

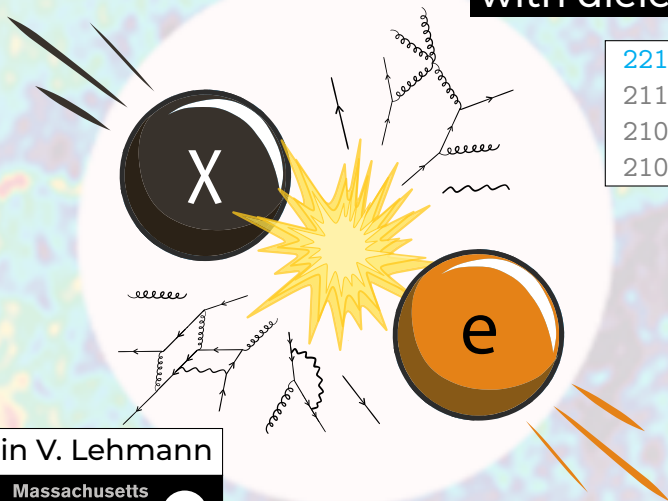
# New **directions** for direct detection with dielectrics

2212.04505

2110.01586

2109.04473

2101.08263



Benjamin V. Lehmann

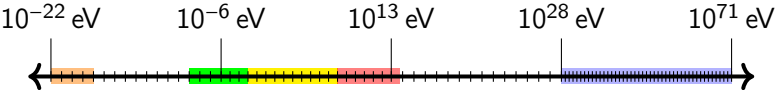


Massachusetts  
Institute of  
Technology

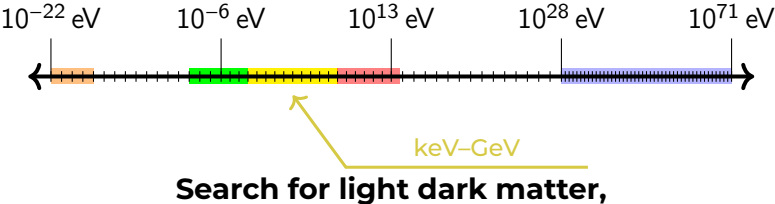


with Christian Boyd, Yonit Hochberg, Yoni Kahn, Eric David Kramer, Noah Kurinsky & To Chin Yu

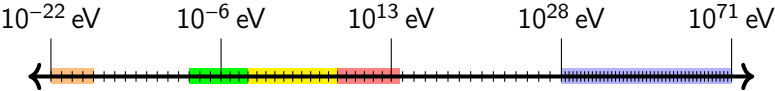
# This talk in one slide



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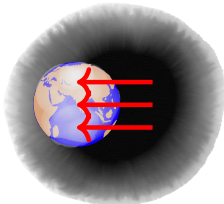
# This talk in one slide



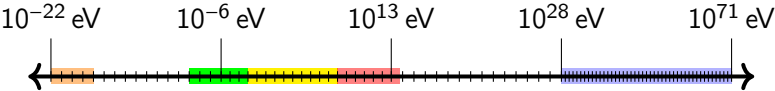
keV-GeV

**Search for light dark matter,  
directionally,**

background



# This talk in one slide

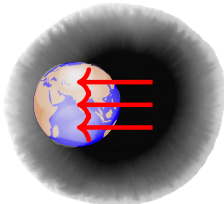


keV-GeV

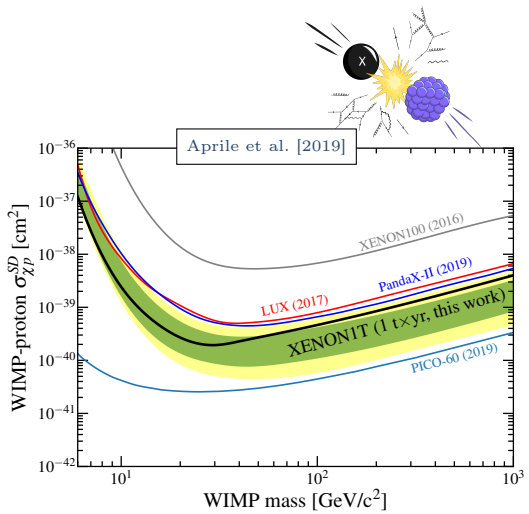
**Search for light dark matter,  
directionally, with anisotropic dielectrics.**

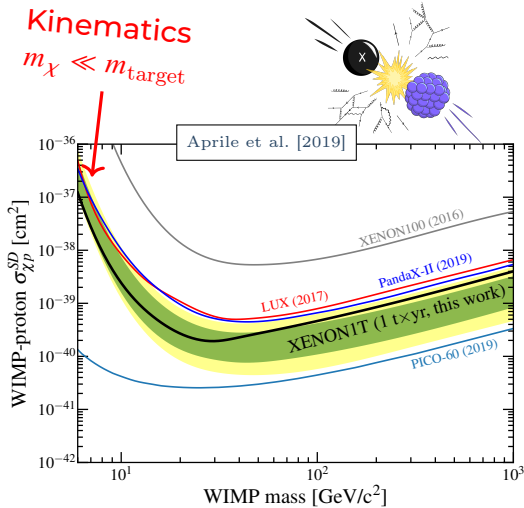
background

common stuff

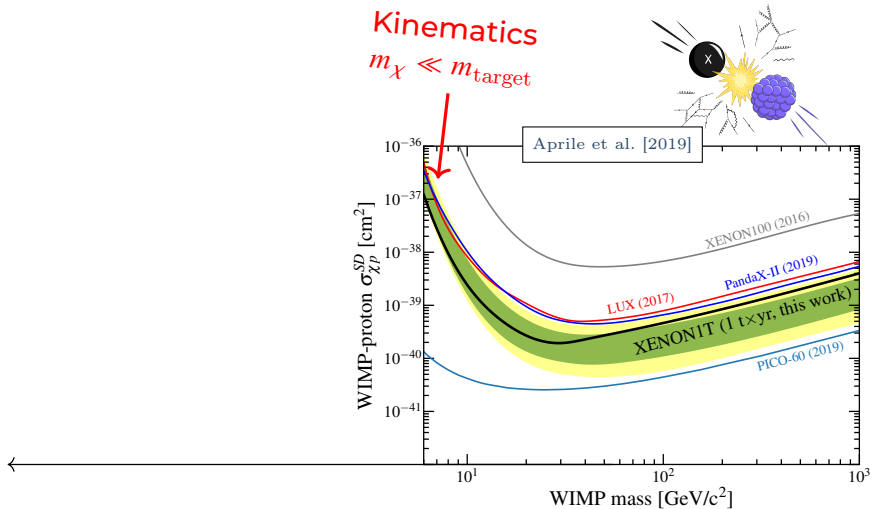


# Sub-GeV DM





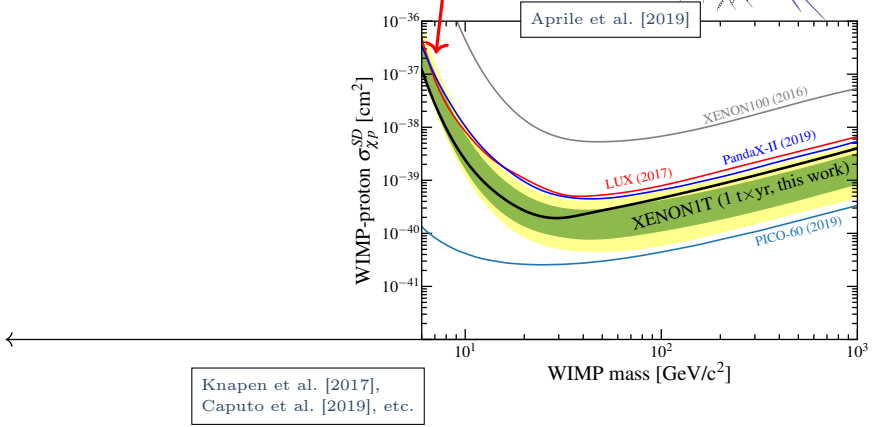
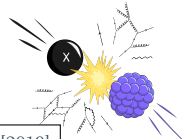
# Sub-GeV DM





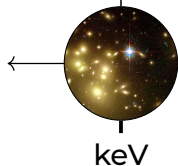
# Sub-GeV DM

Kinematics  
 $m_\chi \ll m_{\text{target}}$



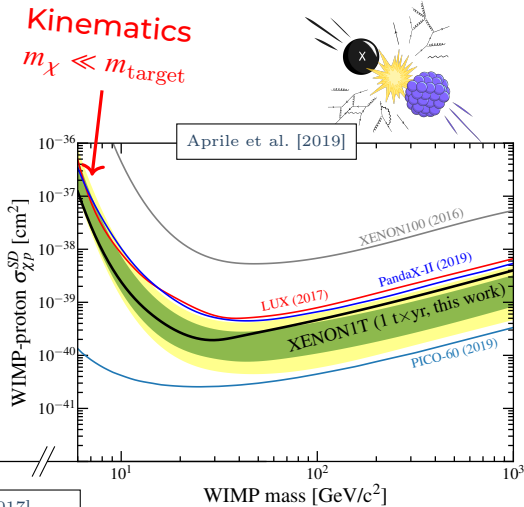
# Sub-GeV DM

Structure limits



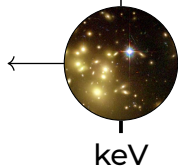
keV

Knapen et al. [2017],  
Caputo et al. [2019], etc.



