Probing turbulence in core-collapse supernovae: prospects for upcoming neutrino detectors

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Pennsylvania State University

Neutrinos in Cosmology and Astrophysics
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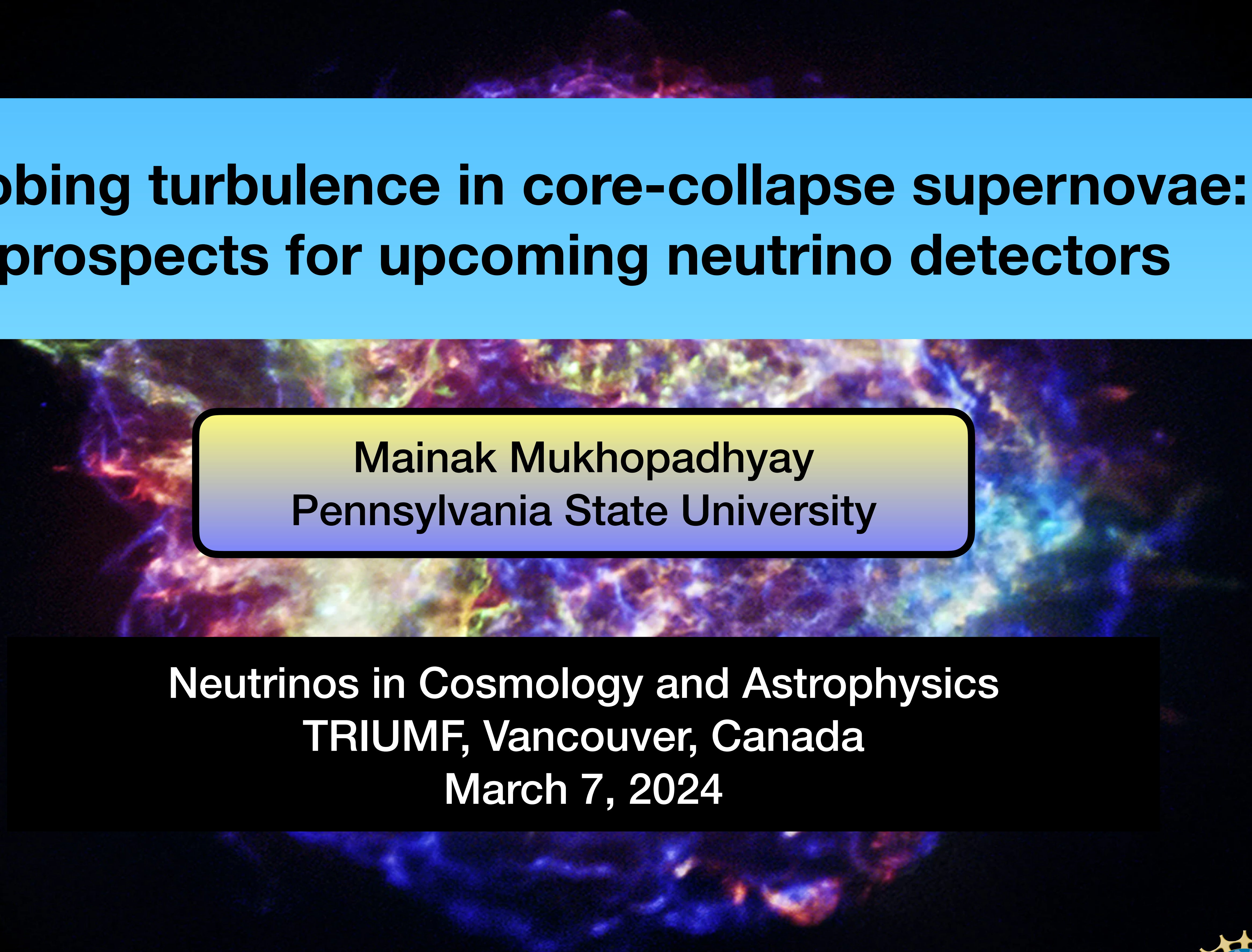


Photo credits: D. Milisavljevic and R.Fesen

On the menu

Hors D'Oeuvres

Review:

CCSNe
Neutrinos from CCSNe

Appetizer

Some details:

Shocks in CCSNe
Neutrino signatures of shock wave propagation

Main course

Questions:

Does the double dip survive in the presence of turbulence?
Will the upcoming neutrino detectors (DUNE and Hyper-K) be able to constrain turbulence in CCSNe?

