Black hole surprises

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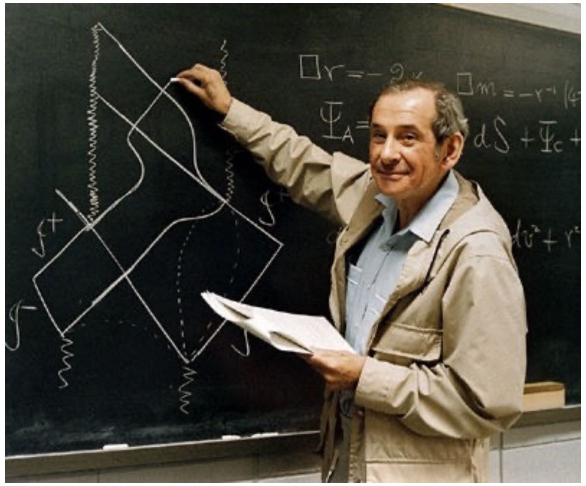

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Event Horizons in Static Vacuum Space-Times

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(Received 27 April 1967)

The following theorem is established. Among all static, asymptotically flat vacuum space-times with closed simply connected equipotential surfaces g_{00}=constant, the Schwarzschild solution is the only one which has a nonsingular infinite-red-shift surface $g_{00}=0$. Thus there exists no static asymmetric perturbation of the Schwarzschild manifold due to internal sources (e.g., a quadrupole moment) which will preserve a regular event horizon. Possible implications of this result for asymmetric gravitational collapse are briefly discussed.

This proved the uniqueness of the Schwarzschild black hole in vacuum GR.

He was well aware of the significance of his result. At the end of his paper he considers a collapsing object and says:

either the body has to divest itself of all quadrupole and higher moments by some mechanism (perhaps gravitational radiation), or else an event horizon ceases to exist.[13]

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Event Horizons in Static Electrovac Space-Times

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Abstract. The following theorem is established. Among all static, asymptotically flat electrovac fields with closed, simply-connected equipotential surfaces g_{00} = const., the only ones which have regular event horizons $g_{00} = 0$ are the Reissner-Nordström family of spherisymmetric solutions with $m \geqq G^{1/2}|e|/c$. In the special case where the gravitational coupling of the electromagnetic energy density is neglected ($G = 0$) all solutions are computed explicitly, thus extending an earlier result of GINZBURG for a magnetic dipole in SCHWARZSCHILD's space-time. Possible implications for gravitational collapse are briefly discussed.

In the 1970’s it was shown that general relativity coupled to simple (linear) matter fields has no other static black hole solutions.

Wheeler: Black holes have no hair. All black holes are characterized by M, Q, J.

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We now know that this is not true.

Examples of black holes with hair

1990's: GR coupled to nonlinear matter fields can have static matter outside BHs, e.g., Einstein-Yang-Mills. Can put BHs inside many solitons.

(Bizon, 1990; Volkov and Gal'tsov, 1990; …)

2000's: In anti-de Sitter space, charged black holes become unstable to forming charged scalar hair at low temperatures.

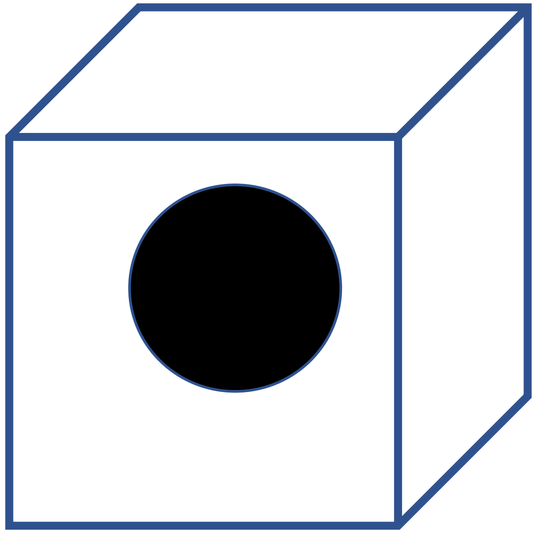
(Gubser, 2008; Hartnoll, Herzog, GH, 2008)