# Sub-GeV DM

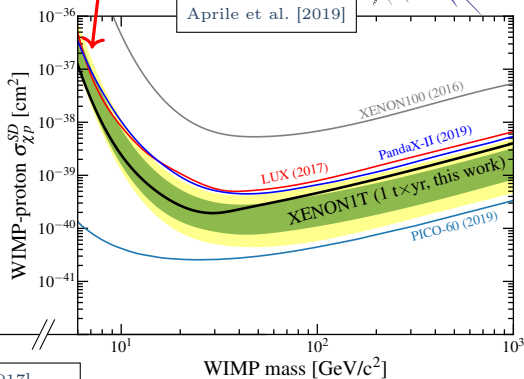
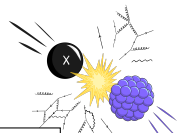
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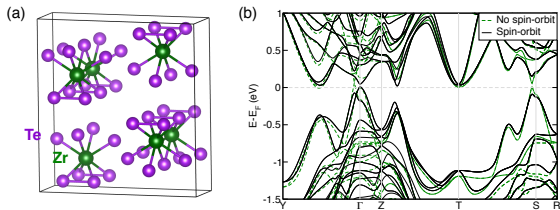
Light fermion  
( $e^-$  scattering)

Knapen et al. [2017],  
Caputo et al. [2019], etc.

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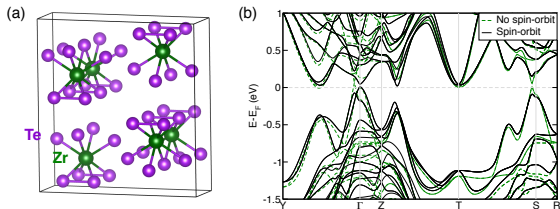
# All is not well with 1-particle language



Hochberg et al. [2018]

**DM does not interact with just one particle.**

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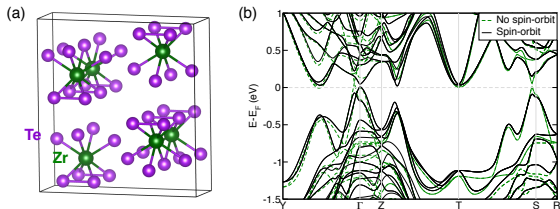


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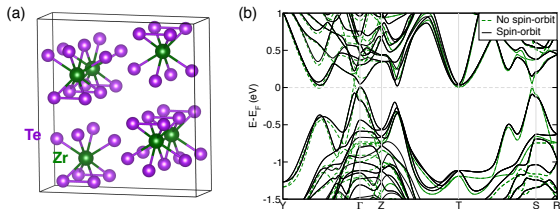


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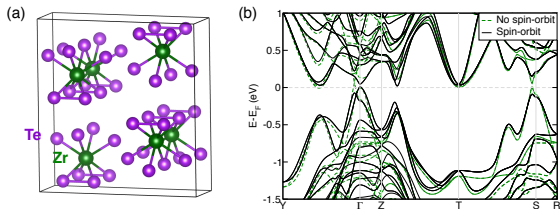
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$\Psi$  and  $\Psi'$  from material physics

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# DM scattering in dielectrics

Electrons are not free: **condensed matter matters**

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Predict scattering rate from **response function**

$$\Gamma = \int \frac{d^3\mathbf{q}}{(2\pi)^3} |V(\mathbf{q})|^2 \left[ \underbrace{2 \frac{q^2}{e^2} \text{Im} \left( -\frac{1}{\epsilon(\mathbf{q}, \omega_{\mathbf{q}})} \right)}_{\text{"Loss function" } \mathcal{W}} \right]$$

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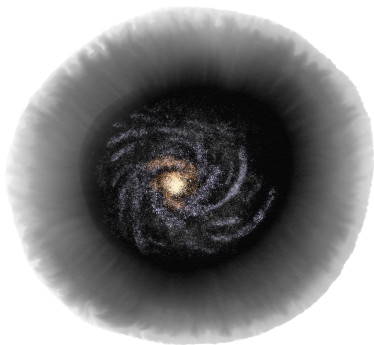
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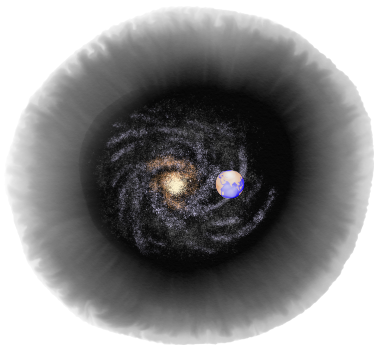
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- 4 **Inclusive:**  $\epsilon$  contains **all** collective modes

# Directional sensitivity

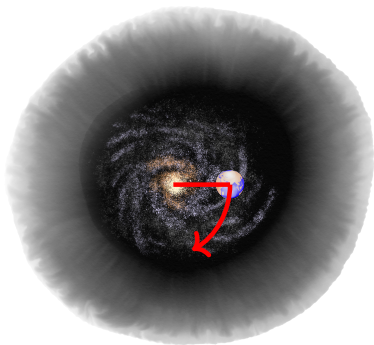


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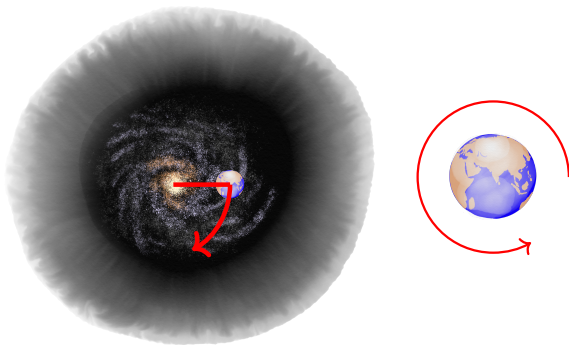




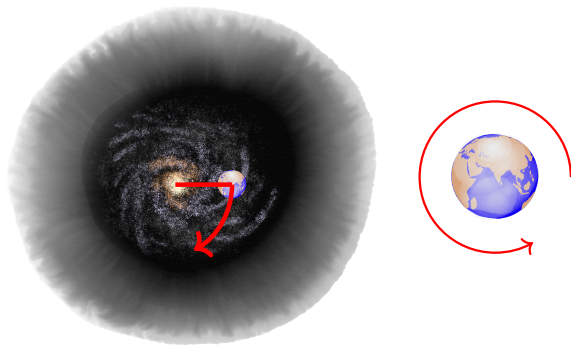
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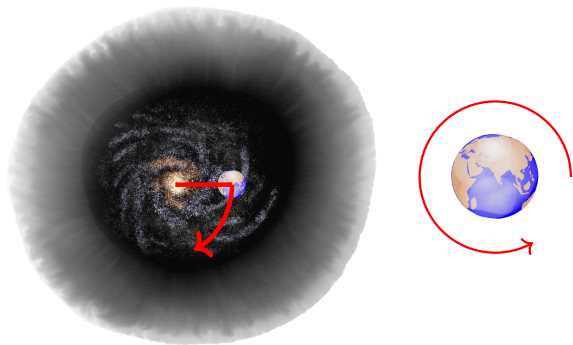


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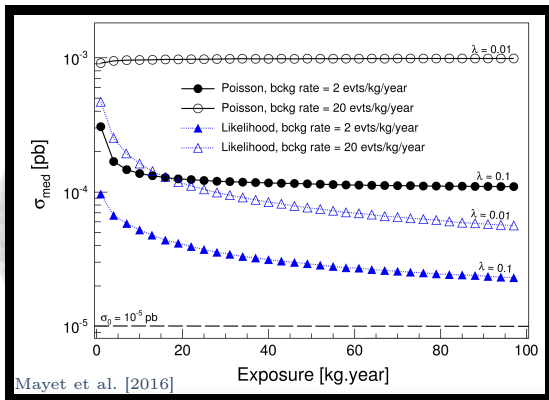
Anisotropic sensitivity  $\rightarrow$  daily modulation in rate

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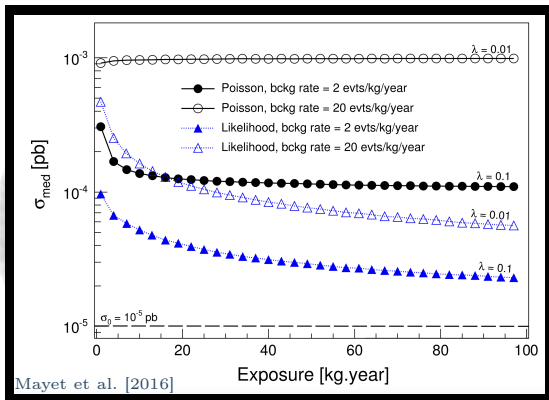
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**Cut through background:** scale with exposure

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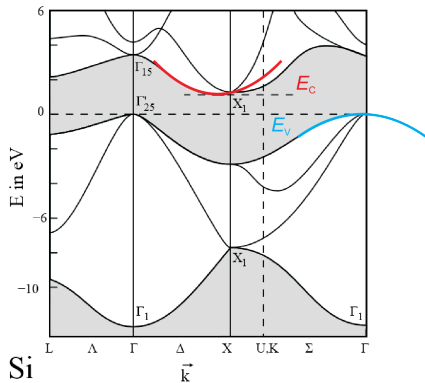
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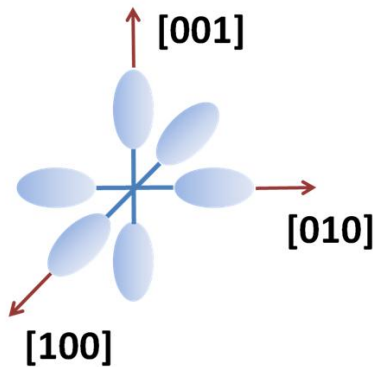
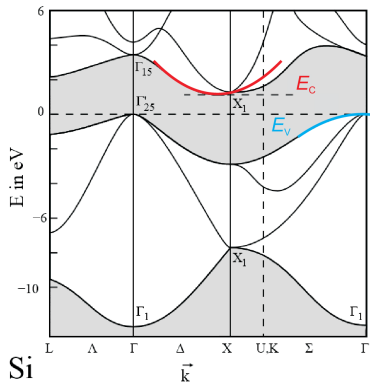
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An experimental challenge?

