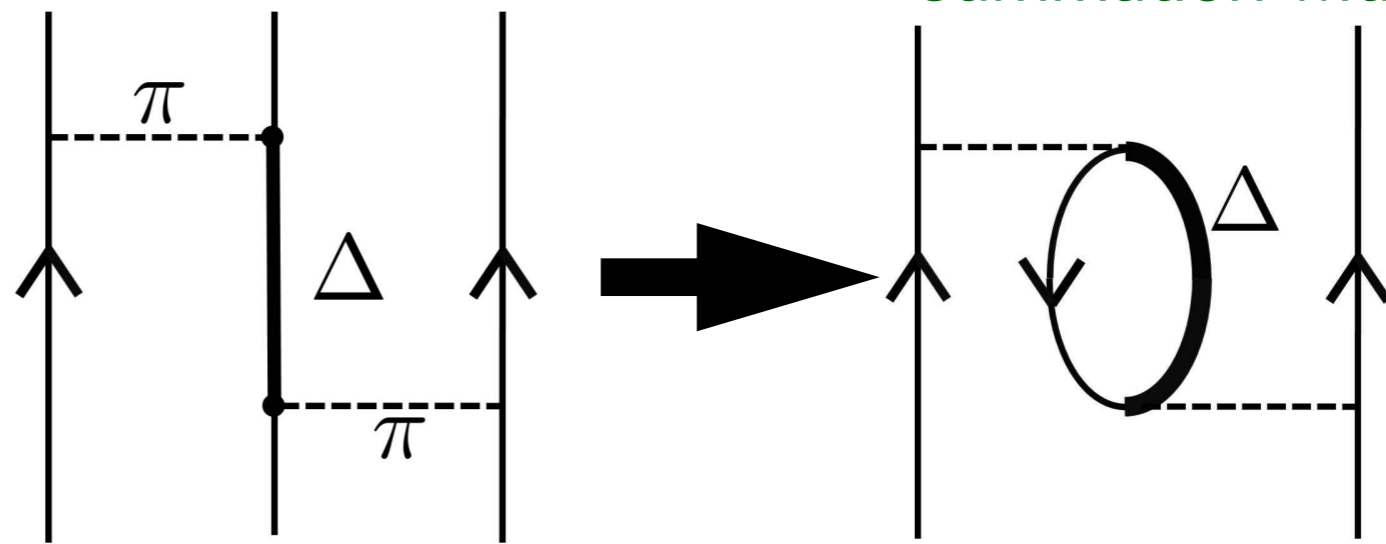
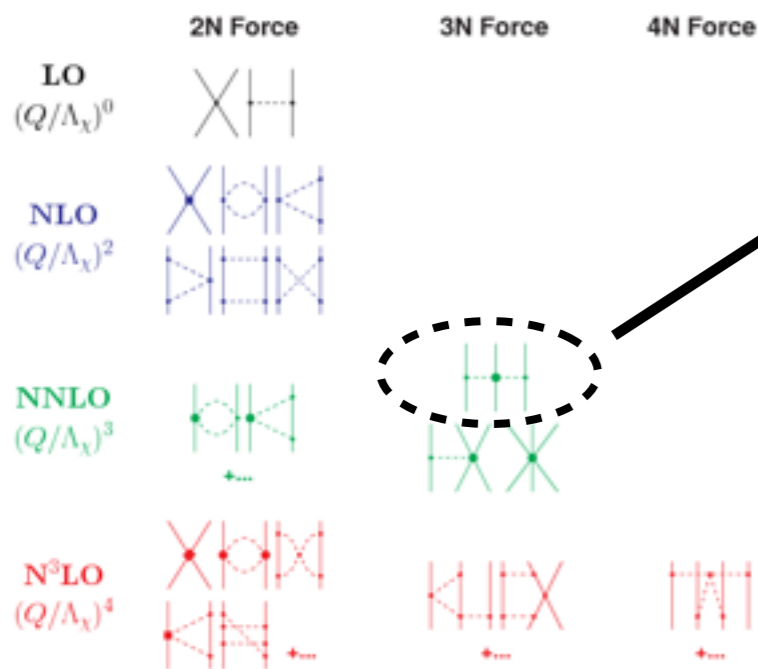
$$\tilde{E} = \frac{V_{ah,cp} V_{pb,hd}}{\epsilon_c - \epsilon_b - \epsilon_p + \epsilon_h}$$

EKK method

$$\tilde{H}_{\text{eff}}^{(n)} = \tilde{H}_{\text{BH}}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E) \{\tilde{H}_{\text{eff}}^{(n-1)}\}^k.$$