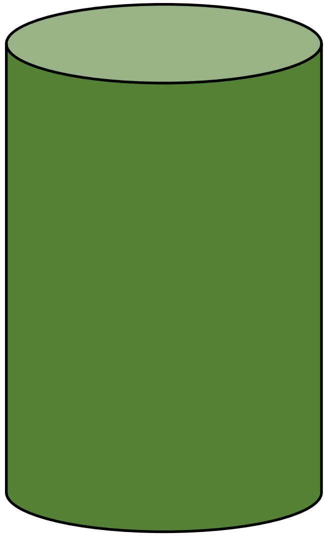
2010's: Kerr can have massive scalar hair due to superradiant instability.

(Herdeiro and Radu, 2014)

Suppose you put a black hole inside a steel cage, or add inhomogeneous boundary conditions in anti-de Sitter (AdS).

What happens to the horizon?



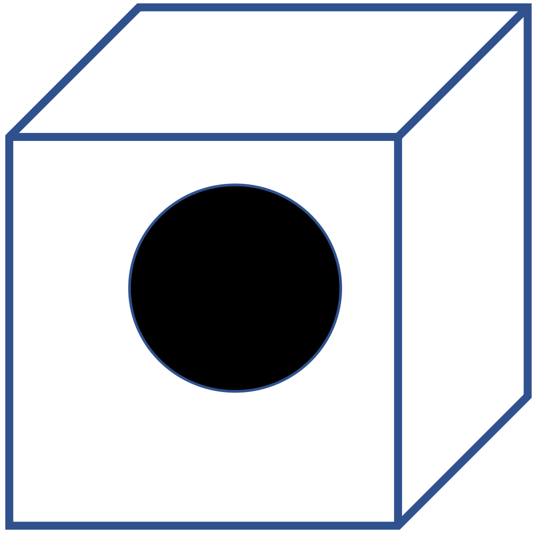


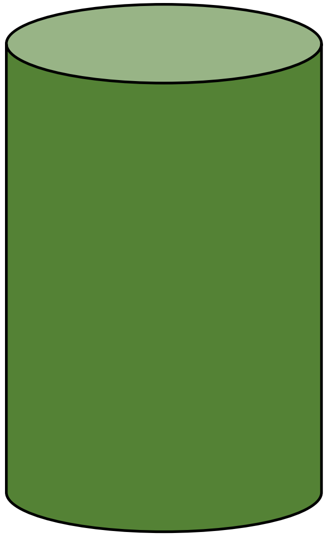
A_t not constant
or
S^2 not round
on boundary

For nonextremal black holes, the horizon becomes distorted but remains smooth.

Suppose you put an extremal black hole inside a steel cage, or add inhomogeneous boundary conditions in anti-de Sitter.

The horizon is now infinitely far away.





A_t not constant

or

S^2 not round

on boundary

GR with $\Lambda = 0$: Horizon is unaffected

GR with $\Lambda < 0$: Horizon becomes singular!

Main Results

(Kolanowski, Remmen, Santos, and GH, 2022, 2023)

- Nonspherical extremal black holes in AdS have a metric that is C^0 but not C^2 at the horizon. The horizon is singular, and tidal forces diverge for ingoing timelike or null geodesics.

- Asymptotically flat, extremal Kerr is very sensitive to higher curvature corrections. Even small higher curvature terms produce singular horizons on extremal Kerr black holes.