# New approach: anisotropic mass

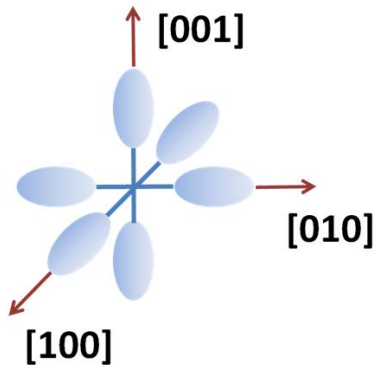
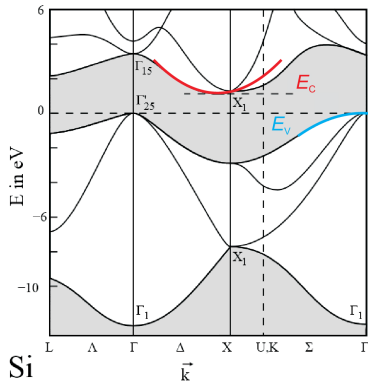


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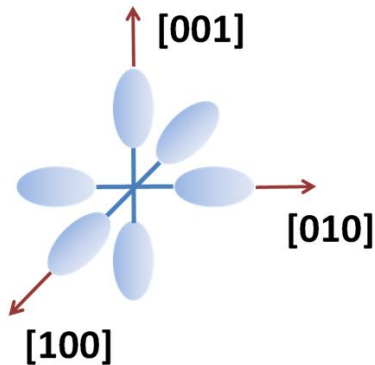
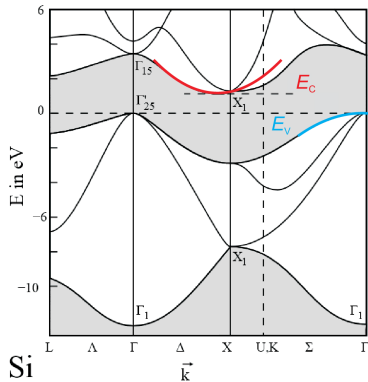
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Toy model: anisotropic  $m_e^*$

$$E_{\mathbf{q}} = \frac{q_x^2}{2m_x} + \frac{q_y^2}{2m_y} + \frac{q_z^2}{2m_z}$$

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What happens to  $\mathcal{W} = \text{Im} \left( -\frac{1}{\epsilon} \right)$ ?

# Technicalities

Where did  $\text{Im}(-1/\epsilon)$  come from?

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Material physics enters Golden Rule via dynamic structure factor

$$S(\mathbf{q}, \omega) = \frac{2\pi}{\text{vol}} \sum_f |\langle f | \hat{n}_{e^-}(-\mathbf{q}) | 0 \rangle|^2 \delta(\omega - [E_f - E_0])$$

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$$\chi(\mathbf{q}, \omega) = \frac{1}{V_{\text{Coul.}}(\mathbf{q})} \frac{1}{\epsilon(\mathbf{q}, \omega)}$$

See e.g. Mahan [2013]

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# Computing $\chi$

See e.g. Mahan [2013]

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**Random phase approximation (RPA)**

$$\chi_{\text{RPA}}(\mathbf{q}, \omega) = \sum_{(\text{geom.})} P^{(1)}(\mathbf{q}, \omega)$$



# The Lindhard function

$$P^{(1)}(\mathbf{q}, \omega) = \chi_0(\mathbf{q}, \omega) = 2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{n_{\text{FD}}(E_{\mathbf{p}+\mathbf{q}}) - n_{\text{FD}}(E_{\mathbf{p}})}{E_{\mathbf{p}+\mathbf{q}} - E_{\mathbf{p}} - \omega - i\Gamma}$$

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$$\epsilon_{\text{RPA}} \stackrel{T \rightarrow 0}{=} 1 + \frac{3\omega_{\text{p}}}{q^2 v_{\text{F}}} \left\{ \frac{1}{2} + \frac{k_{\text{F}}}{4q} \left[ 1 - \left( \frac{q}{2k_{\text{F}}} - \frac{\omega + i\Gamma}{qv_{\text{F}}} \right)^2 \right] \text{Log} \left( \frac{\frac{q}{2k_{\text{F}}} - \frac{\omega + i\Gamma}{qv_{\text{F}}} + 1}{\frac{q}{2k_{\text{F}}} - \frac{\omega + i\Gamma}{qv_{\text{F}}} - 1} \right) \right. \\ \left. + \frac{k_{\text{F}}}{4q} \left[ 1 - \left( \frac{q}{2k_{\text{F}}} + \frac{\omega + i\Gamma}{qv_{\text{F}}} \right)^2 \right] \text{Log} \left( \frac{\frac{q}{2k_{\text{F}}} + \frac{\omega + i\Gamma}{qv_{\text{F}}} + 1}{\frac{q}{2k_{\text{F}}} + \frac{\omega + i\Gamma}{qv_{\text{F}}} - 1} \right) \right\}$$

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Material parameters:

# The Lindhard function

$$P^{(1)}(\mathbf{q}, \omega) = \chi_0(\mathbf{q}, \omega) = 2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{n_{\text{FD}}(E_{\mathbf{p}+\mathbf{q}}) - n_{\text{FD}}(E_{\mathbf{p}})}{E_{\mathbf{p}+\mathbf{q}} - E_{\mathbf{p}} - \omega - i\Gamma}$$

→ Lindhard dielectric function

$$\epsilon_{\text{RPA}} \stackrel{T \rightarrow 0}{=} 1 + \frac{3\omega_p}{q^2 v_F} \left\{ \frac{1}{2} + \frac{k_F}{4q} \left[ 1 - \left( \frac{q}{2k_F} - \frac{\omega + i\Gamma}{qv_F} \right)^2 \right] \text{Log} \left( \frac{\frac{q}{2k_F} - \frac{\omega + i\Gamma}{qv_F} + 1}{\frac{q}{2k_F} - \frac{\omega + i\Gamma}{qv_F} - 1} \right) \right. \\ \left. + \frac{k_F}{4q} \left[ 1 - \left( \frac{q}{2k_F} + \frac{\omega + i\Gamma}{qv_F} \right)^2 \right] \text{Log} \left( \frac{\frac{q}{2k_F} + \frac{\omega + i\Gamma}{qv_F} + 1}{\frac{q}{2k_F} + \frac{\omega + i\Gamma}{qv_F} - 1} \right) \right\}$$

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plasma frequency  $\omega_p \sim O(1) \times E_F$ ,

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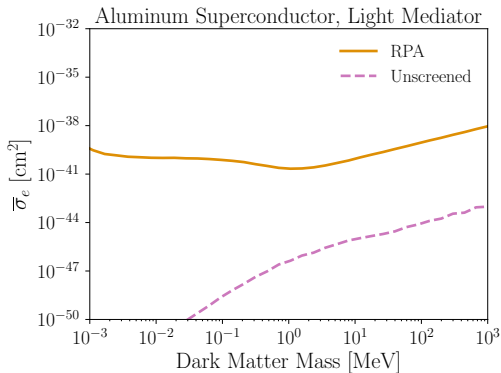
plasmon width  $\Gamma \sim \mathcal{O}(0.01-0.1) \times \omega_p$

# Understanding $\chi$ — screening

*Heuristically identical to E&M*

# Understanding $\chi$ — screening

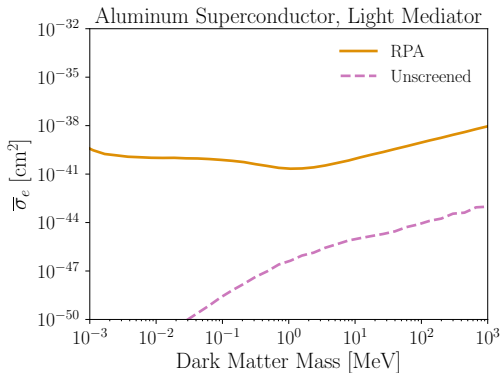
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# Understanding $\chi$ — screening