Dessert

Caveats
Questions

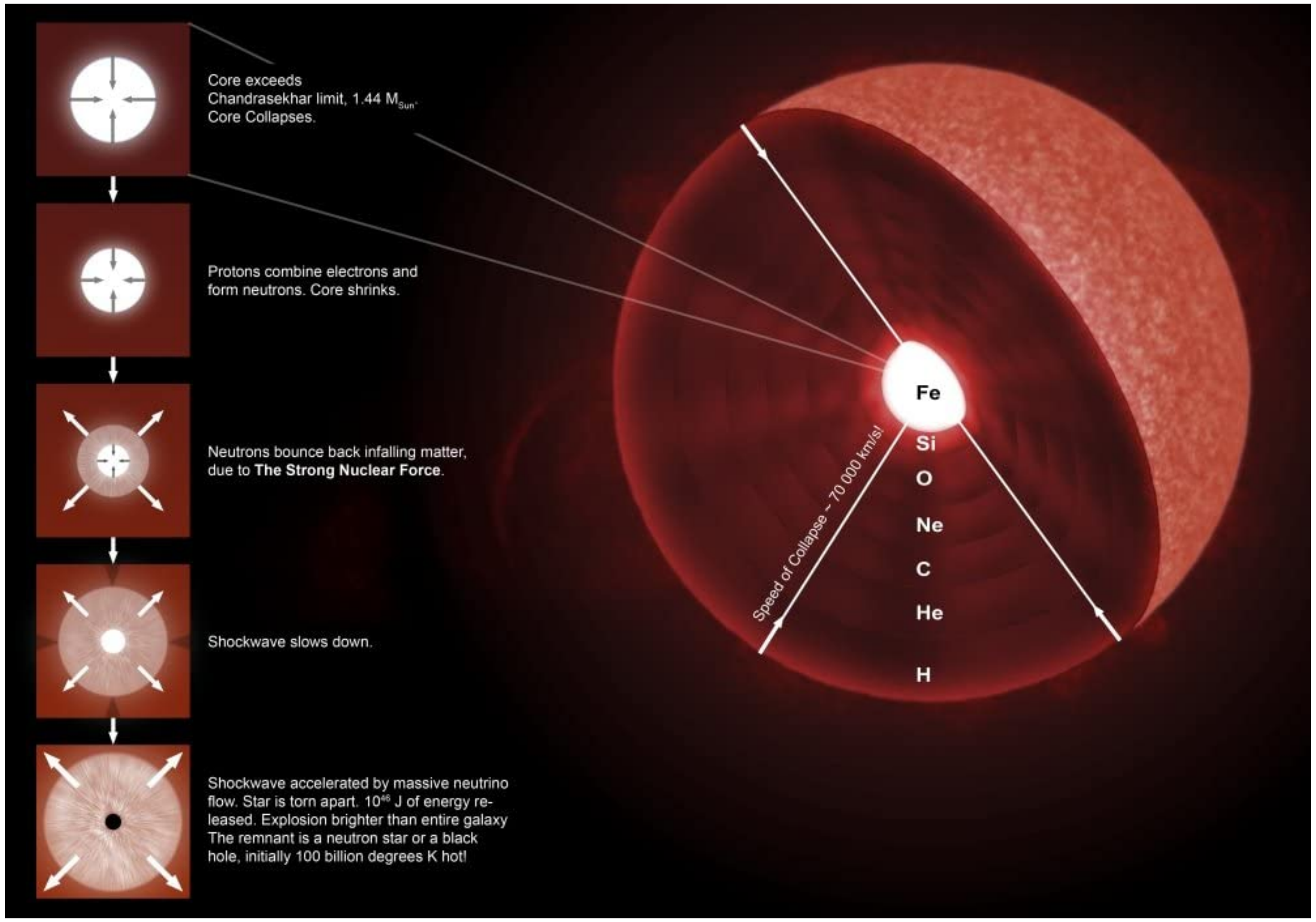
Based on

On probing turbulence in core-collapse supernovae in upcoming neutrino detectors

MM, M. Sen (arXiv: 2310.08627).
(accepted for publication in JCAP).

Core-collapse supernovae (CCSNe)

CCSN: death of a massive (>10 M_{sun}) star



Red or Blue supergiants: advanced stages of nuclear burning

↓

Fe core: Fusion turns off: loss of pressure

↓

Core collapses

↓

Collapsed core: very dense (nuclear densities): Incompressible

↓

Infalling matter bounces off: Shockwave produced

↓

Shockwave stalls

Shockwave re-energized ← Shockwave stalls

Star explodes: Supernova
Neutron star forming collapse (NSFC) ← Shockwave re-energized

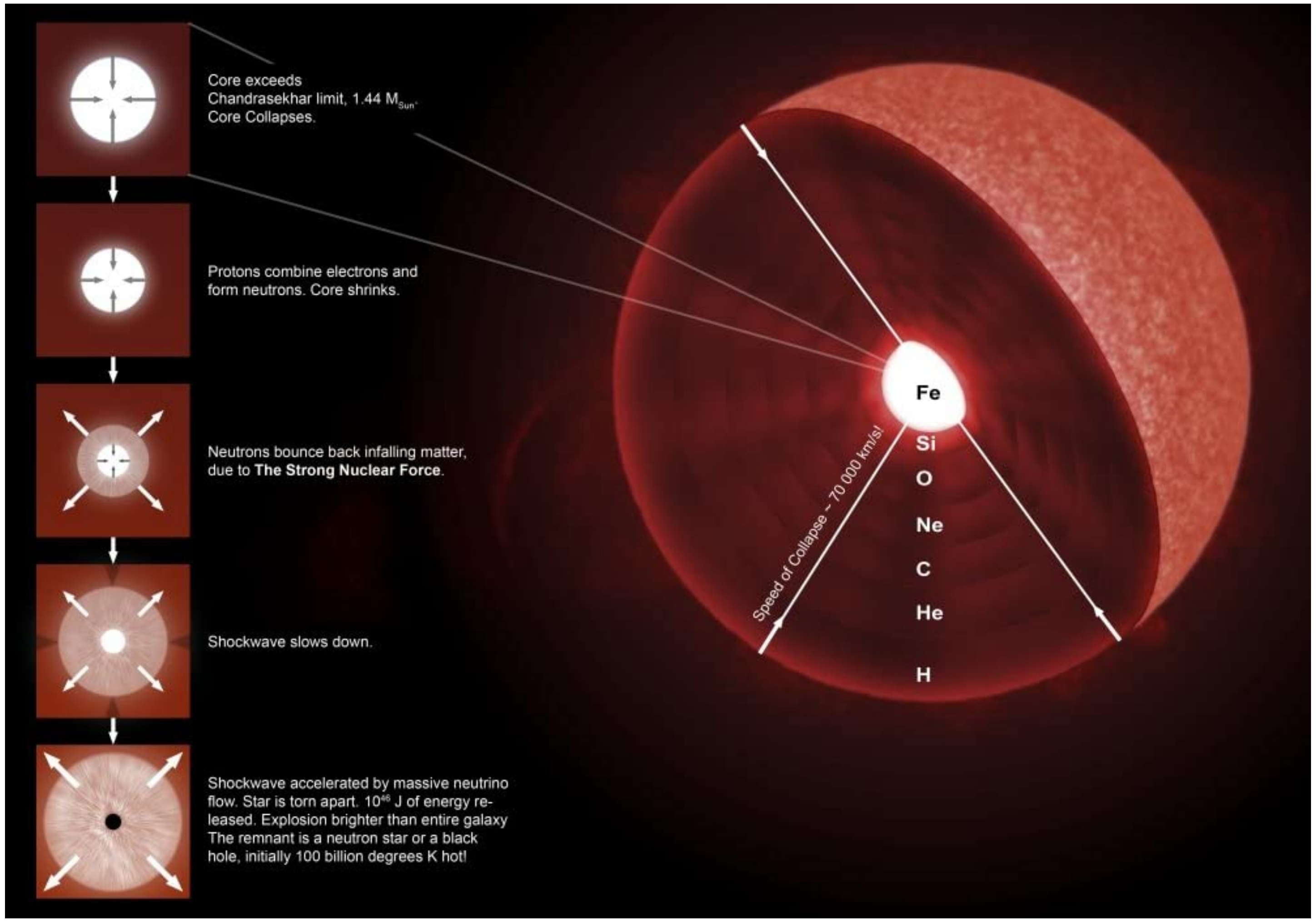
Shockwave dies down ← Shockwave stalls

Failed Supernova
Black hole forming collapse (BHFC) ← Shockwave dies down

Credits: https://images-na.ssl-images-amazon.com/images/I/61yf26rpIXL._AC_SL1000_.jpg

Core-collapse supernovae (CCSNe)

