DECAY OF A BOUND MUON TO A BOUND ELECTRON

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In Collaboration with

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WNPPC, Feb. 13 - 16, 2020

Why Muons are important?

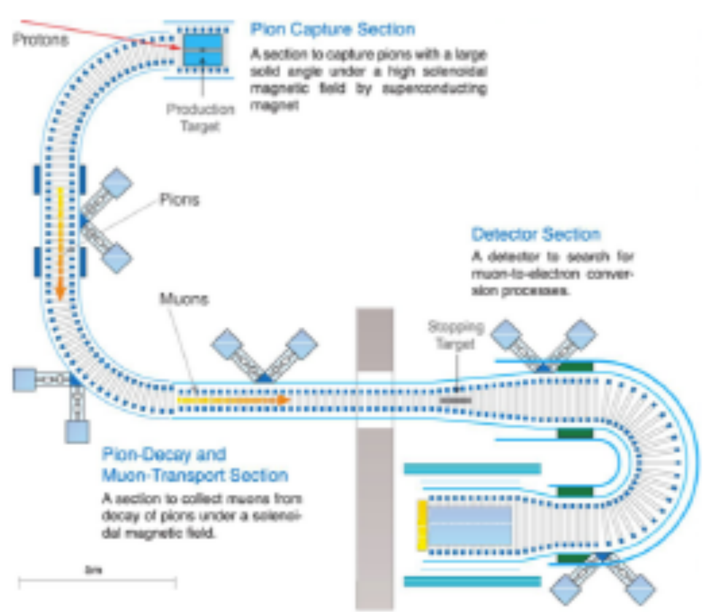
Great tool to search for the New physics. They are easy to produce, long lived and massive enough.

They contributed in different anomalies.