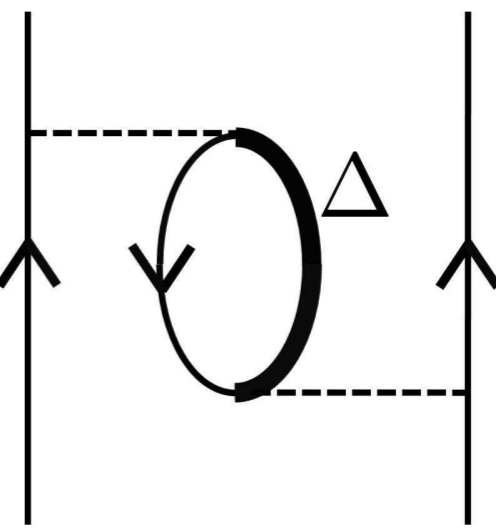
# 3N interaction ( $\Delta$ -hole interaction)

main contribution of 3N



Fujita-Miyazawa type  
**3N** interaction

summation with hole state



Effective  
2N interaction

- Adding up effective 2N interaction derived from 3N interaction to EKK 2N effective interaction [1]
- This is one of the lowest order interaction from 3N force and for higher order we are working on...

[1] T. Otsuka, T. Suzuki, J. D. Holt, A. Schwenk, and Y. Akaishi, Phys. Rev. Lett. 105, 032501 (2010).

# Microscopically derived Hamiltonian vs empirical Hamiltonian

	Microscopic	Empirical
Example	EKK+3N(multi-shell), In-medium SRG(single-shell)	USD, GXPF1, KB3, sdpf-MU, sdpf-U-mix, etc.
Reproduction of the known data	Good	Very good
Theoretical meaning	Clear	Sometimes not clear
# of parameters	Only a few practical parameters	Basically the same as number of matrix

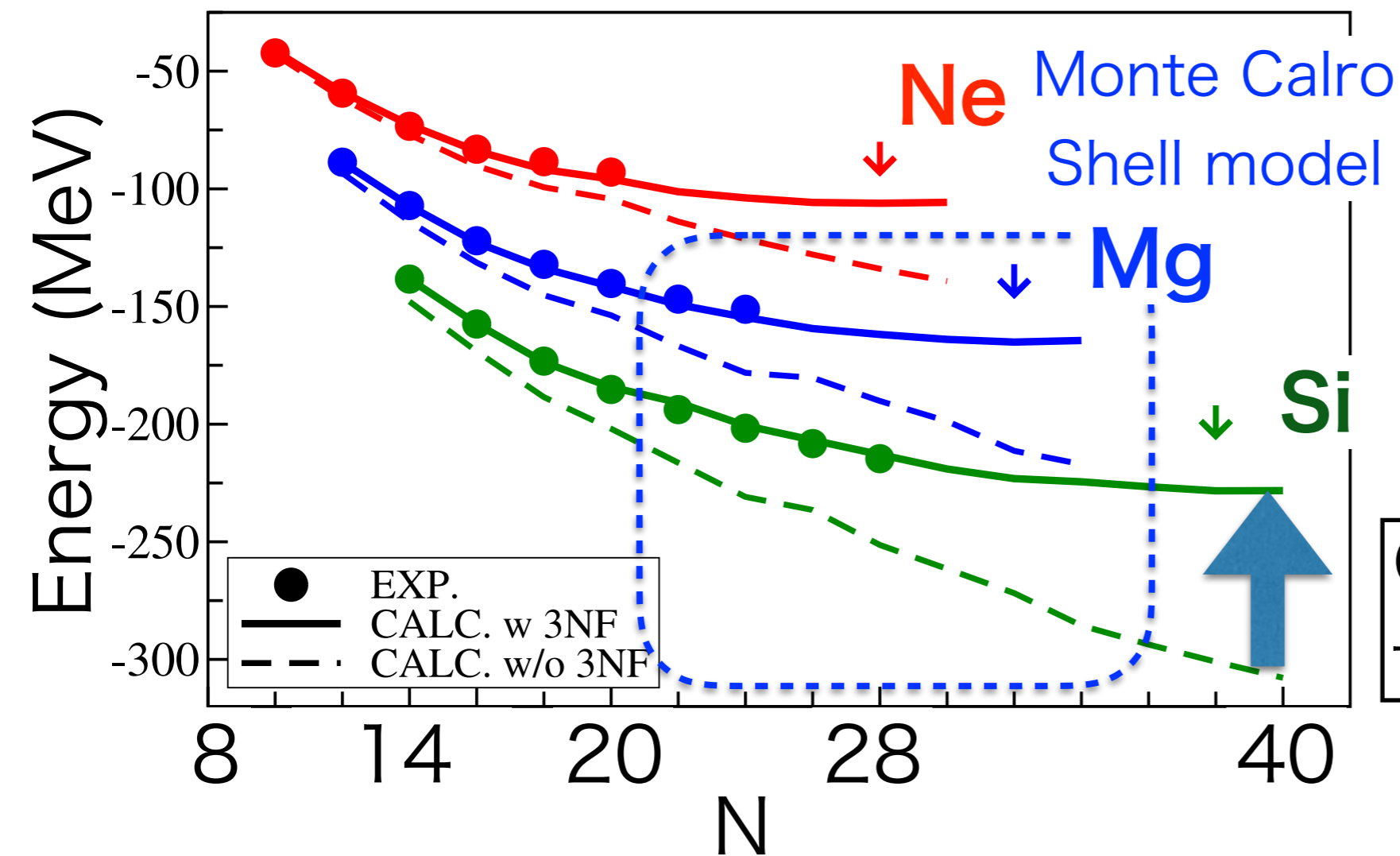
# of matrix elements in sdpf-shell: 2116