CCSN: death of a massive (>10 M_{sun}) star



Red or Blue supergiants: advanced stages of nuclear burning

↓

Fe core: Fusion turns off: loss of pressure

↓

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↓

Collapsed core: very dense (nuclear densities): Incompressible

↓

Infalling matter bounces off: Shockwave produced

↓

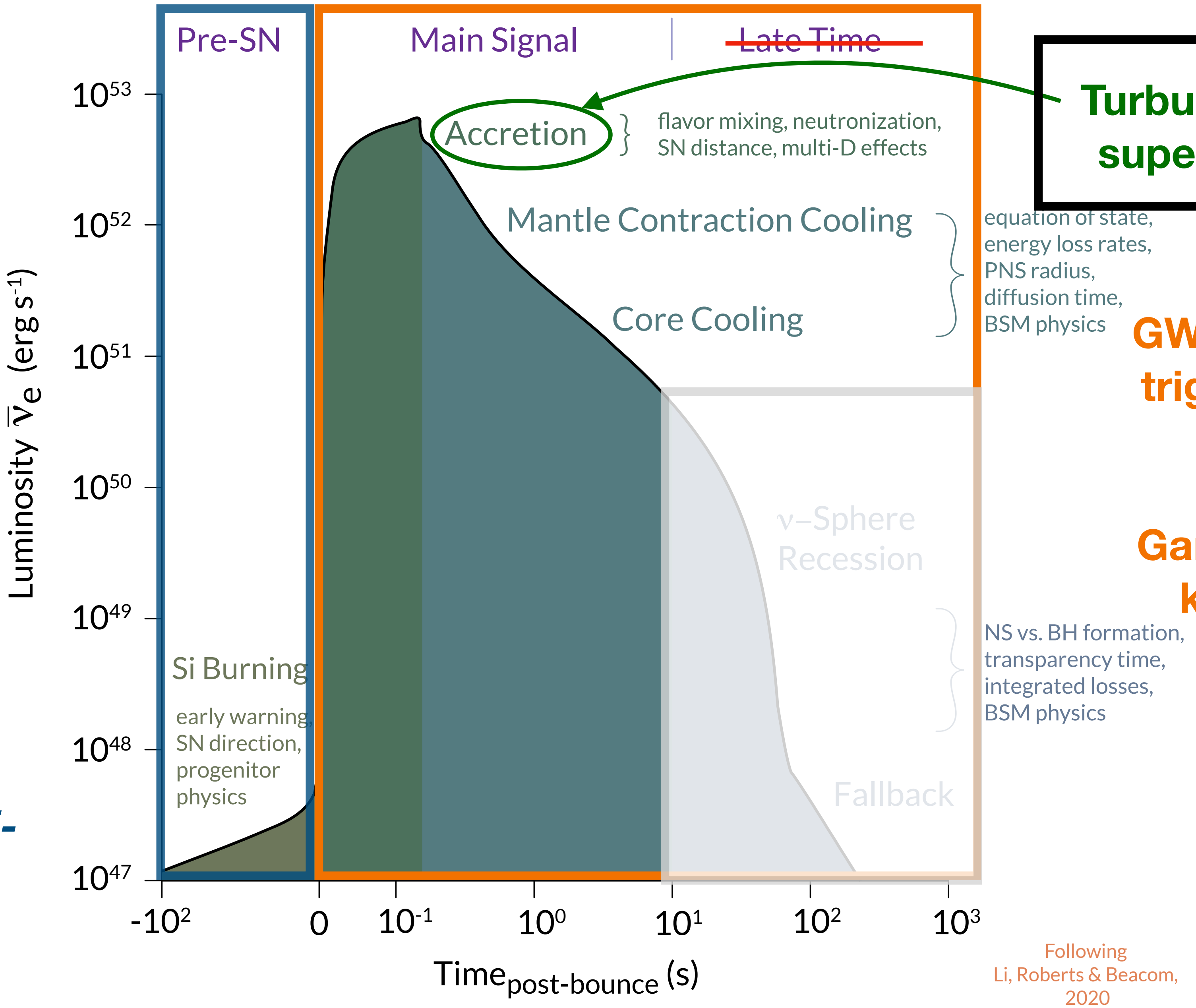
Star explodes: Supernova

Neutrinos emitted right after the collapse: collapsed core cools

Shockwave stalled: accelerated by neutrinos

Giunti and Kim. Fundamentals of Neutrino Physics and Astrophysics (2007)
Janka, Langanke, Marek and Martinez-Pinedo, et.al. Phys. Rept., 442:38–74, (2007)

Supernova neutrinos



Following Li, Roberts & Beacom, 2020

Turbulence in supernovae

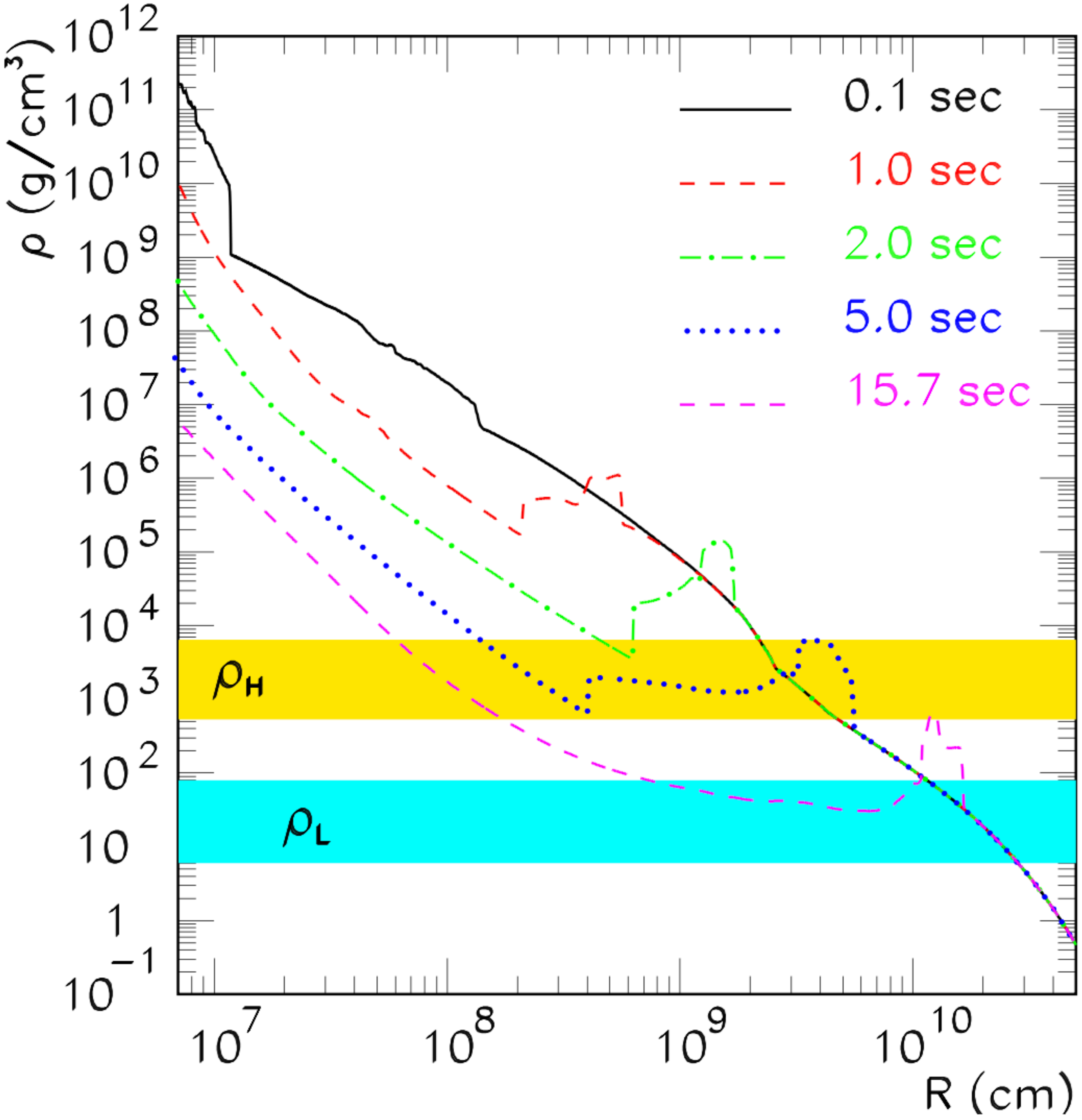
GW Memory, memory-triggered SN neutrino detection