There is a new era of experiments with muons: e.g. $\mu \to e$

COMET Phase 1 J-PARC — 7e-15

COMET Phase 2 J-PARC — 2.6e-17



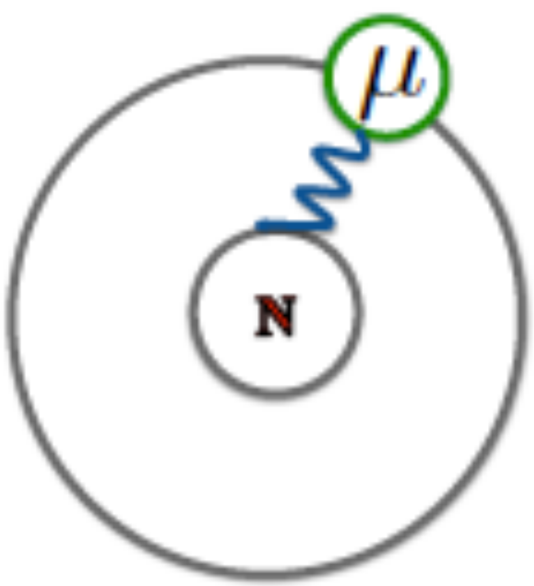
Mu2e Fermilab — 2e-17

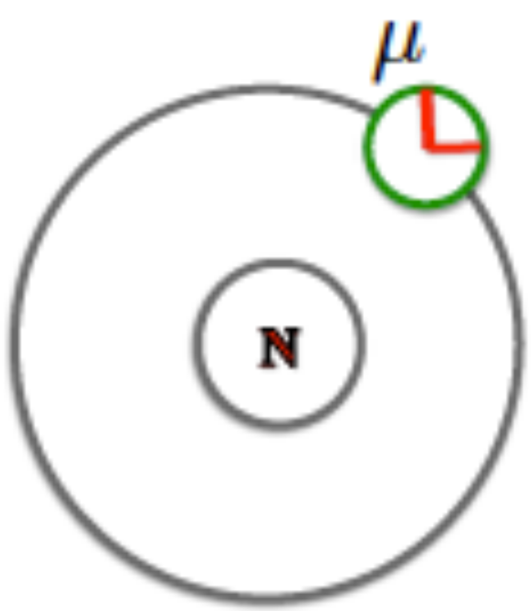
For comparison,

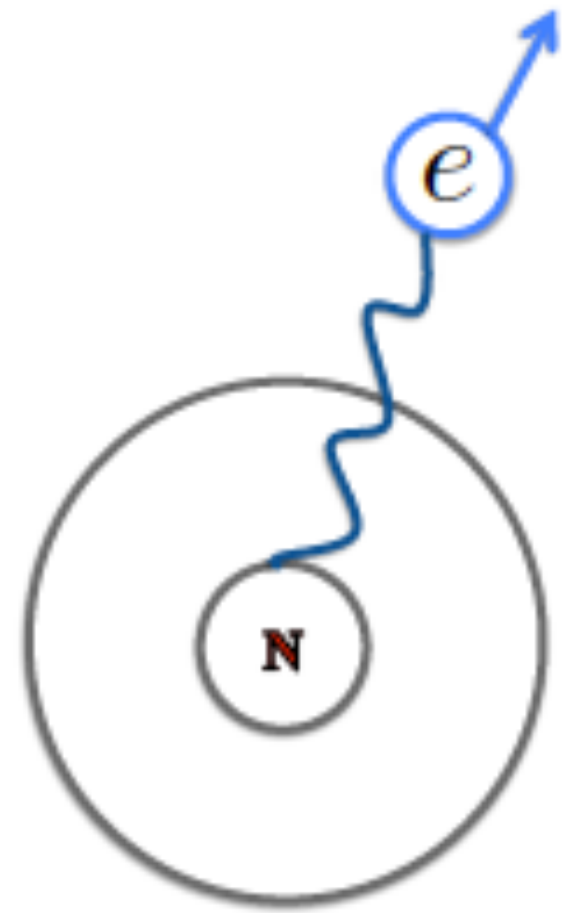
$BR(\mu \to e\gamma) <$ 4e-13

A. Czarnecki's talk at: Lake Louise Winter Inst. 2016.

Distinct Features of a Bound Muon

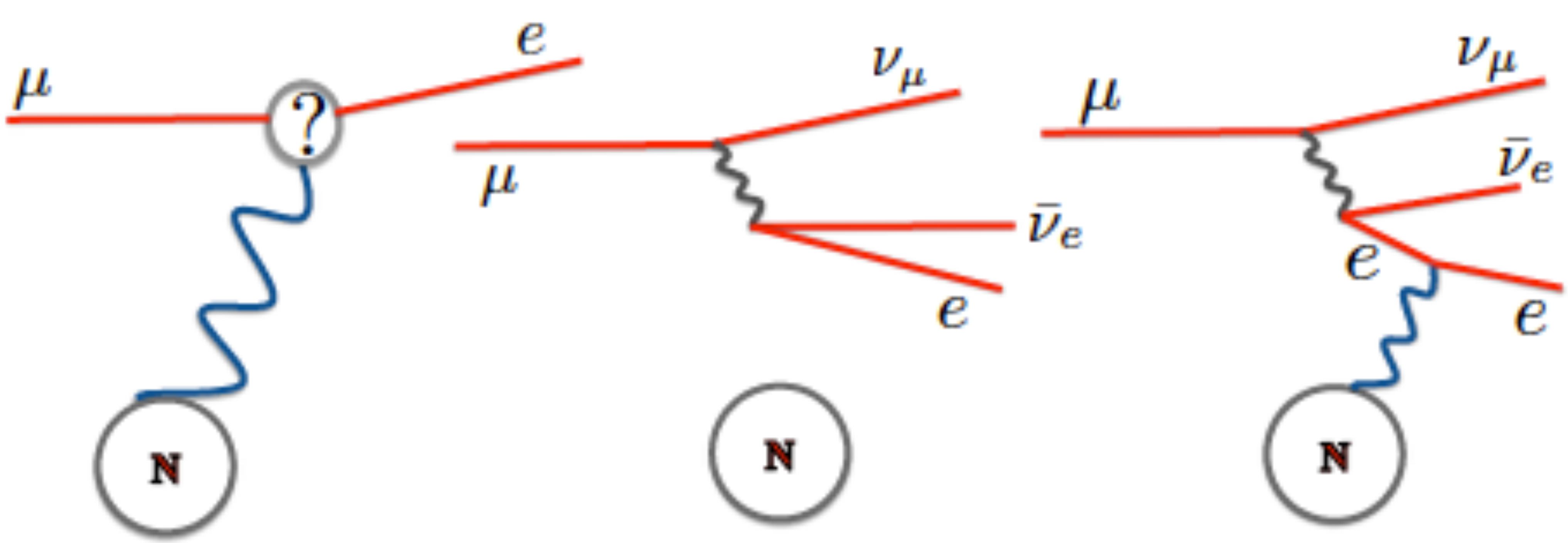






Phase Space Decreases	Time Dilation	Coulomb Attraction
Slows Down	Slows Down	Speeds Up