It is generally difficult to fit the matrix elements empirically by hand !

Island of inversion by microscopically derived  
Hamiltonian



# Ground state energies and dripline



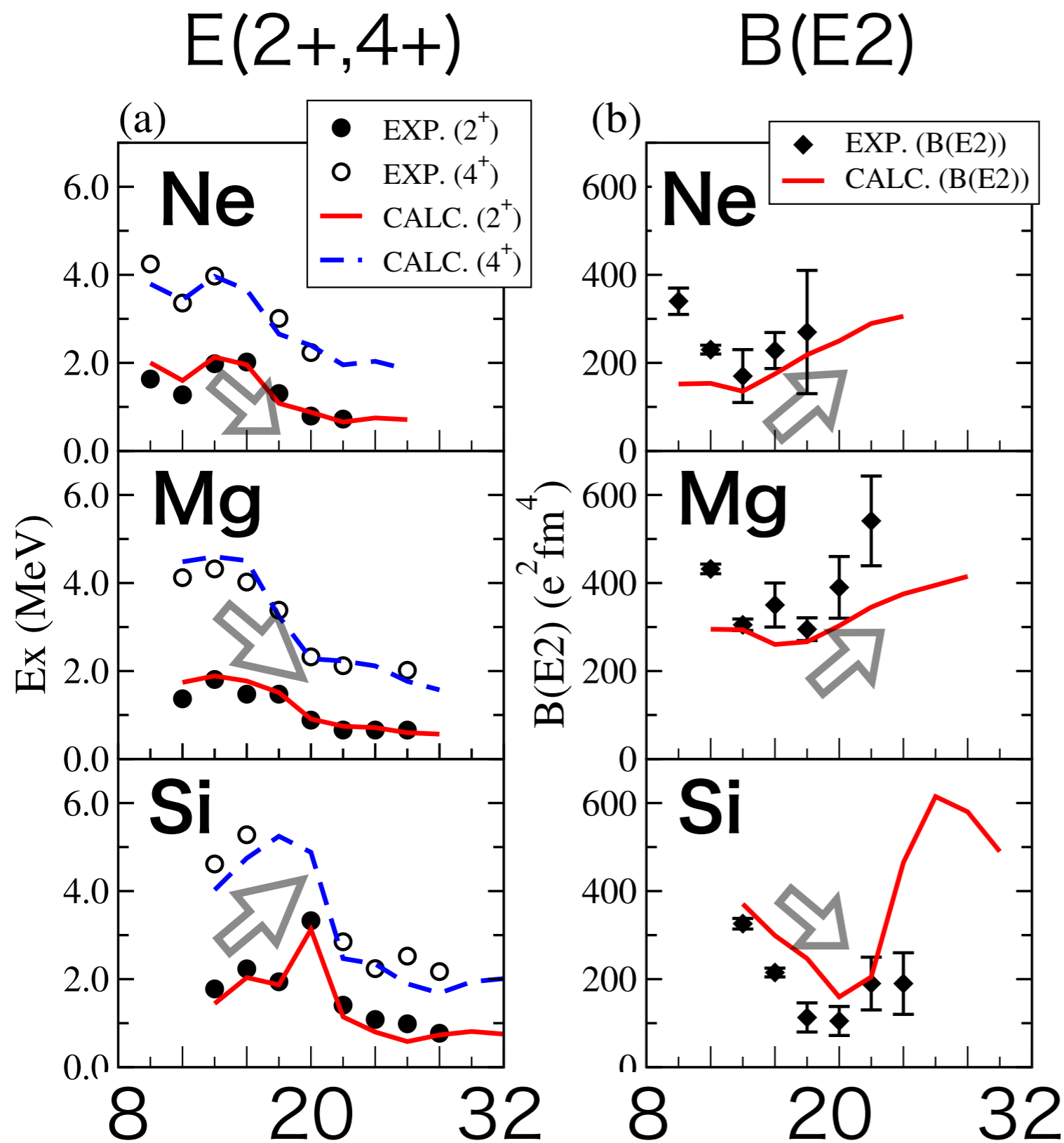
## Resource:

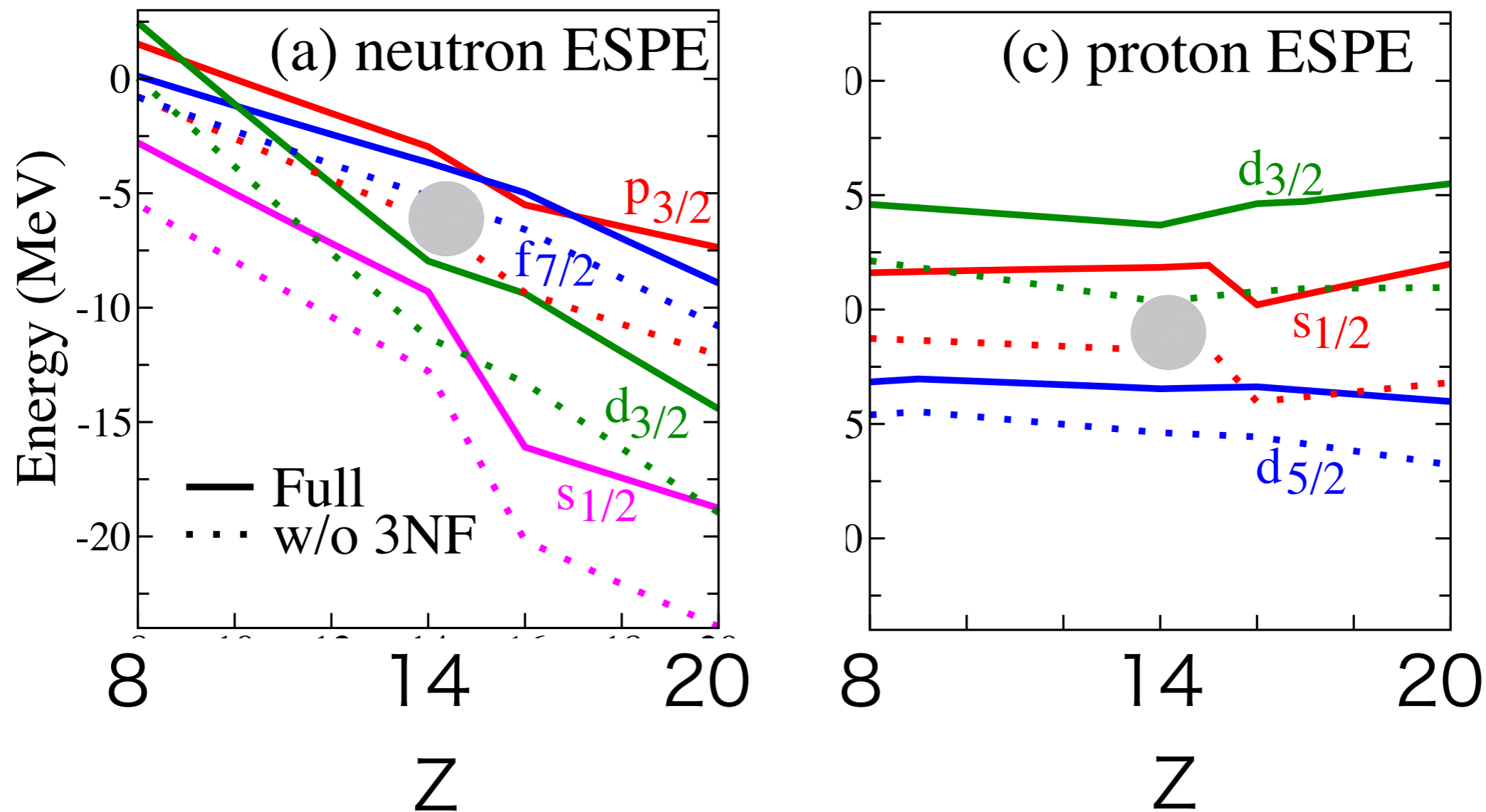
~ $10^6$  node\*hour  
in K-computer for  
around **100 states**