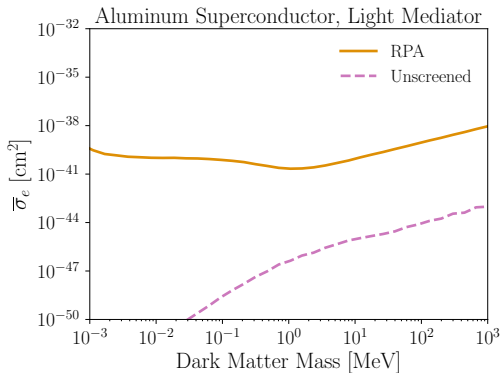
*Heuristically identical to E&M*



**Screening is a property of material response**

# Understanding $\chi$ — screening

*Heuristically identical to E&M*



**Screening is a property of material response**  
Scalar and vector interactions are screened identically

# Understanding $\chi$ — plasmons

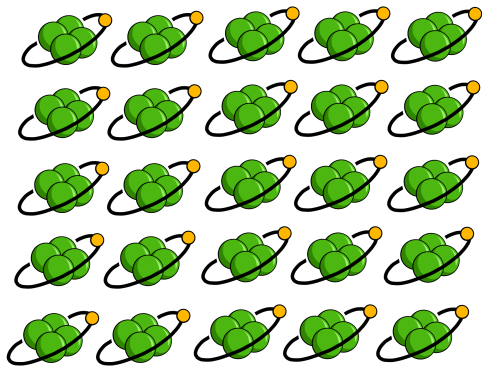
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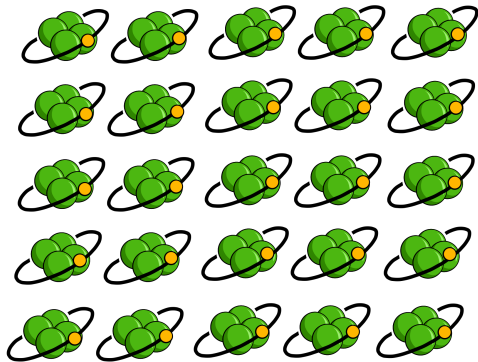


# Understanding $\chi$ — plasmons



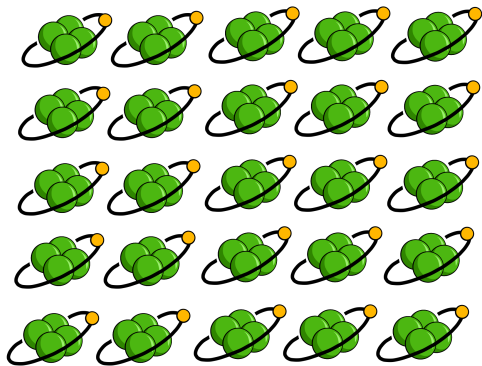
A **collective oscillation** of electrons

# Understanding $\chi$ — plasmons



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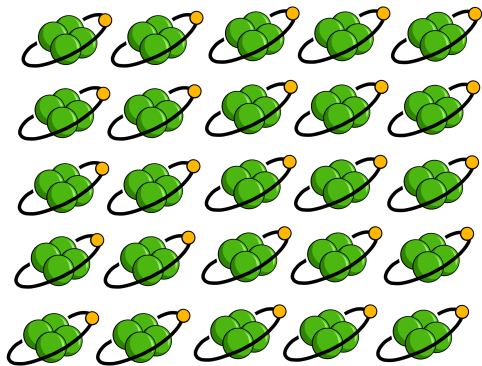
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A **collective oscillation** of electrons



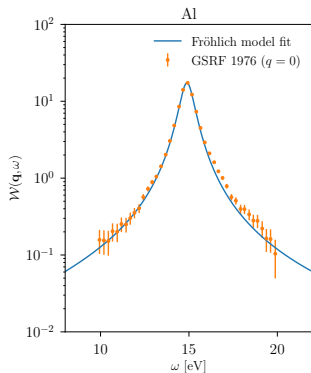
# Understanding $\chi$ — plasmons



A **collective oscillation** of electrons

Shows up as a resonance in the **loss function**

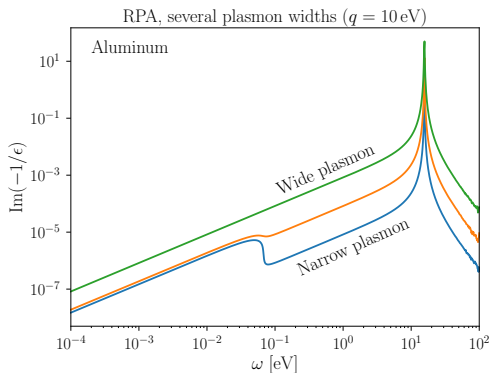
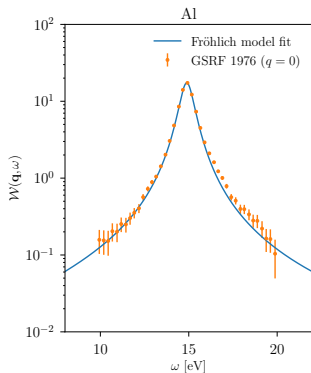
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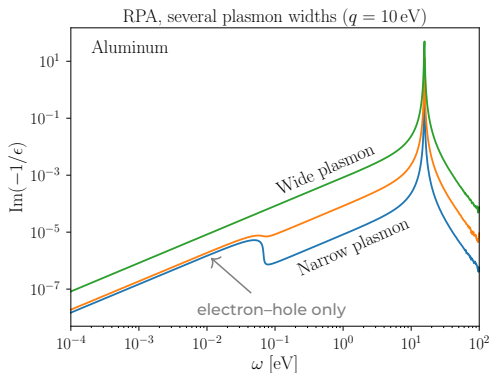
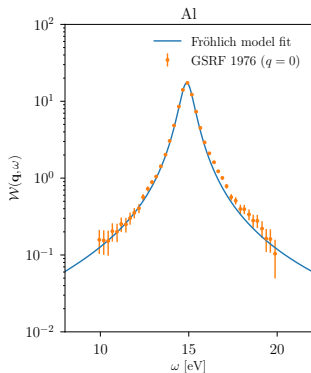
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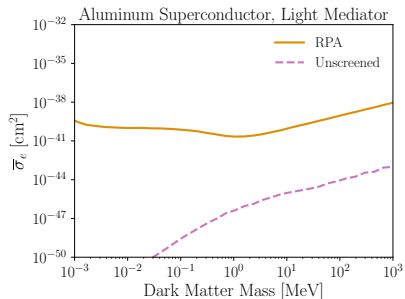


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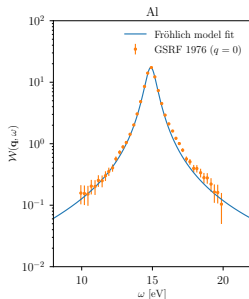
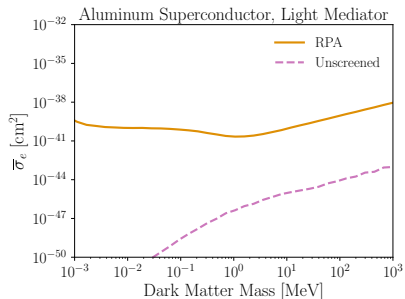
# Understanding $\chi$ — Lindhard's lessons

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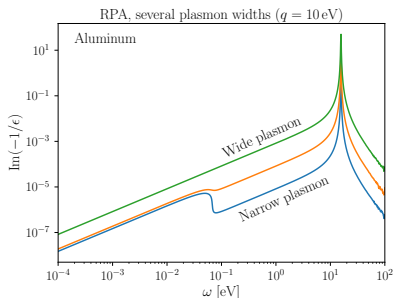
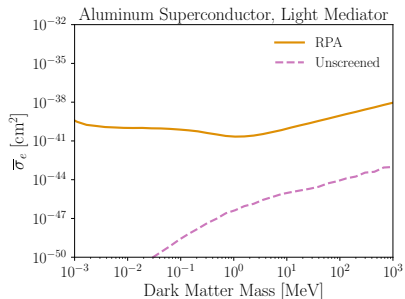
- 1 Screening is generic, not model-dependent

# Understanding $\chi$ — Lindhard's lessons



- 1 Screening is generic, not model-dependent
- 2 Resonances (plasmons) enhance the scattering rate

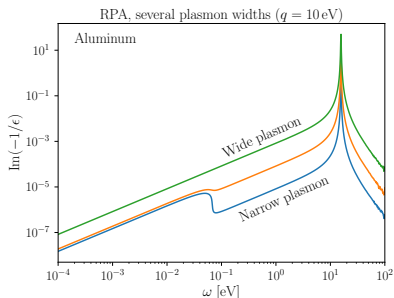
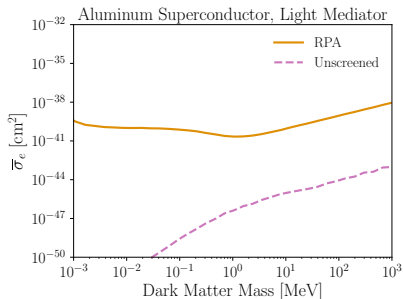
# Understanding $\chi$ — Lindhard's lessons