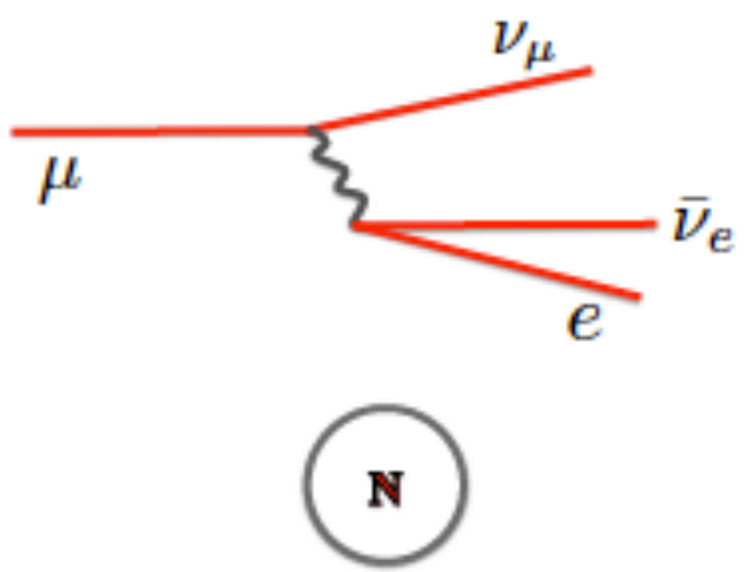
The Standard Model decay provide background for $\mu \to e$ conversion. In case of Mu2e experiment: (cLFV)