Λ < 0: Massless scalar field

Reissner-Nordstrom AdS:

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega^2$$

where

$$f(r) = \frac{r^2}{L^2} + 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

The horizon r_+ is the largest zero of f, and at an extremal horizon:

$$f''(r_+) = \frac{6}{L^2} + \frac{2Q^2}{r_+^4}$$

Consider a static, massless scalar test field, and expand in modes:

$$\phi = \sum_{\ell,m} \phi_{\ell m} Y_{\ell m}$$

These satisfy: $$(f\phi'_{\ell m})' + \frac{2f\phi'_{\ell m}}{r} - \frac{\ell(\ell+1)}{r^2}\phi_{\ell m} = 0$$

Near the extremal horizon, try $\phi_{\ell m} \sim (r - r_+)^\gamma$

Find a solution if $$\gamma = \frac{1}{2}\left[\sqrt{1 + \frac{4\ell(\ell+1)}{1 + 6\, r_+^2/L^2}} - 1\right]$$

If $\ell = 1$, we have

$0 < \gamma < 1$ for all $r_+ > 0$

So T_{rr} diverges

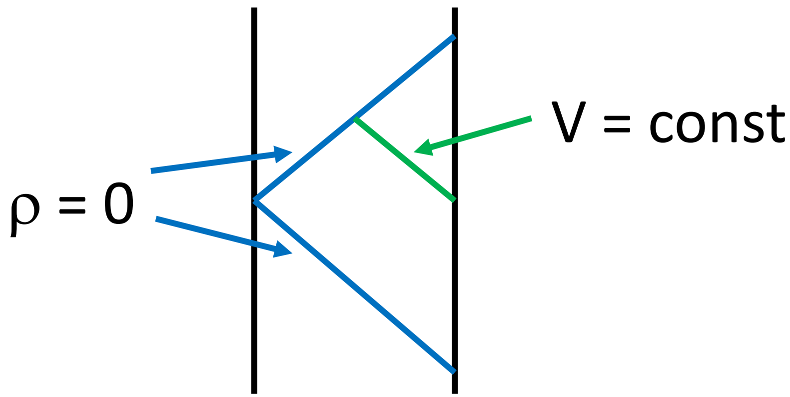
Comments

- If r_+ is big enough, $\gamma < 1$ for higher ℓ's
- The larger r_+ , the smaller γ, so T_{rr} becomes more divergent.
- The result is derived locally. The only role of asymptotic conditions is to source nonspherical modes.
- $\Lambda = 0$ follows by taking L to infinity. Find $\gamma = \ell$ (always integer)
- $\Lambda > 0$ follows by analytically continuing L^2 to - L^2. Find that the field is C^1 but not C^2 (for small r_+).

Linearized Einstein-Maxwell

Start with $AdS_2 \times S^2$ in ingoing null coordinates:

$$ds^2 = -\rho^2 dv^2 + 2dvd\rho + d\Omega^2$$



Since this is highly symmetric, we can expand static perturbations in terms of modes on both AdS_2 and S^2. First set

$$\delta g = \rho^\gamma \left(\delta F\, \rho^2\, \mathrm{d}v^2 + 2\,\rho\, \delta h_a\, \mathrm{d}v\, \mathrm{d}x^a + \delta q_{ab}\, \mathrm{d}x^a\, \mathrm{d}x^b\right)$$

And similarly for Maxwell perturbation. Then expand coefficients in spherical harmonics.

Get a set of algebraic equations for the coefficients. Solutions exist if the exponent is

$$\gamma_\pm = \frac{1}{2}\left[-1 + \sqrt{\frac{4\ell(\ell+1) + 5\sigma \pm 4\sqrt{\sigma^2 + 2\ell(\ell+1)(1+\sigma)}}{\sigma}}\right]$$