- 1 Screening is generic, not model-dependent
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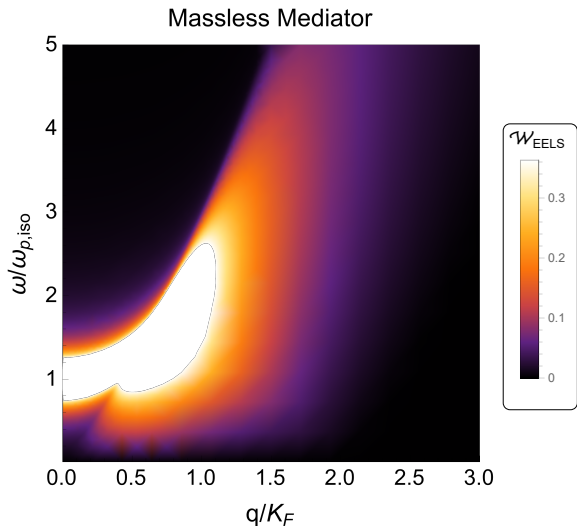
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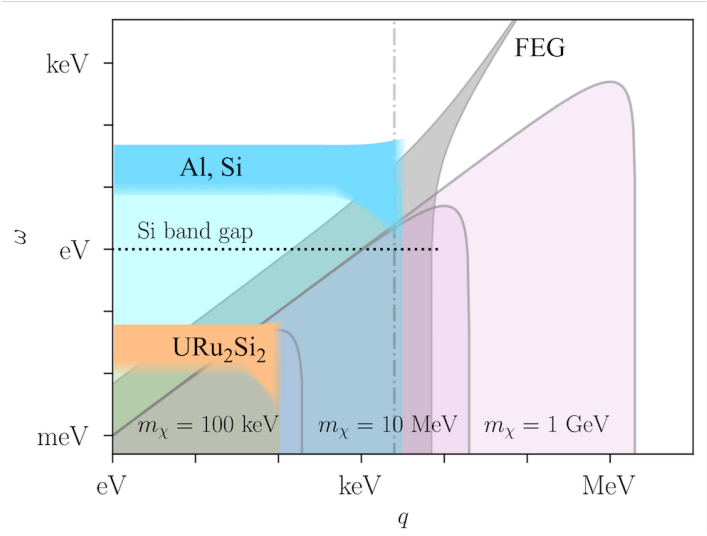
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We can use analytical forms of  $\epsilon$  for heuristics

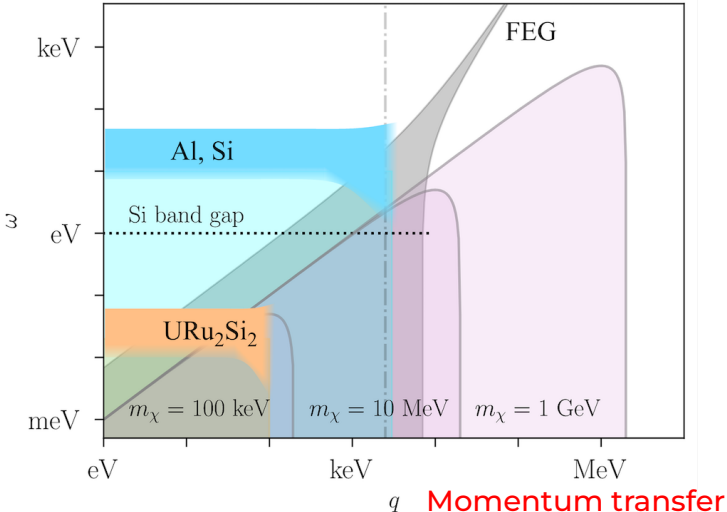
# Visualizing the loss function



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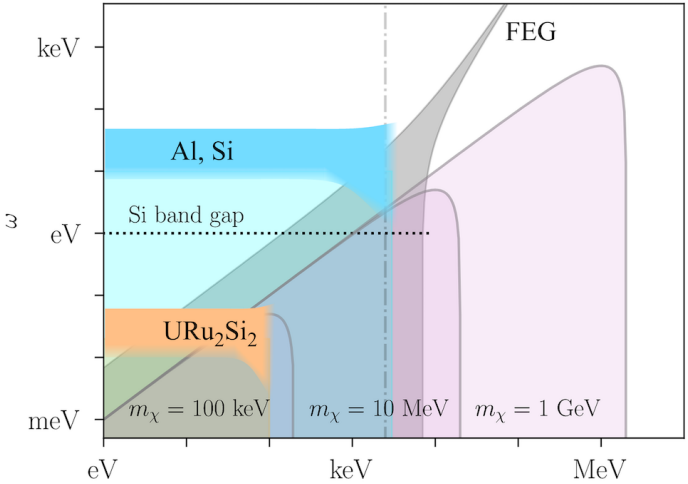


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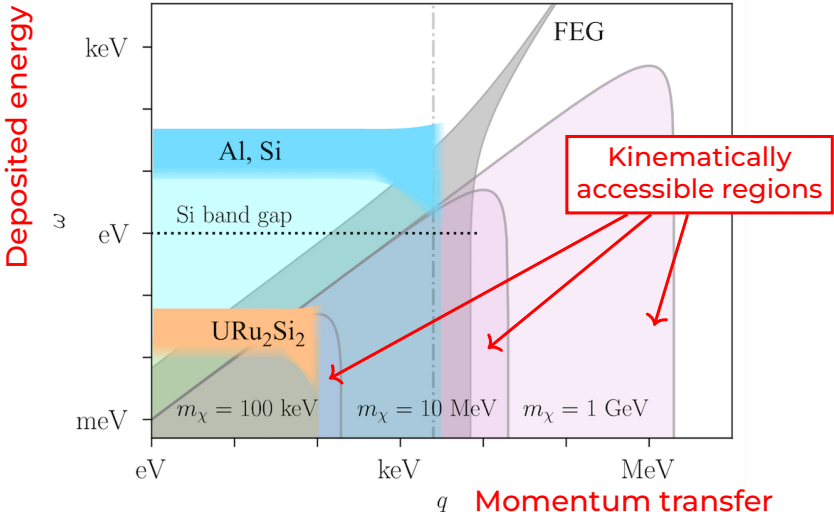
# Visualizing the loss function

Deposited energy

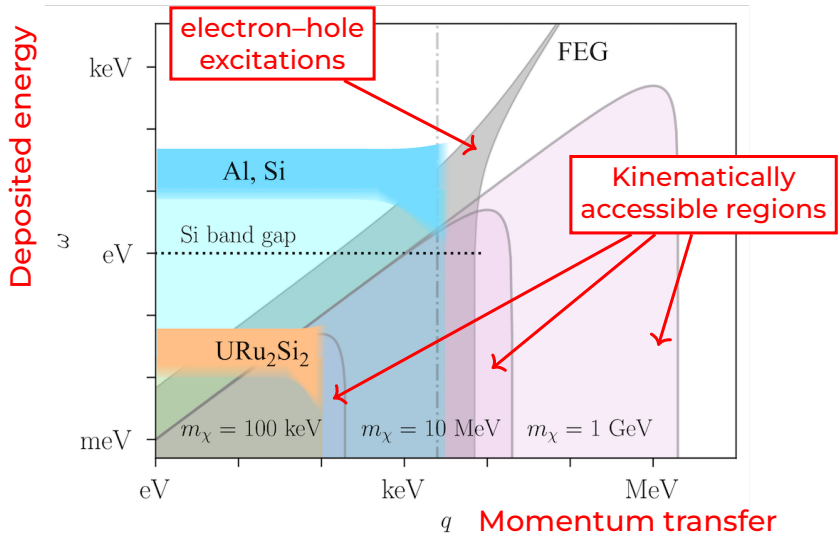


$q$  Momentum transfer

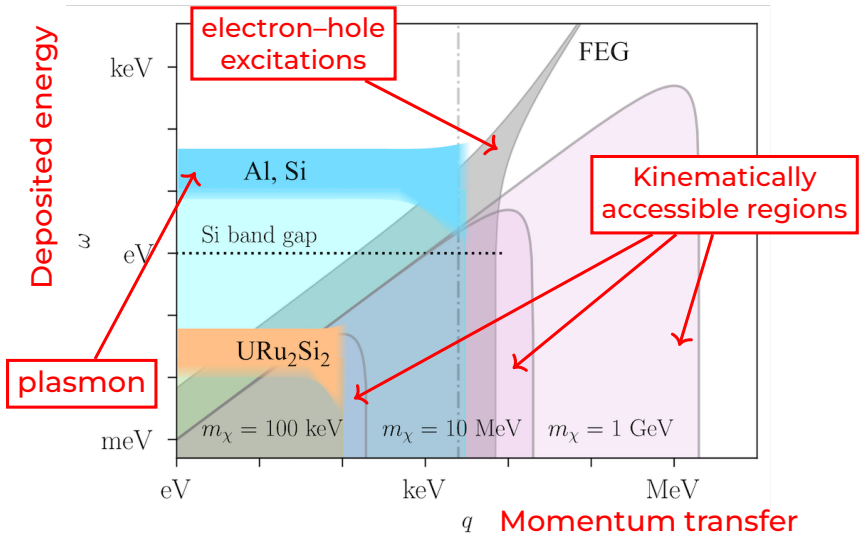
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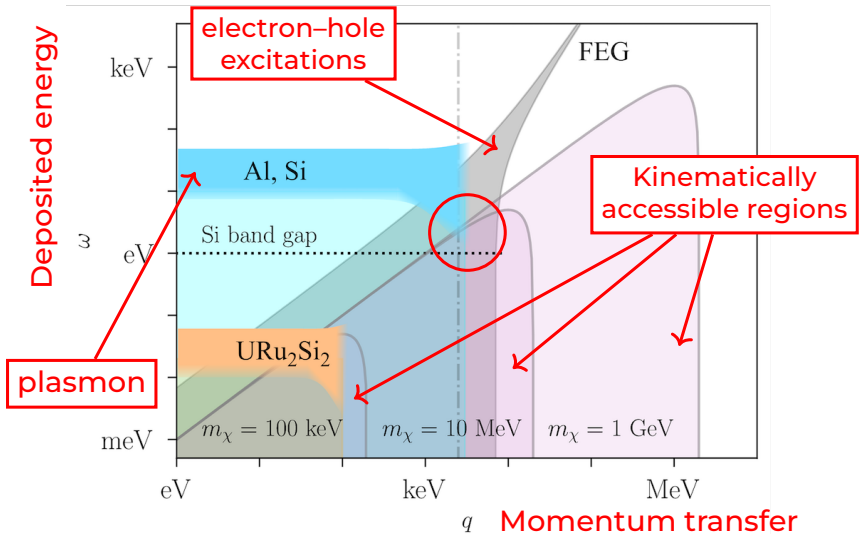