Background

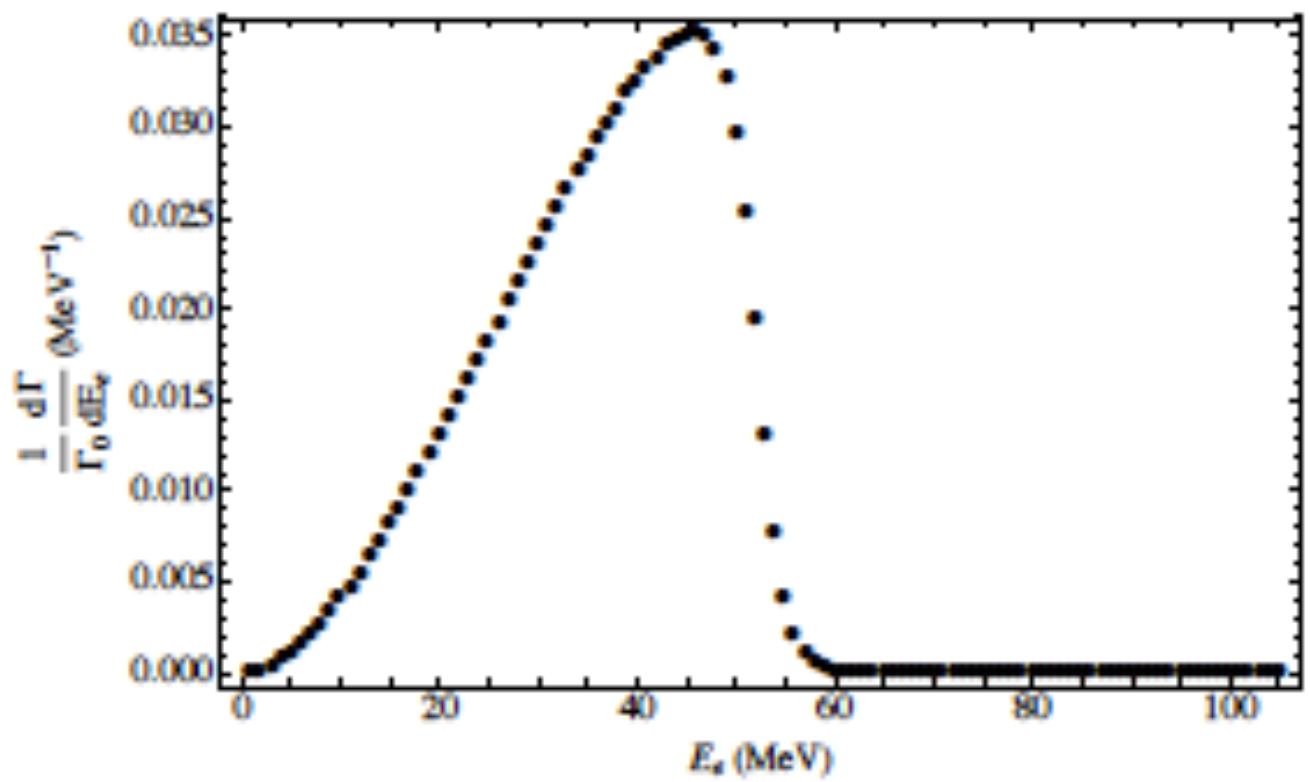
Background

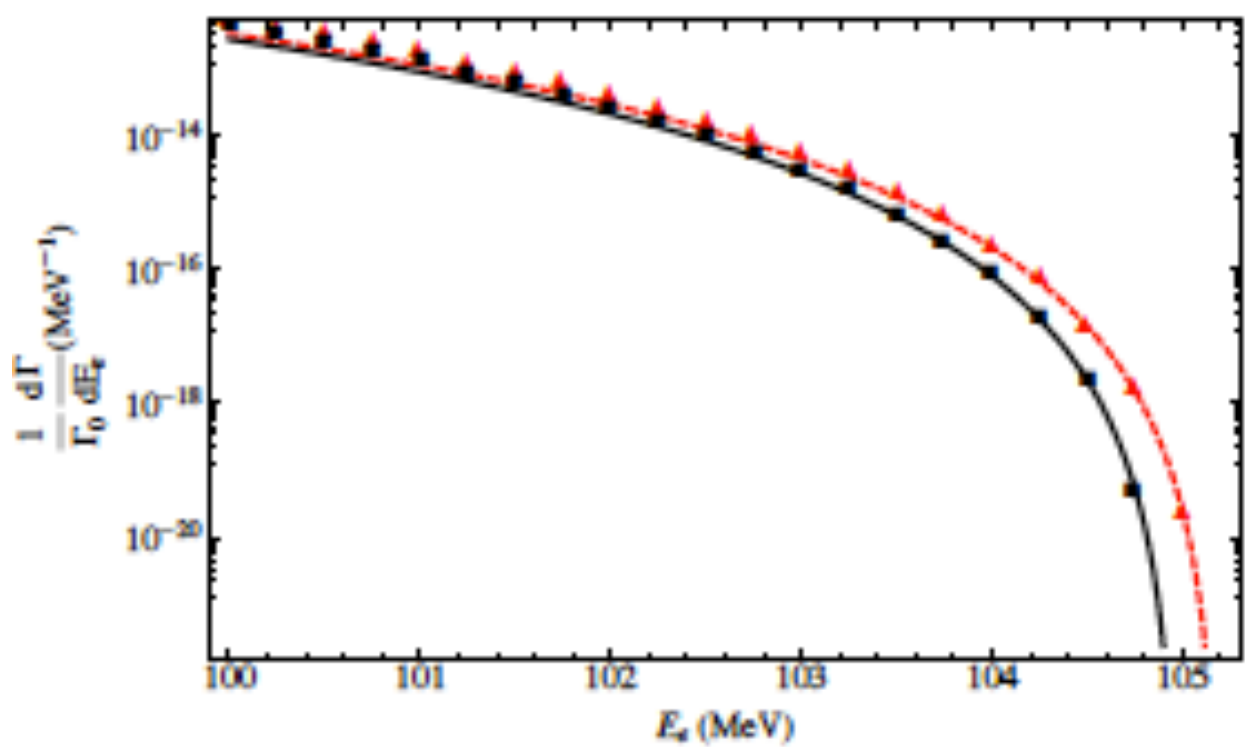
Decay in Orbit DIO



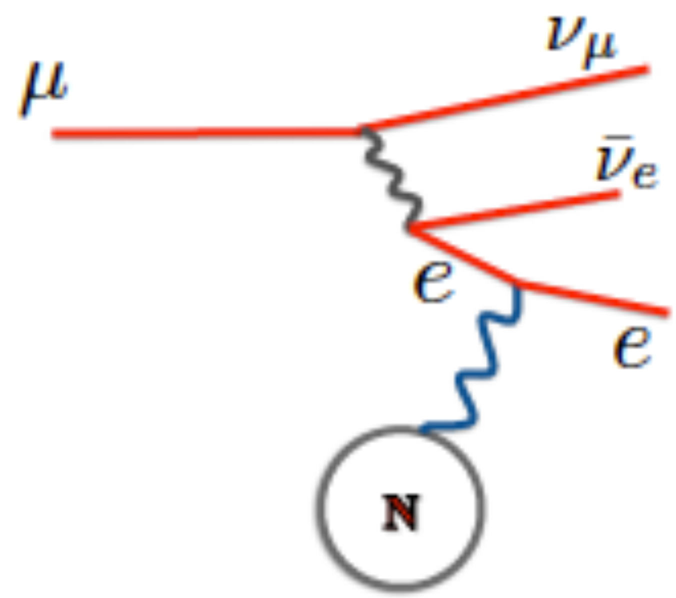
A. Czarnecki et al.

Phys.Rev. D84 (2011) 013006