Gamma-ray echo: 511 keV photons from neutrinos

Directional pointing, early alert, *testing stellar evolution, multi-messenger observations, exotic physics - DM (ALPs)*

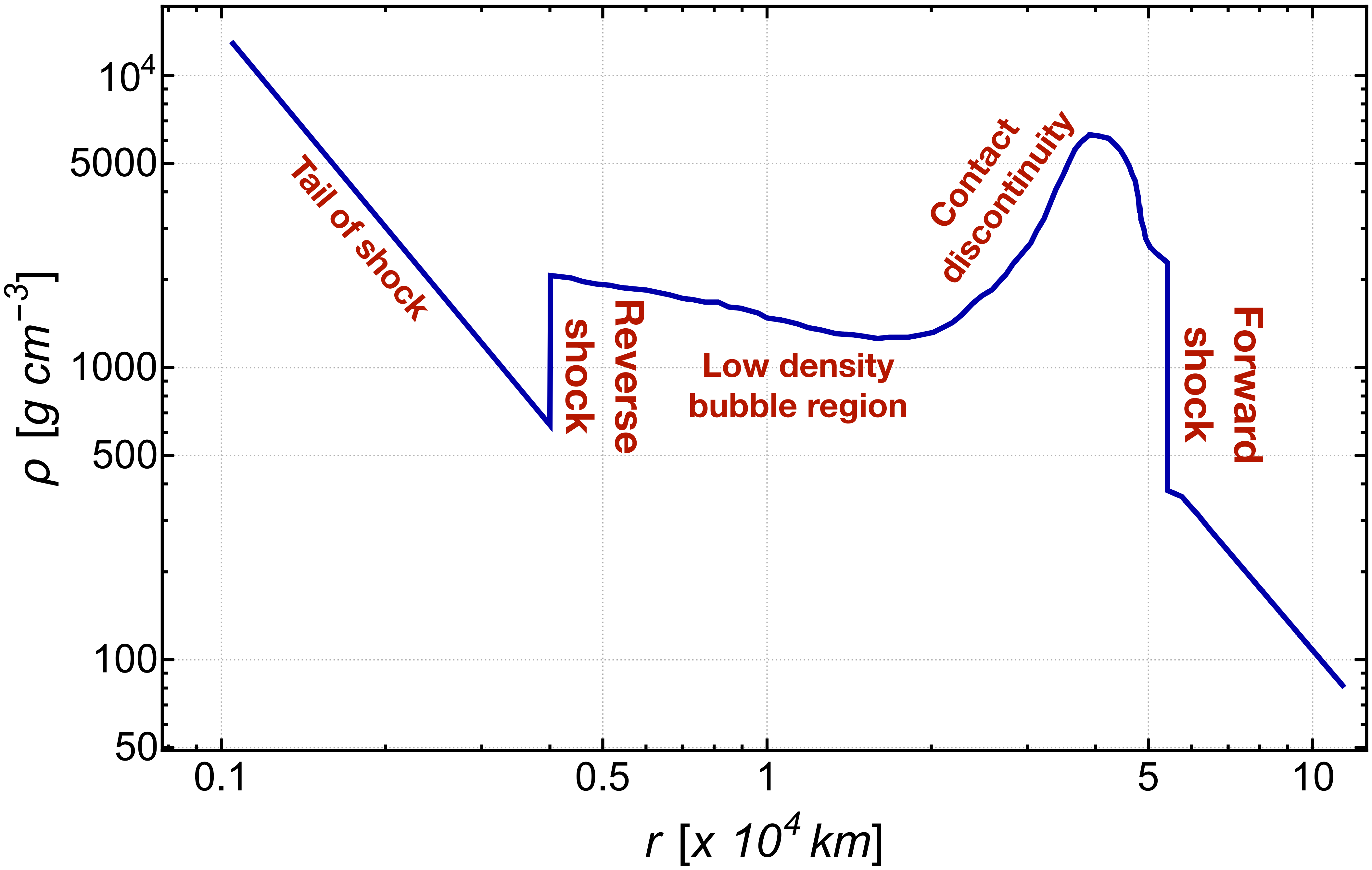
Graphics by: Frank Timmes

Shocks in CCSNe: Density profile



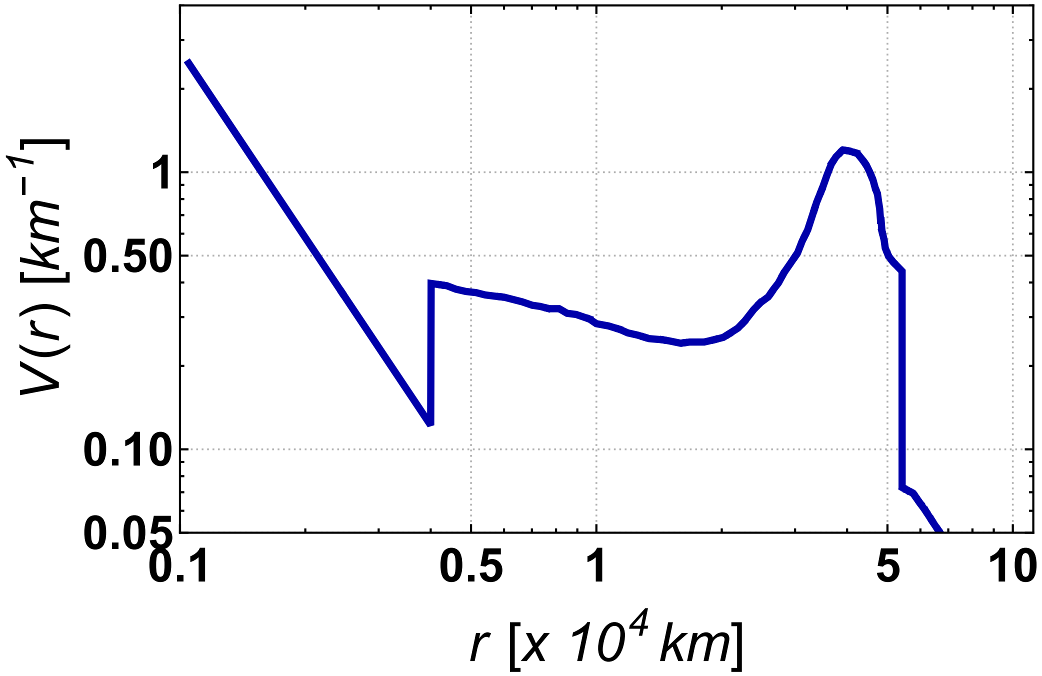
R. Tomas, M. Kachelriess, G. Raffelt, A. Dighe, H. T. Janka, and L. Scheck, JCAP 09, 015

Shocks in CCSNe: Matter profile (t = 5.0 s)