where $\sigma \equiv 1 + \frac{6\, r_+^2}{L^2}$

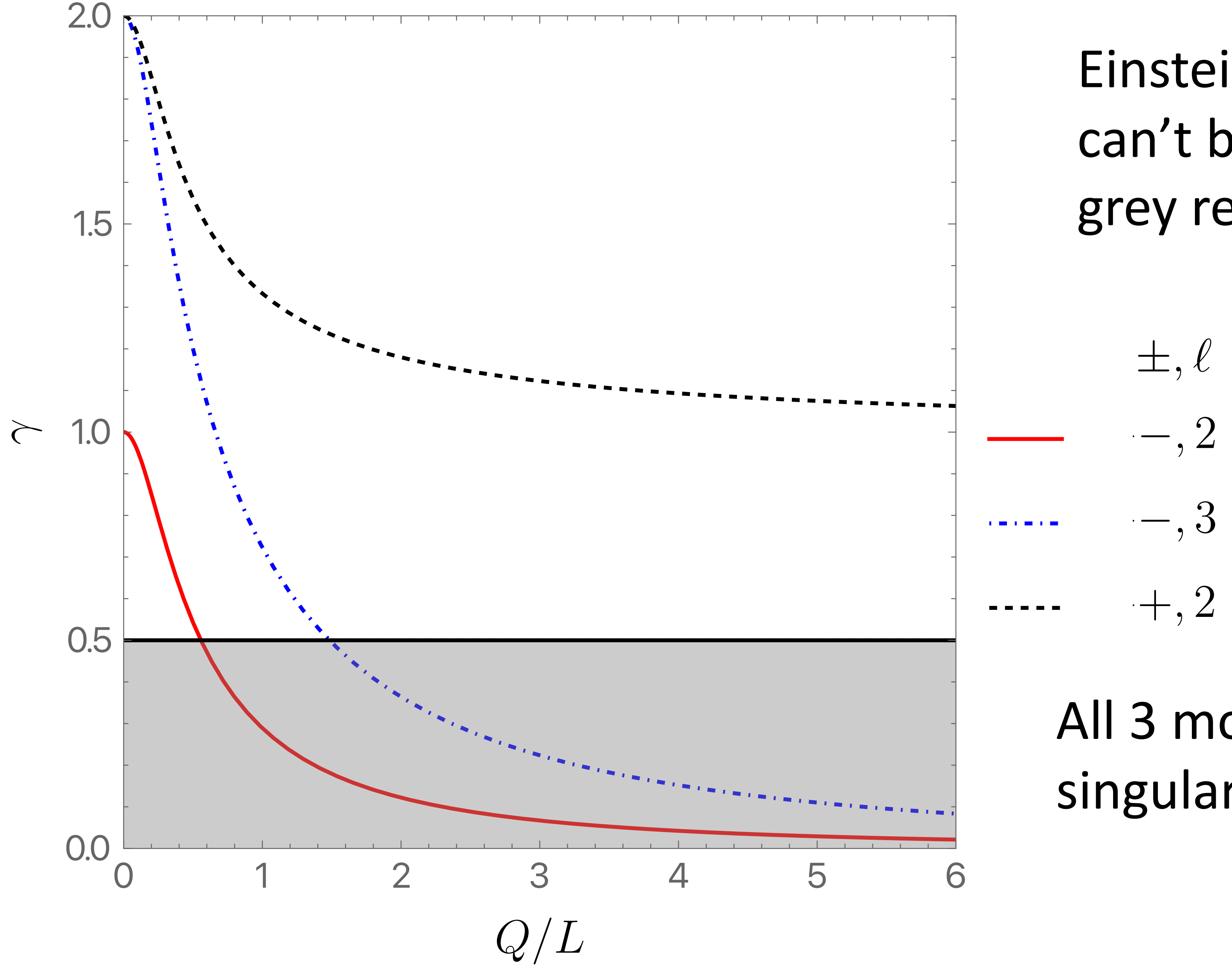
ρ components of the curvature scale like:

$$\delta C_{\rho a \rho b} \sim \gamma(\gamma-1)\rho^{\gamma-2}$$

$$\delta R_{\rho\rho} \sim \gamma(\gamma-1)\rho^{\gamma-2},$$

So if $1 \neq \gamma < 2$

linearized solutions are singular.



Einstein's equation can't be defined in grey region.

All 3 modes lead to singular horizons.

We will show that nonlinear effects do not remove these singularities.

Since generic, nonspherical boundary conditions will include these low ℓ modes:

generic nonspherical extremal black holes in AdS are singular.

(For $\Lambda = 0$, $\gamma_+ = \ell + 1$, $\gamma_- = \ell - 1$, so horizon is nonsingular.
For $\Lambda > 0$, γ is larger, but still less than 2 for small Q.)