A small region of the final state phase space namely where the daughter electron is emitted with such low energy:



Bound muon to a bound electron: A focus here.

Nucleus is taken to be Spinless;

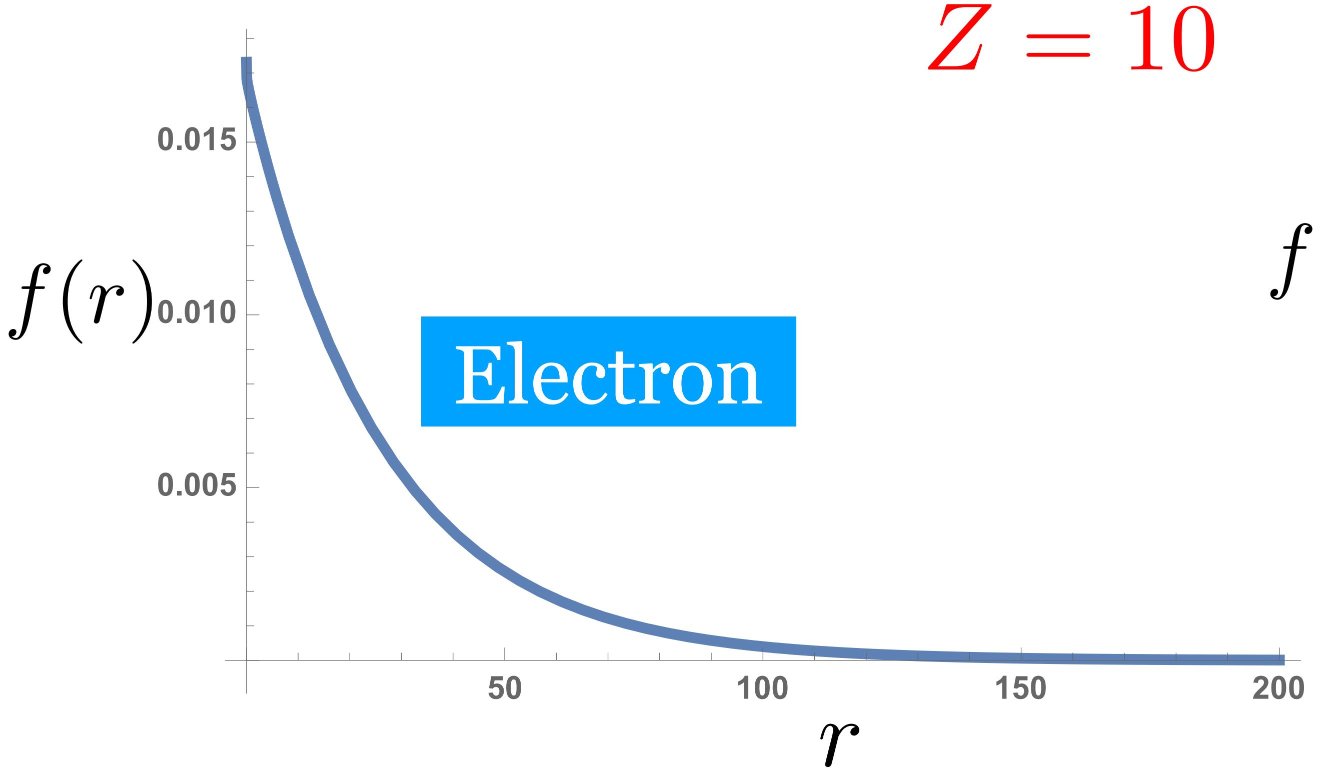
Both muon and electron are in the Ground State.