R. Tomas, M. Kachelriess, G. Raffelt, A. Dighe, H. T. Janka, and L. Scheck, JCAP 09, 015

Shocks in CCSNe: Signatures in neutrinos



Neutrinos

↓

Oscillate due to the matter profile (The MSW effect)

↓

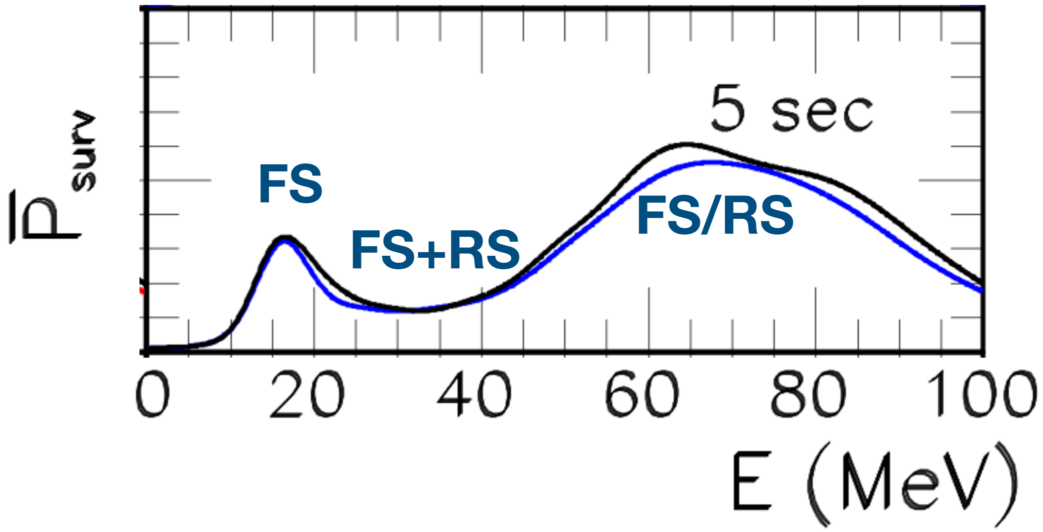
Presence of shocks: Breaks adiabaticity

↓

Can be used to test for presence of shocks

↓

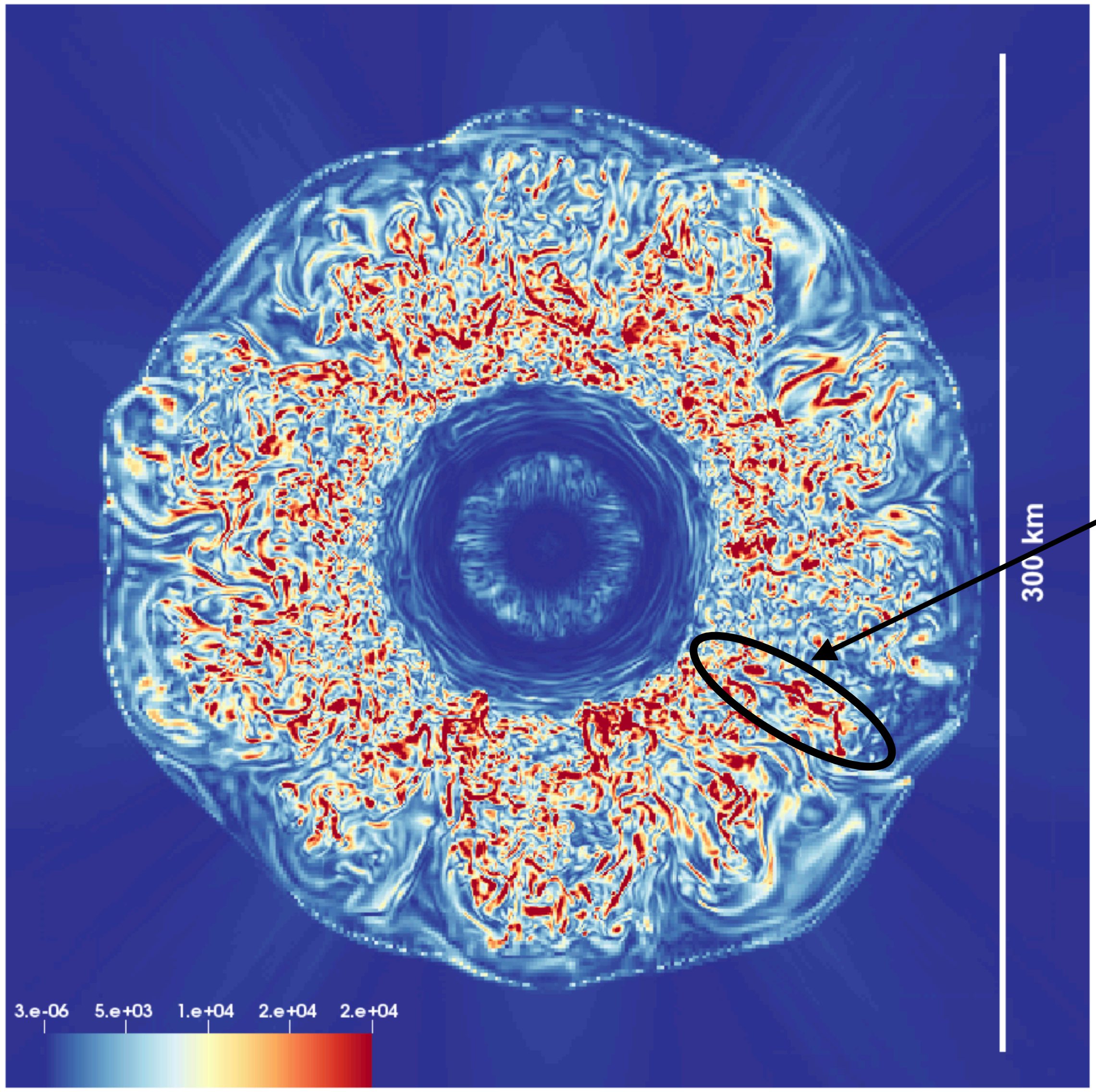
Can also be used to constrain turbulence