# Visualizing the loss function





# Visualizing the loss function



# Maximizing the rate

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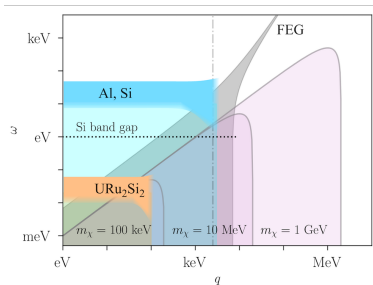
Small  $q$ : Can DM hit the plasmon peak?

$$(\omega \sim \omega_p)$$

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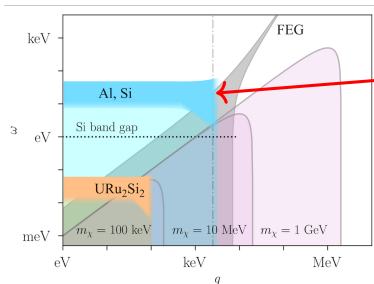
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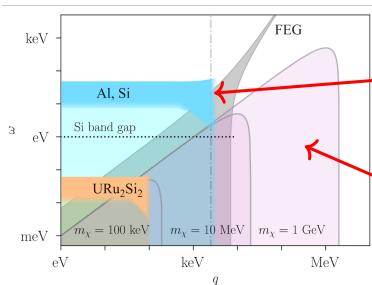
plasmon trails off

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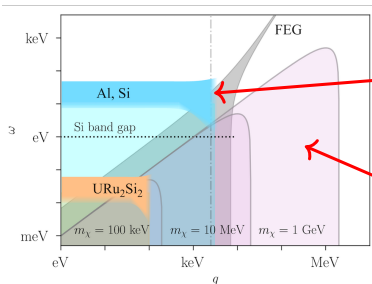
$$\omega < qv_\chi$$

kinematics

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Small  $q$ : Can DM hit the plasmon peak?

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plasmon trails off

$$q_c \approx \omega_p / v_F$$

Only possible if  $v_F \lesssim v_\chi$

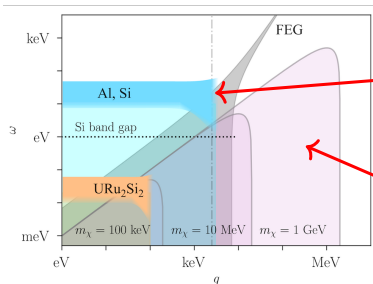
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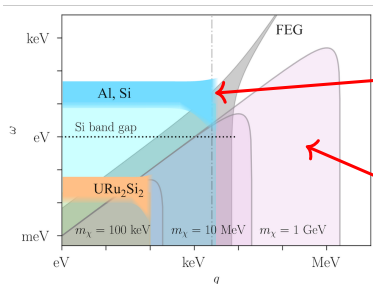
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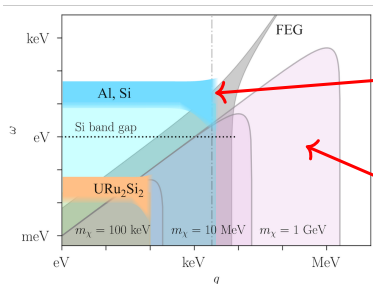
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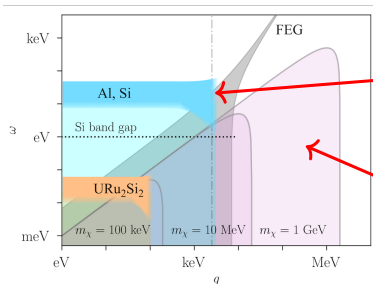
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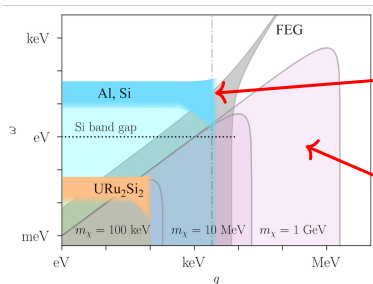
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**Find a target with a low Fermi velocity**

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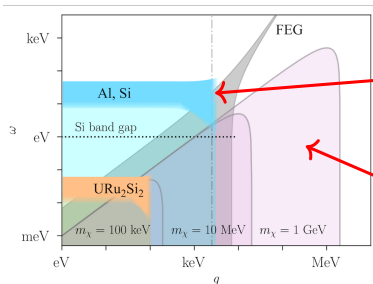
**Find a target with a low Fermi velocity**

*Dirac materials*

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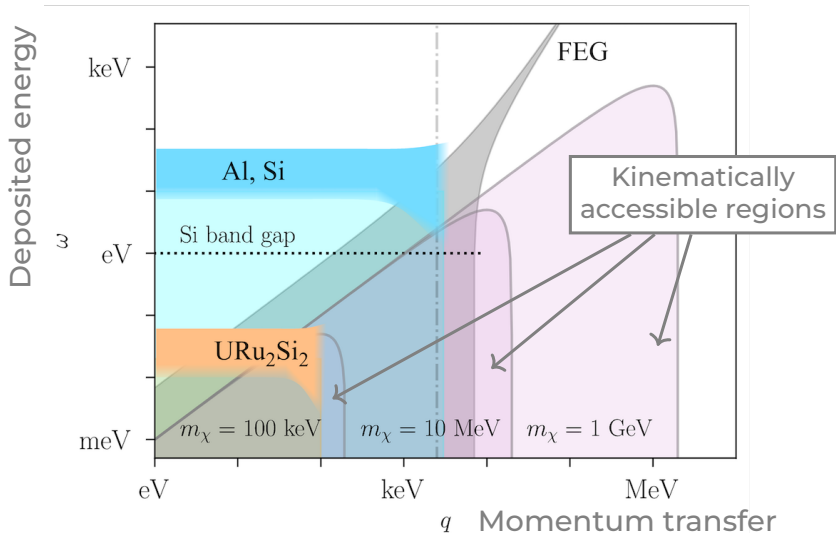
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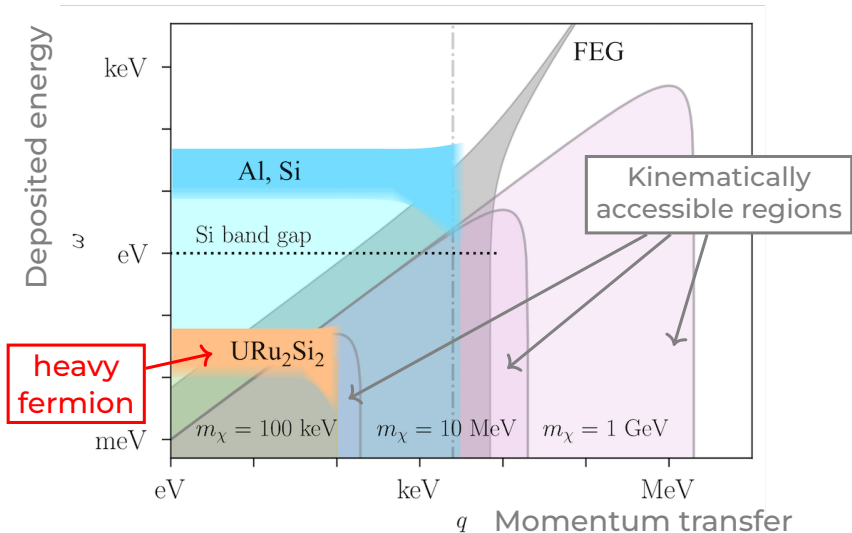
*Dirac materials*