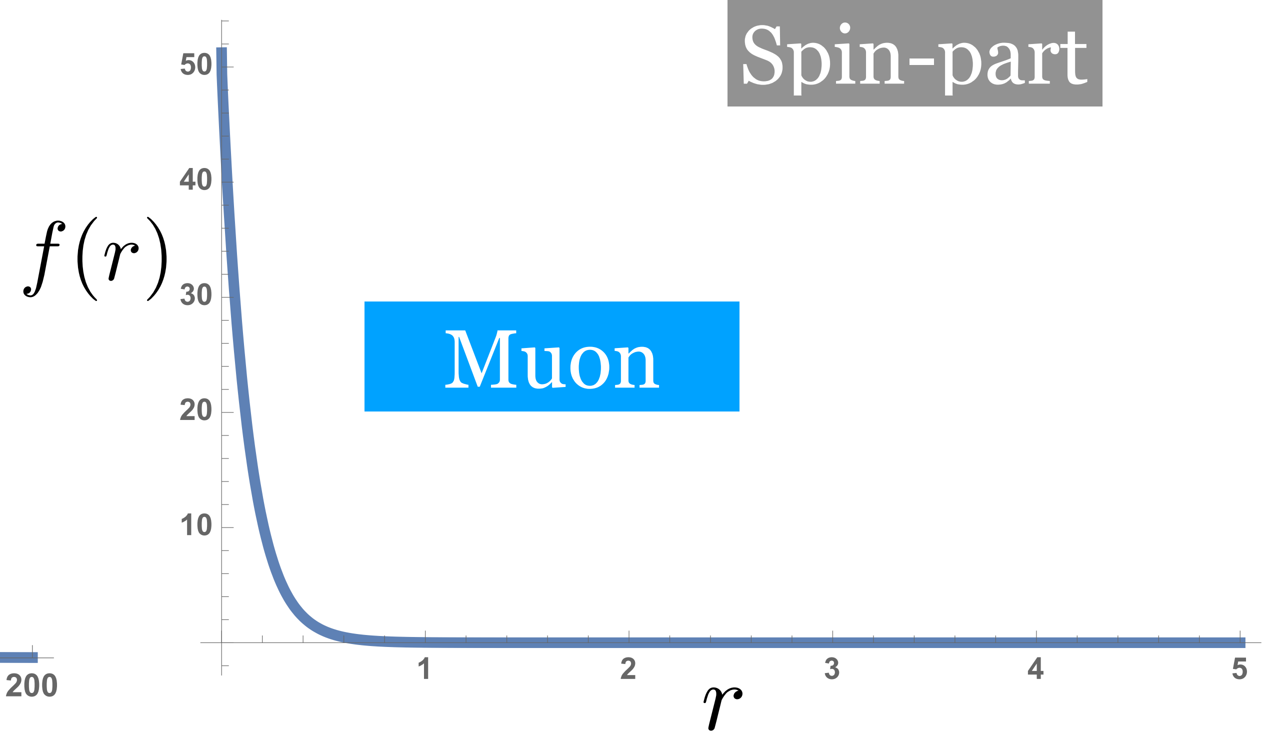
Full Dirac wave functions Functions for 1s electron and muon are

$$\Phi\left(\vec{r}\right) = \psi_{n=1,j=\frac{1}{2},\uparrow(\downarrow)}\left(r,\theta,\phi\right) = \frac{f\left(r\right)}{\sqrt{4\pi}}u_{+(-)},$$