The Double-dip feature

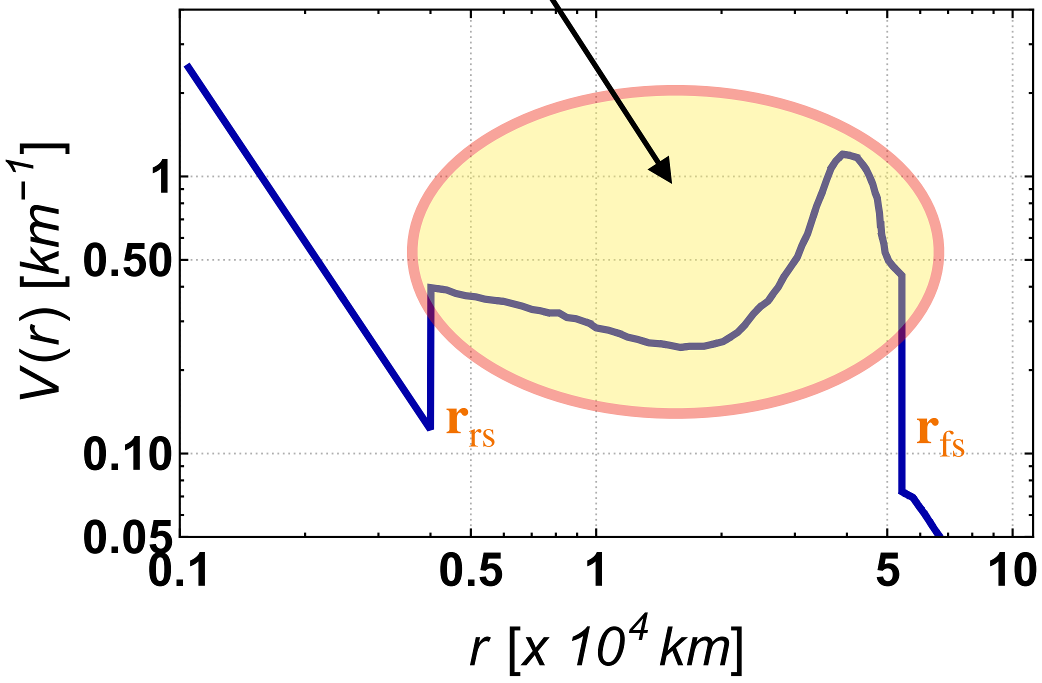
R. Tomas, M. Kachelriess, G. Raffelt, A. Dighe, H. T. Janka, and L. Scheck, JCAP 09, 015

Turbulence in CCSN



Amplitude of vorticity

Turbulence develops



Due to instabilities:

Standing accretion shock instability (SASI)
Neutrino-driven convection

D. Radice, E. Abdikamalav, C. Ott, P. Mosta, S. M. Couch, and L. F. Roberts, J. Phys. G: Nucl. Part. Phys. 45 (2018) 053003

References (non-exhaustive….)

Neutrino signatures on shock wave:

R.C. Schirato and G.M. Fuller, astro-ph/0205390.
K. Takahashi, K. Sato, H.E. Dalhed and J.R. Wilson, Astropart. Phys. 20 (2003) 189.
C. Lunardini and A.Y. Smirnov, JCAP 06 (2003) 009.
G.L. Fogli, E. Lisi, A. Mirizzi and D. Montanino, JCAP 0504 (2005) 002.
B. Dasgupta and A. Dighe, Phys. Rev. D 75 (2007) 093002.
S. Galais, J. Kneller, C. Volpe and J. Gava, Phys. Rev. D81 (2010) 053002.

Double-dip:

G.L. Fogli, E. Lisi, D. Montanino and A. Mirizzi, Phys. Rev. D68 (2003) 033005.

R. Tomas, M. Kachelriess, G. Raffelt, A. Dighe, H.T. Janka and L. Scheck, JCAP 09 (2004) 015.

J.P. Kneller, G.C. McLaughlin and J. Brockman, Phys. Rev. D 77 (2008) 045023.

Turbulence :

G.L. Fogli, E. Lisi, A. Mirizzi and D. Montanino, JCAP 06 (2006) 012.
A. Friedland and A. Gruzinov, astro-ph/0607244 (2006).
J.P. Kneller and C. Volpe, Phys. Rev. D 82 (2010) 123004.
T. Lund and J.P. Kneller, Phys. Rev. D 88 (2013) 023008.
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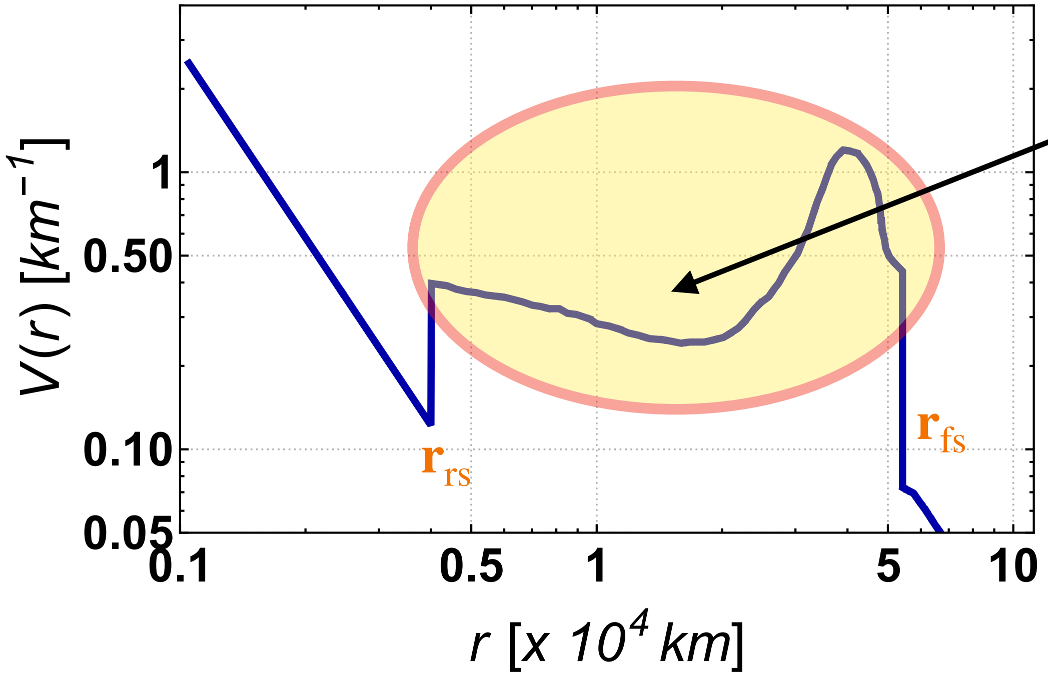
K.M. Patton, J.P. Kneller and G.C. McLaughlin, Phys. Rev. D 91 (2015) 025001.
J.P. Kneller and M. de los Reyes, J. Phys. G 44 (2017) 084008.
Y. Yang and J.P. Kneller, J. Phys. G 45 (2018) 045201.
S. Abbar, Phys. Rev. D 103 (2021) 045014.