Nonlinear solutions

We numerically found static, charged black holes in AdS with

boundary metric (conformal to) round S^2 x R

and $$A_t|_{bdy} = 2 + .1\cos\theta$$

We cooled them down and monitored the maximum of $C_{\rho\phi\rho\phi}$ on the horizon.

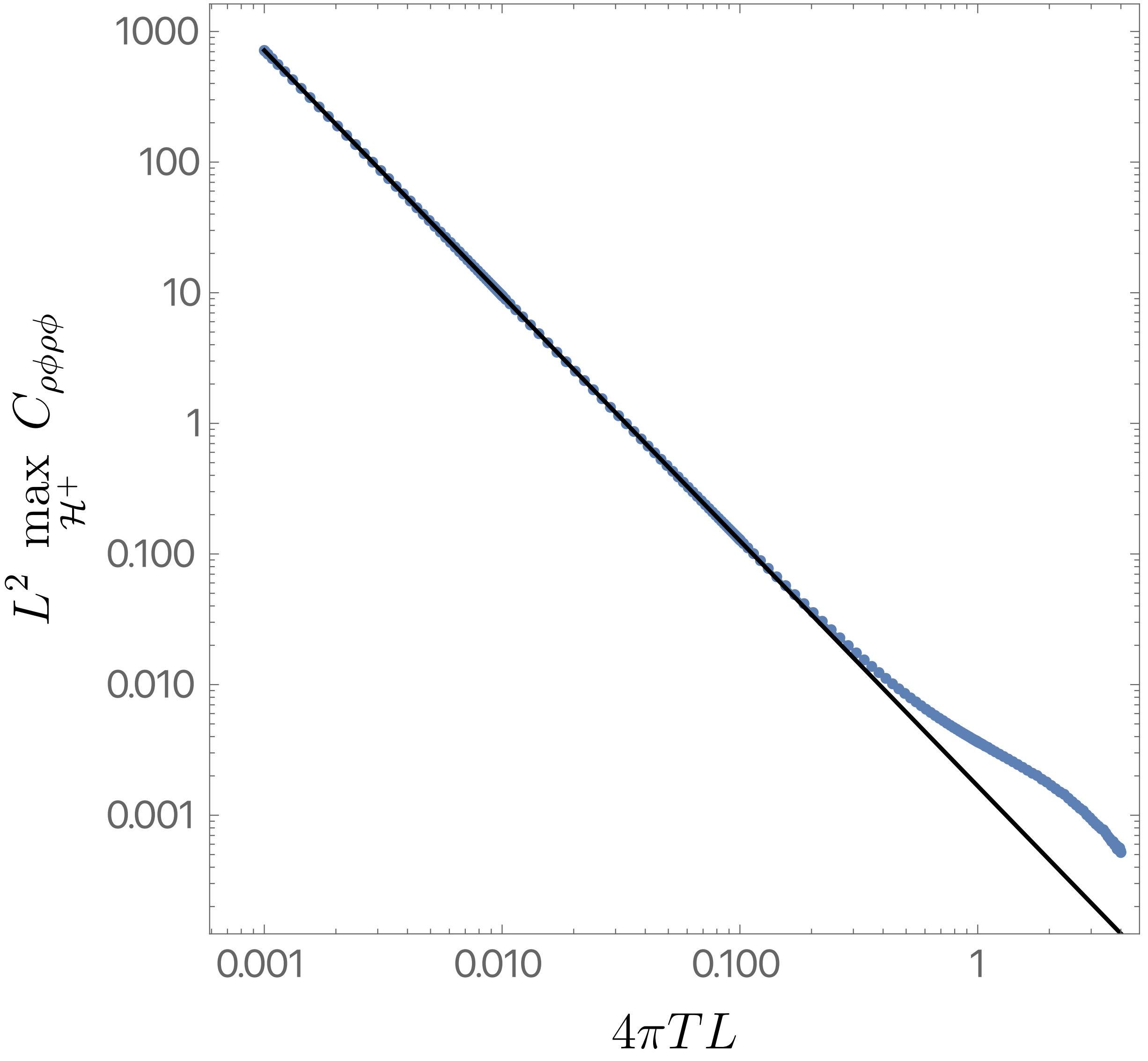
Scaling argument

Consider a near extremal black hole. Near the horizon, the metric is

$$ds^2 = -(\rho^2 - b^2)dt^2 + \frac{d\rho^2}{\rho^2 - b^2} + d\Omega^2$$

and $T = b/2\pi$. Away from the horizon the nonspherical perturbations should behave like the extremal solution: $C_{\rho\phi\rho\phi} \sim \rho^{(\gamma - 2)}$

This should hold until $\rho \sim T$. So we expect $C_{\rho\phi\rho\phi} \sim T^{(\gamma - 2)}$



Find max $C_{\rho\phi\rho\phi} \sim a_0\, T^{(\gamma-2)}$

with $\gamma = .1224$

Predicted value from lowest ℓ=2 mode in linear analysis and scaling argument: $\gamma = .1220$

Also, $a_0 = .0017$, consistent with ℓ=2 mode being generated nonlinearly.

These singularities affect BH thermodynamics

The BH entropy acquires anomalous scaling with T:

The perturbation decays like ρ^{γ} but S_{BH} doesn't change to first order.
The leading correction to S_{BH} comes at second order and scales like $\rho^{2\gamma}$.

Scaling argument relates ρ to T: $S_{BH} = S_0 + S_2\, T^{2\gamma}$

The specific heat at constant charge scales like $C_Q = T\frac{\mathrm{d}S}{\mathrm{d}T} \propto T^{2\gamma}$

So if $\gamma < ½$ this is larger than the usual linear T behavior.

This provides an easy way to detect

these singularities in a dual field theory!

$\Lambda = 0$

In GR, extremal Kerr black holes are not affected by stationary, axisymmetric perturbations (e.g. sourced by distant matter).

But the low energy effective action probably includes higher curvature terms:

$$S = \int_{\mathcal{M}} \mathrm{d}^4x \sqrt{-g} \Big(R + \eta\, R^3 + \lambda R^4 + \cdots \Big)$$