Spin-part

$Z = 10$



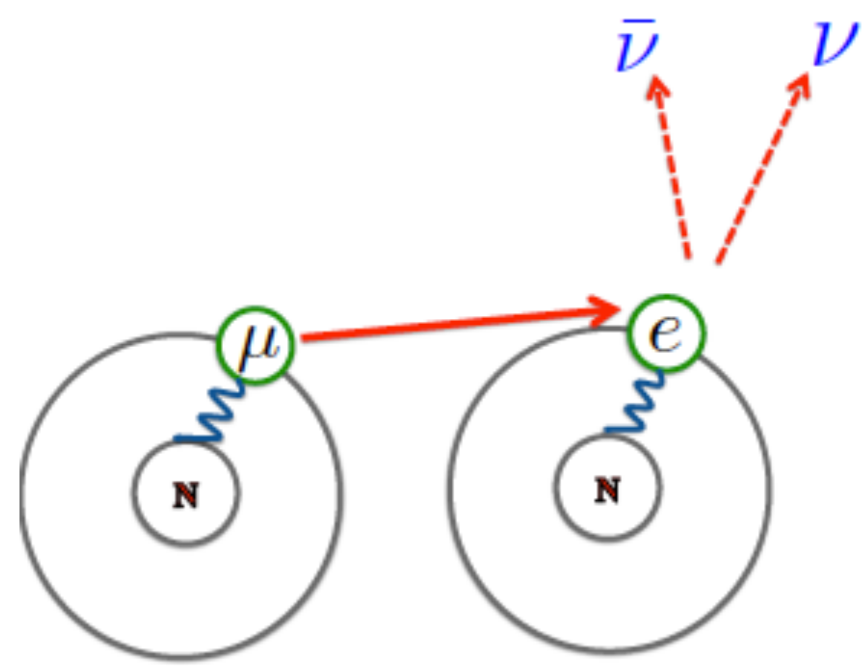


$$J^{\rho} = \int d^3r' e^{-i\vec{q}\cdot\vec{r}'}\bar{\Phi}_e\left(\vec{r}'\right)\gamma^{\rho}P_L\Phi_{\mu}\left(\vec{r}'\right)$$

$$= \int d^3r' e^{-i\vec{q}\cdot\vec{r}'} f_e\left(r'\right) f_{\mu}\left(r'\right)\left\langle e\left|\gamma^{\rho}L\right|\mu\right\rangle$$

Equal mass limit

$$\frac{\Gamma\left[(Z\mu^-) \to (Ze^-)\bar{\nu}_e\nu_\mu\right]}{\Gamma\left[\mu^- \to e^-\bar{\nu}_e\nu_\mu\right]} \to 1 - 3(Z\alpha)^2, \quad Z\alpha \ll 1$$



Binding shows strong influence then just the time dilation factor $\sim (1-(Z\alpha)^2/2)$

Überall, Phys. Rev. 119, 365 (1960)

This result agrees with:

PHYSICAL REVIEW D VOLUME 52, NUMBER 7 1 OCTOBER 1995

Atomic alchemy: Weak decays of muonic and pionic atoms into other atoms

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(Received 9 May 1994)

But, is it the case for the arbitrary muon and electron masses?

Let’s Check!

Approaches	$Z = 10$	$Z = 80$
Alchemy:	1.42×10^{-8}	2.96×10^{-5}
Our:	1.32×10^{-8}	1.83×10^{-5}

The difference grows with Z and for Z = 80 it is around 38%.

So, the Answer is NO.

Reason!

$$J^{\mu}_{rs} = \int D^3 k_1 \bar{\tilde{\Phi}}_s \left(e; \vec{k}_2\right) \gamma^{\mu} L \tilde{\Phi}_r \left(\mu; \vec{k}_1\right)$$

Projectors

$$P_A = \frac{1}{2E}\left(m + E\beta - \vec{k}\cdot\vec{\gamma}\right)\beta \qquad P_B = \frac{1}{2E}\left(E\beta - m - \vec{k}\cdot\vec{\gamma}\right)\beta$$

$$P_A\tilde{\Phi} = A\sqrt{\frac{1}{2E}}u\left(\vec{k}, u_z\right) \qquad P_B\tilde{\Phi} = B\sqrt{\frac{1}{2E}}v\left(-\vec{k}, u_z\right)$$

Only this part is taken in Alchemy.

We have verified that if we take both positive and negative energies parts, the Alchemy results matches with our formalism.