Picture Credits: Stephane Andre

Generating turbulence



Turbulence develops

$$V(r) = \big(1 + F(r)\big)V_0(r)$$

Random field

Matter potential

$$V_0(r) = \sqrt{2}G_F n_e(r)\mathrm{diag}(1,0)$$

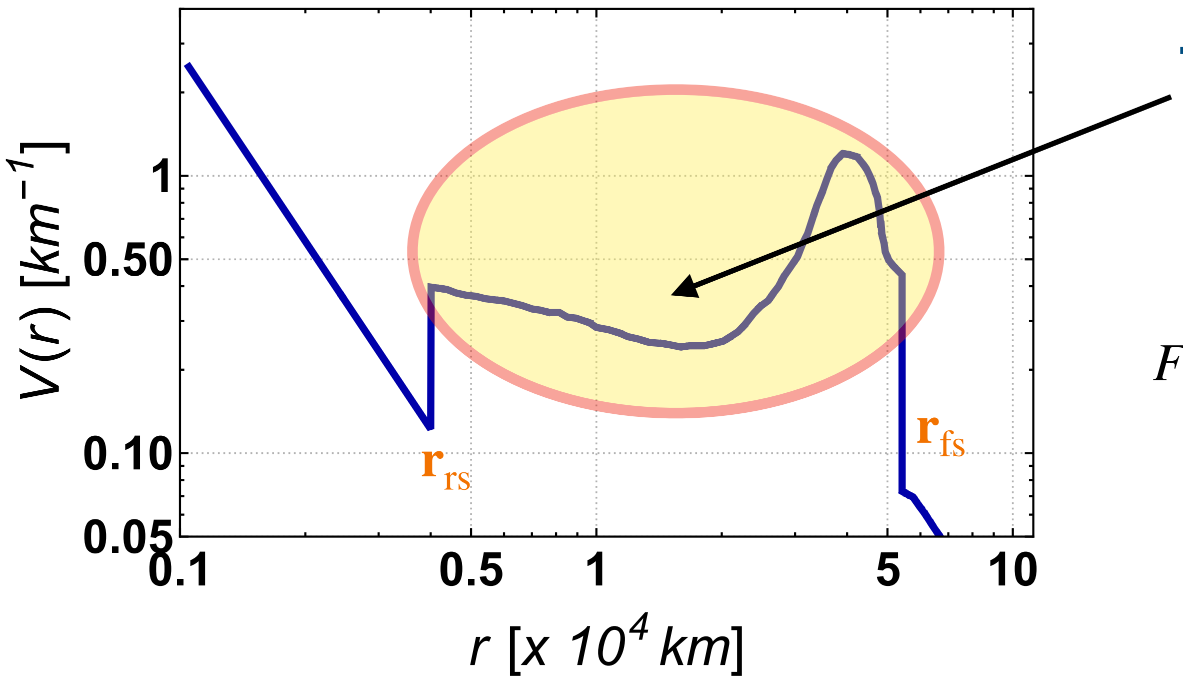
$$F(r) = \begin{cases} C_* \tanh\left(\dfrac{r - r_{\rm rs}}{\lambda}\right)\tanh\left(\dfrac{r_{\rm fs} - r}{\lambda}\right) \times F_{\rm rand}(r),\ r_{\rm rs} \le r \le r_{\rm fs}\,, \\ 0,\ \text{elsewhere} \end{cases}$$

$\mathbf{C_*}$: Amplitude of turbulence

λ: length scale associated with the turbulence ~ 100 km

J. P. Kneller and C. Volpe, PRD 82, 123004 (2010)
T. Lund and J.P. Kneller, PRD 88, 023008 (2013)
A. J. Majda and P. R. Kramer, Phys. Rep. 314, 237 (1999)
P.R. Kramer, O. Kurbanmuradov, and K. Sabelfeld, J. Comp. Phys., 229, 897 (2007)