These come from integrating out massive classical degrees of freedom, or from quantum loop corrections.

The near horizon extreme Kerr geometry (NHEK) takes the form

(Bardeen, GH, 1999)

$$ds^2 = J\ A(\theta)\left[-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + d\theta^2 + B(\theta)(d\phi + \rho\, dt)^2\right]$$

We found the leading corrections to this solution coming from the R^3 and R^4 terms (preserving the SO(2,1) x U(1) symmetry).

Then studied what happens when stationary, axisymmetric perturbations are added. As before, they behave like ρ^γ near the horizon.

Without the R^3 or R^4 corrections, perturbations can be described in terms of Legendre polynomials, $P_\ell(\cos\theta)$, and $\gamma^{(0)}(\ell) = \ell$, starting with $\ell = 2$.

With the corrections, we find $\gamma = \gamma^{(0)} + \eta\,\gamma^{(6)} + \lambda\,\gamma^{(8)}$

with $\gamma^{(6)} = a/J^2 > 0$ and $\gamma^{(8)} = -\,b/J^3 < 0$

As before, if $\gamma < 2$, there are curvature singularities.

η gets contributions at one loop from massive particles: (Goon, 2016)

$\eta > 0$ for bosons and $\eta < 0$ for fermions

Proportional to $1/m^2$, so largest for the lightest particle. In our universe, that is the neutrino, so $\eta < 0$ which implies $\gamma < 2$.

Causality or unitarity both require $\lambda > 0$, so this also implies $\gamma < 2$.

(Gruzinov and Kleban, 2006; Remmen et al 2015)

So generic stationary, axisymmetric perturbations produce a singular extremal Kerr horizon.

The full asymptotically flat Kerr metric includes the $\ell = 2$ correction to the near horizon geometry.

So extreme Kerr develops a singular horizon when R^3 or R^4 corrections are included.

True even with very small coefficients!

Main Results

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