*Heavy-fermion materials*

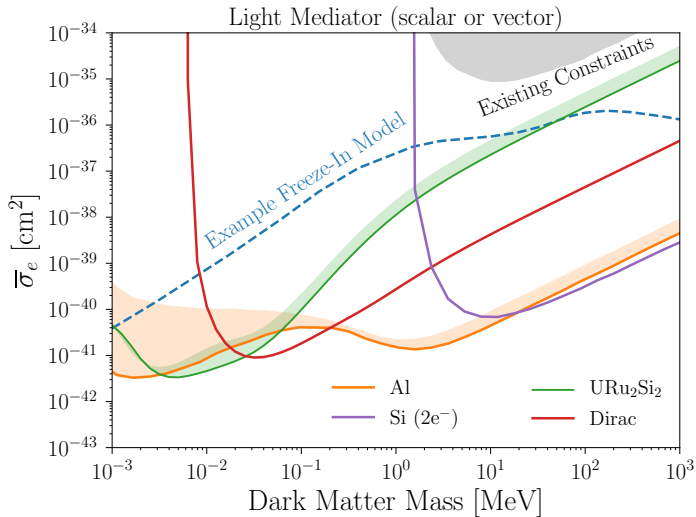
# Heavy fermion materials



# Heavy fermion materials



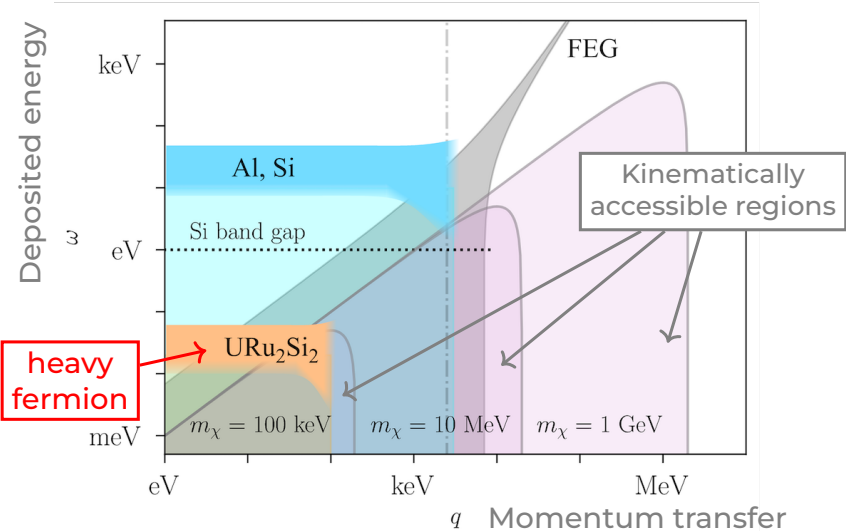
# Projected reach



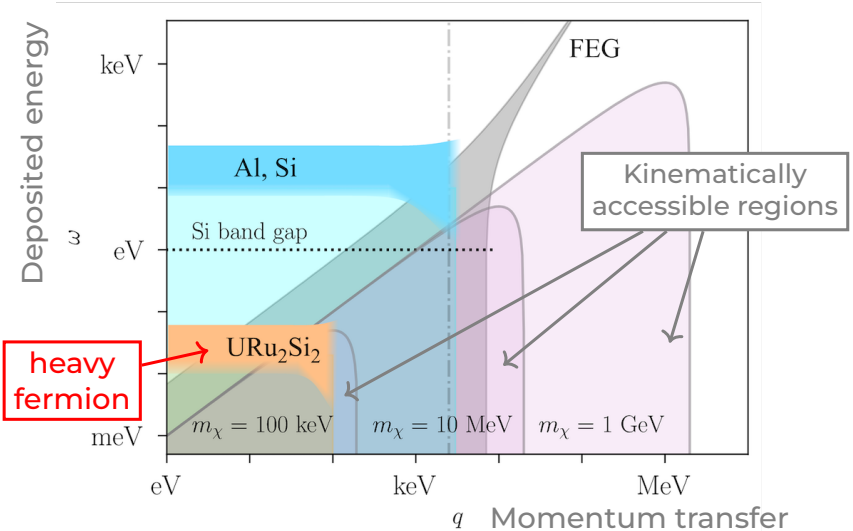


Back to anisotropy

# An anisotropic plasmon?

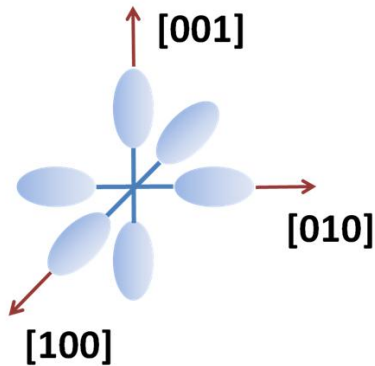
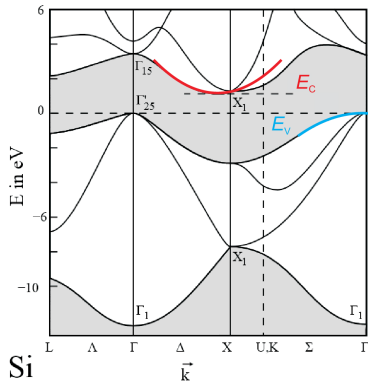


# An anisotropic plasmon?



**Heavy fermion in only one direction?**

# New approach: anisotropic mass



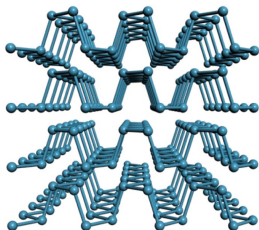
Toy model: anisotropic  $m_e^*$

$$E_{\mathbf{q}} = \frac{q_x^2}{2m_x} + \frac{q_y^2}{2m_y} + \frac{q_z^2}{2m_z}$$

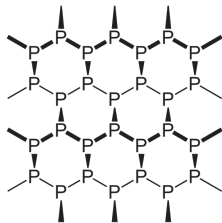
What happens to  $\mathcal{W} = \text{Im} \left( -\frac{1}{\epsilon} \right)$ ?



## Black phosphorus

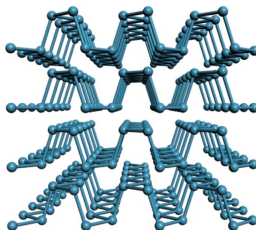


$$m_{\text{heavy}}/m_{\text{light}} \approx 9$$

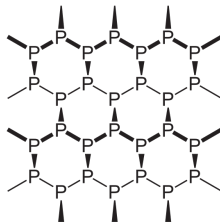


# Real materials

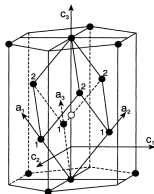
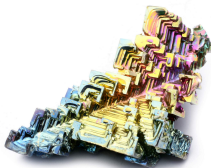
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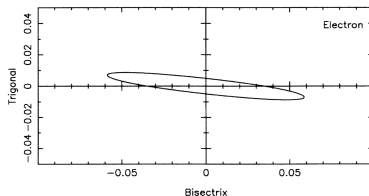
$$m_{\text{heavy}}/m_{\text{light}} \approx 9$$



## Bismuth



$$m_{\text{heavy}}/m_{\text{light}} \approx 230$$



**Isotropic case.**



**Isotropic case.** 
$$\chi_{\text{RPA}}^{\text{iso}} = \sum_{(\text{geom.})} P^{(1)} = \frac{\chi_0^{\text{iso}}(\mathbf{q}, \omega)}{1 - (e^2/q^2)\chi_0^{\text{iso}}(\mathbf{q}, \omega)}$$

**Isotropic case.**  $\chi_{\text{RPA}}^{\text{iso}} = \sum_{(\text{geom.})} P^{(1)} = \frac{\chi_0^{\text{iso}}(\mathbf{q}, \omega)}{1 - (e^2/q^2)\chi_0^{\text{iso}}(\mathbf{q}, \omega)}$

$$\chi_0^{\text{iso}}(\mathbf{q}, \omega) = 2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{n_{\text{FD}}(E_{\mathbf{p}+\mathbf{q}}^{\text{iso}}) - n_{\text{FD}}(E_{\mathbf{p}}^{\text{iso}})}{E_{\mathbf{p}+\mathbf{q}}^{\text{iso}} - E_{\mathbf{p}}^{\text{iso}} - \omega - i\delta}$$

**Isotropic case.**  $\chi_{\text{RPA}}^{\text{iso}} = \sum_{(\text{geom.})} P^{(1)} = \frac{\chi_0^{\text{iso}}(\mathbf{q}, \omega)}{1 - (e^2/q^2)\chi_0^{\text{iso}}(\mathbf{q}, \omega)}$

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**Anisotropic case.**  $E_{\mathbf{q}}^{\text{iso}} = \frac{q^2}{2m} \longrightarrow E_{\mathbf{q}}^{\text{ani}} = \frac{q_x^2}{2m_x} + \frac{q_y^2}{2m_y} + \frac{q_z^2}{2m_z}$

# Anisotropic response function

**Isotropic case.**  $\chi_{\text{RPA}}^{\text{iso}} = \sum_{(\text{geom.})} P^{(1)} = \frac{\chi_0^{\text{iso}}(\mathbf{q}, \omega)}{1 - (e^2/q^2)\chi_0^{\text{iso}}(\mathbf{q}, \omega)}$

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Transform back to isotropic in  $k$ -space

# Anisotropic response function

**Isotropic case.**  $\chi_{\text{RPA}}^{\text{iso}} = \sum_{(\text{geom.})} P^{(1)} = \frac{\chi_0^{\text{iso}}(\mathbf{q}, \omega)}{1 - (e^2/q^2)\chi_0^{\text{iso}}(\mathbf{q}, \omega)}$

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**Anisotropic case.**  $E_{\mathbf{q}}^{\text{iso}} = \frac{q^2}{2m} \longrightarrow E_{\mathbf{q}}^{\text{ani}} = \frac{q_x^2}{2m_x} + \frac{q_y^2}{2m_y} + \frac{q_z^2}{2m_z}$

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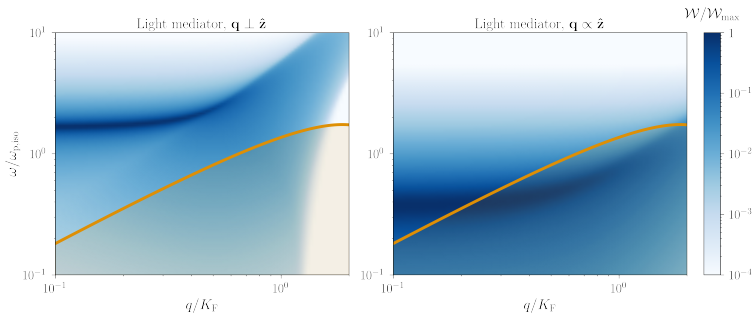
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$$\chi_0^{\text{iso}}(\mathbf{q}, \omega) \longrightarrow \chi_0^{\text{ani}}(\mathbf{q}, \omega) = \chi_0^{\text{iso}}(\mathbf{Q}(\mathbf{q}), \omega)$$