Contribution  
from 3N force

- Experimental ground state energies are well reproduced
- Contribution of 3N force is significant in neutron-rich nuclei
- Combination of **Microscopic theory** and **Large scale calc.**

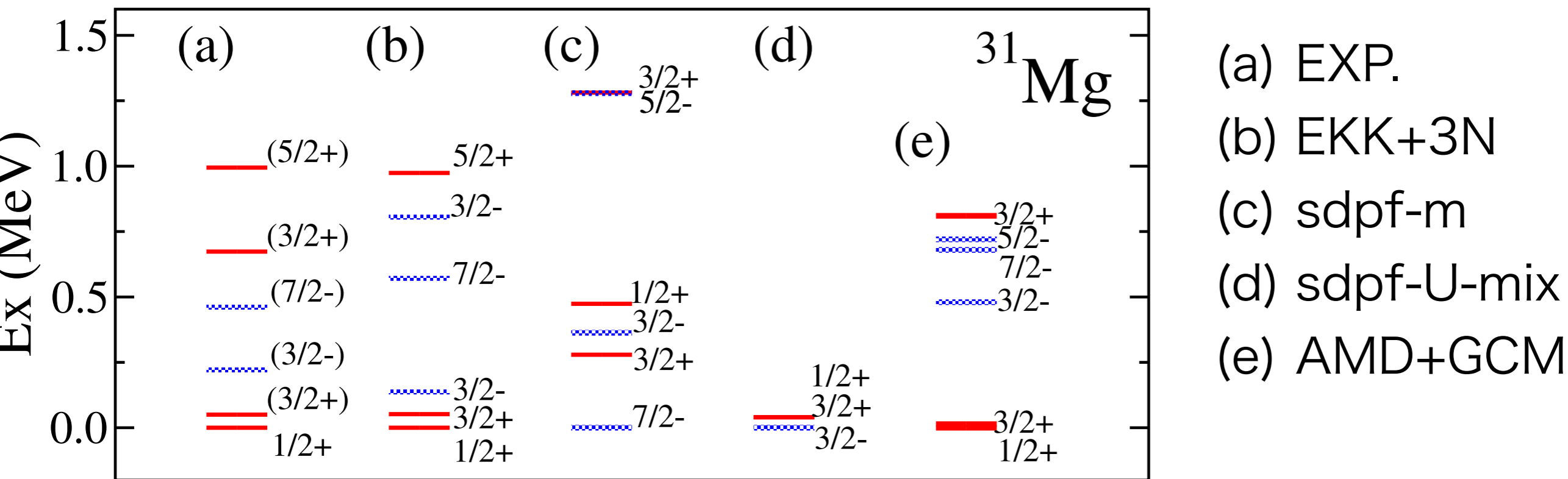
# E(2+,4+) and B(E2)





- Both neutron and proton gap is appearing when  $Z=14$
- Proton-neutron force drive the  $N=20$  gap at  $Z=14$
- at  $Z=20$ ,  $f_{7/2}$  and  $p_{3/2}$  levels are consistent with empirical pf-shell interactions (gxpf1a, KB3 etc.)

# $^{31}\text{Mg}$



## points

- onset of island of inversion
- ordering of levels reproduced
- mixing of different np-nh contribution

ex)  $1/2+$  0hw: 2% 2hw: 66% 4hw: 30% 6hw: 2%  
 $3/2-$  1hw: 33% 3hw: 56% 5hw: 11%

# Summary and conclusion

- The physics in island of inversion is well described by microscopically derived effective Hamiltonian
  - Ground state energies,  $E(2+,4+)$ ,  $B(E2)$  of even-even nuclei, positive and negative parity states of  $^{31}\text{Mg}$
- MBPT is the theory to construct the effective Hamiltonian starting from nuclear force.
- **EKK method** is introduced to derive the effective interaction for the shell model which is applicable to **multi-shell** system.
- **EKK** and **3N** combination is the powerful tool to explore the wide area of the nuclear chart

# Collaborators

- Takaharu Otsuka (Univ. Tokyo)
- Noritaka Shimizu (CNS)
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- Kazuo Takayanagi (Sofia Univ.)
- Toshio Suzuki (Nihon Univ.)
- Morten Hjorth-Jensen (Oslo Univ.)