# Anisotropic loss function

$$m_z/m_{xy} = 20, \quad m_x m_y m_z = m_e^3$$

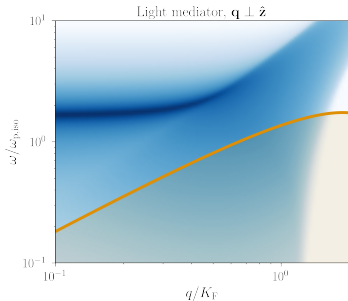


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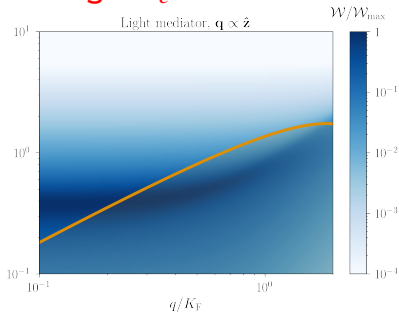
$$m_z/m_{xy} = 20,$$

$$m_x m_y m_z = m_e^3$$

Low- $m_e$  direction



High- $m_e$  direction



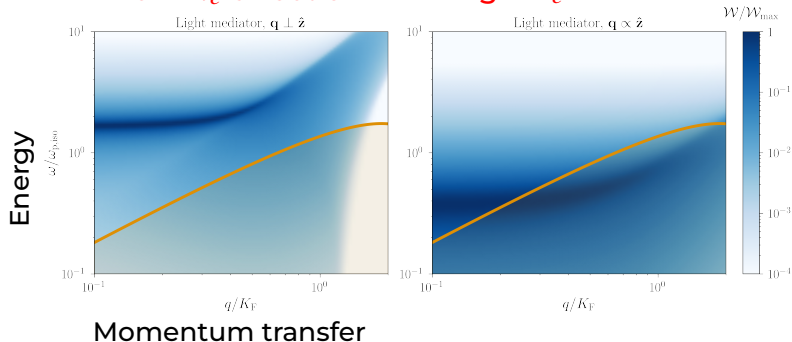
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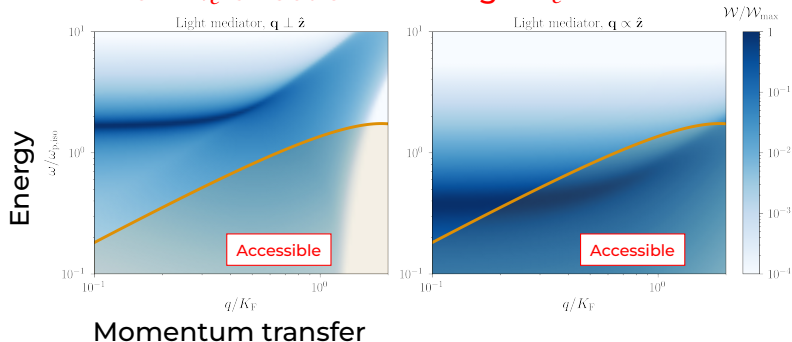
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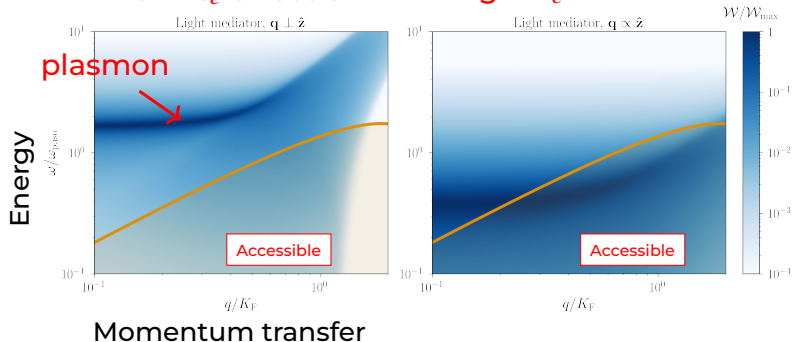
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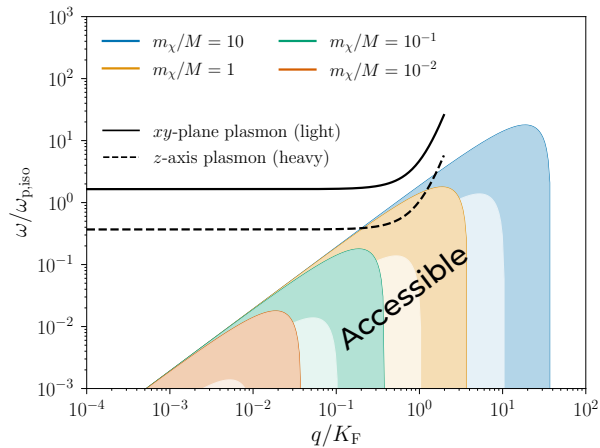
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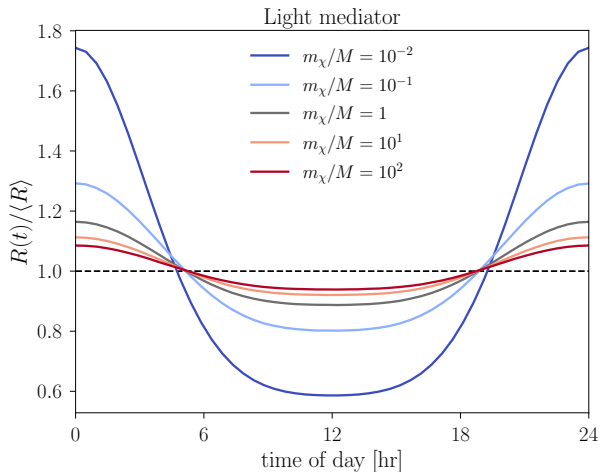
High- $m_e$  direction



# Anisotropic plasmon threshold



# Daily modulation in the rate

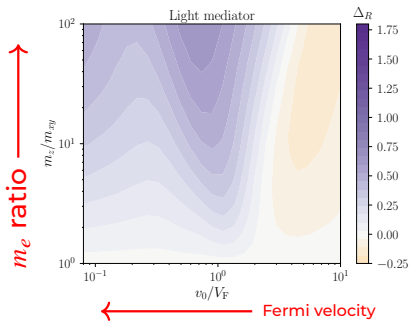




$$\Delta_R = \frac{R(\text{midnight}) - R(\text{noon})}{\langle R \rangle}$$

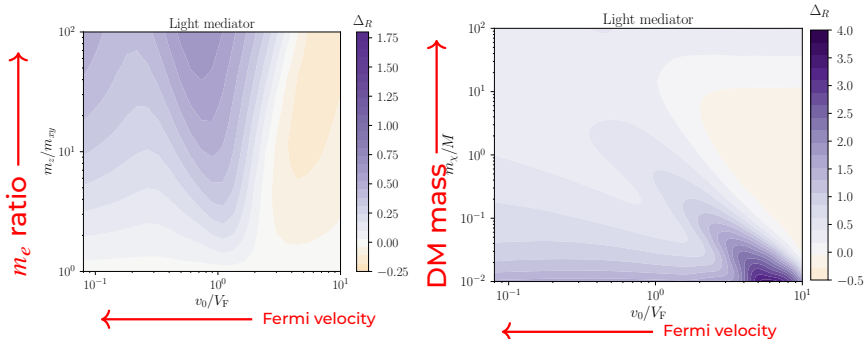
# Modulation features

$$\Delta_R = \frac{R(\text{midnight}) - R(\text{noon})}{\langle R \rangle}$$

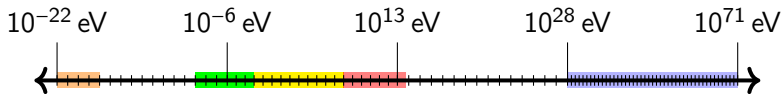


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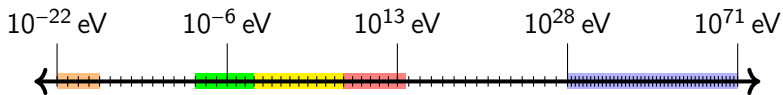


# Conclusions



**Search for light dark matter,  
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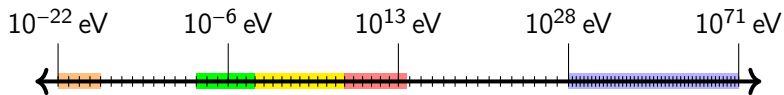
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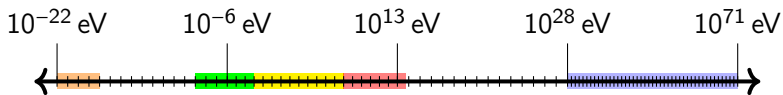
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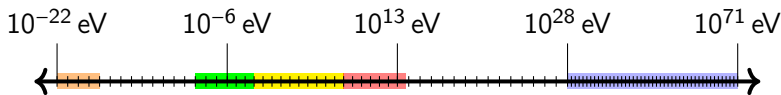
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