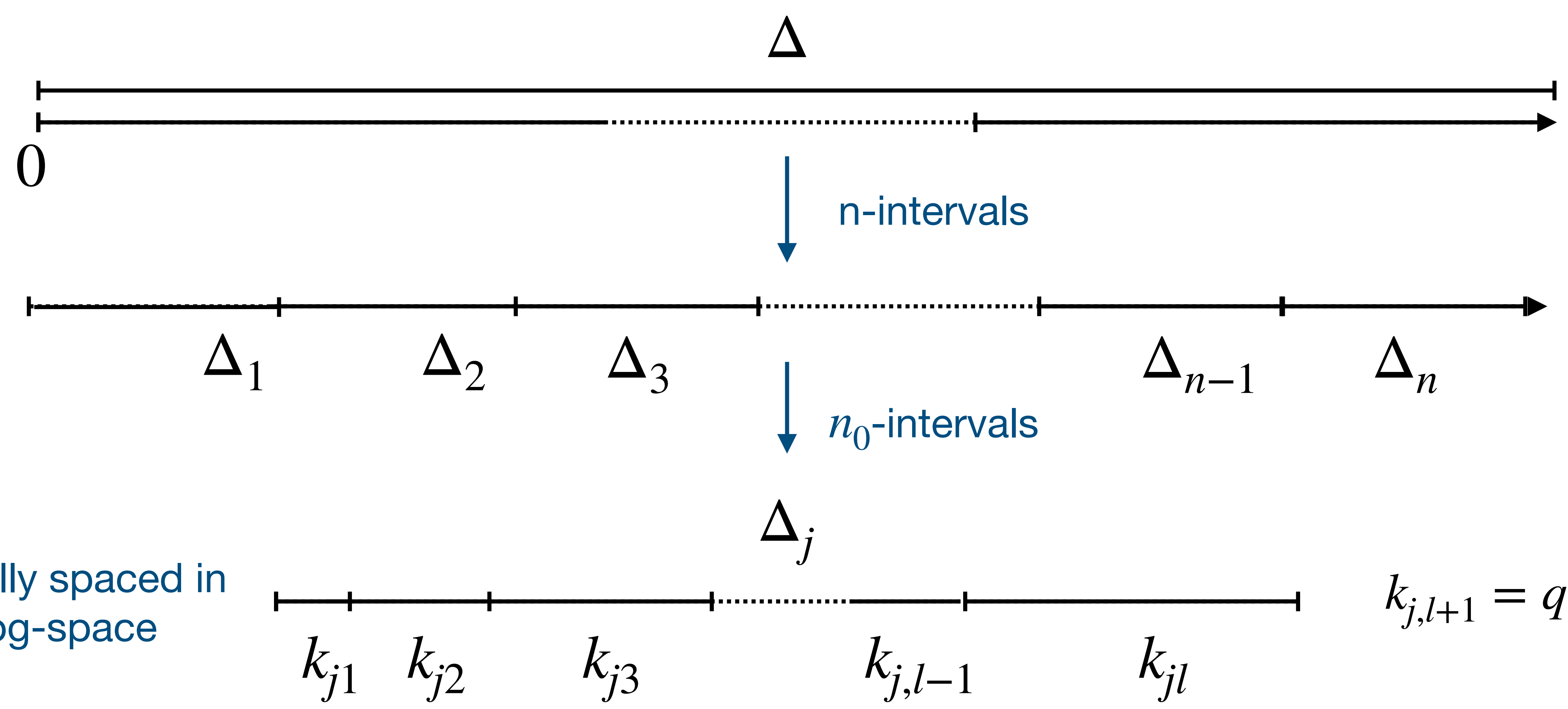
Generating turbulence



Turbulence develops

$$V(r) = \big(1 + F(r)\big)\, V_0(r)$$

$$F(r) = \begin{cases} C_* \tanh\left(\frac{r - r_{\rm rs}}{\lambda}\right) \tanh\left(\frac{r_{\rm fs} - r}{\lambda}\right) \times F_{\rm rand}(r), & r_{\rm rs} \le r \le r_{\rm fs}\,, \\ 0, \text{ elsewhere} \end{cases}$$



k-space

Δ

0

n-intervals

Δ_1 Δ_2 Δ_3 Δ_{n-1} Δ_n

n_0-intervals

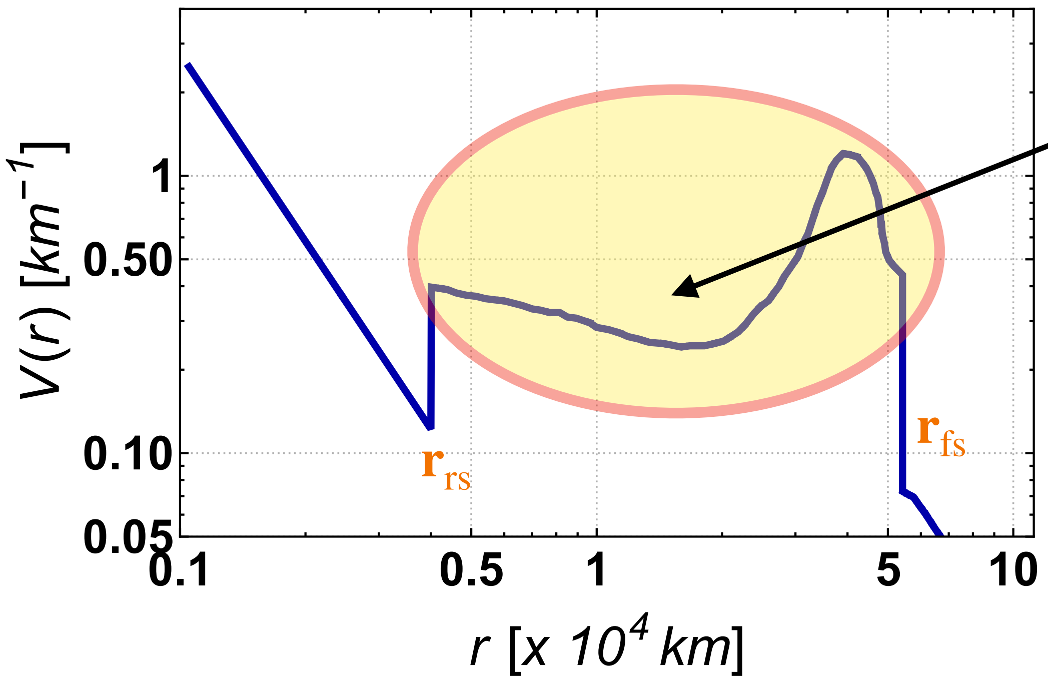
Δ_j

Equally spaced in log-space

k_{j1} k_{j2} k_{j3} $k_{j,l-1}$ k_{jl}

$$k_{j,l+1} = q k_{j,l}$$

Generating turbulence



Turbulence develops

$$V(r) = \big(1 + F(r)\big)\, V_0(r)$$

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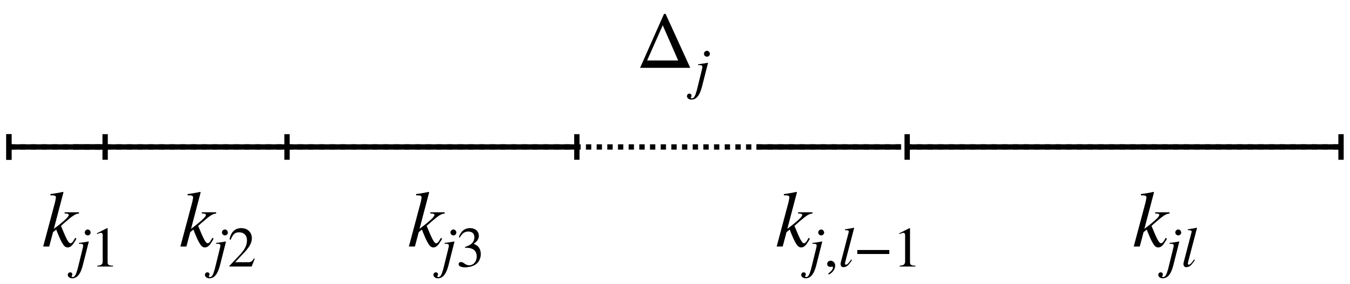
k-space



Selection of k’s according to

$$p_j(k) = \begin{cases} 2\ E(k)/\sigma_j^2, \text{ for } k \in \Delta_j\,, \\ 0, \text{ for } k \notin \Delta_j\,, \end{cases}$$

Spectral Index

$$\sigma_j^2 = 2 \int_{\Delta_j} E(k) dk$$

Power spectrum

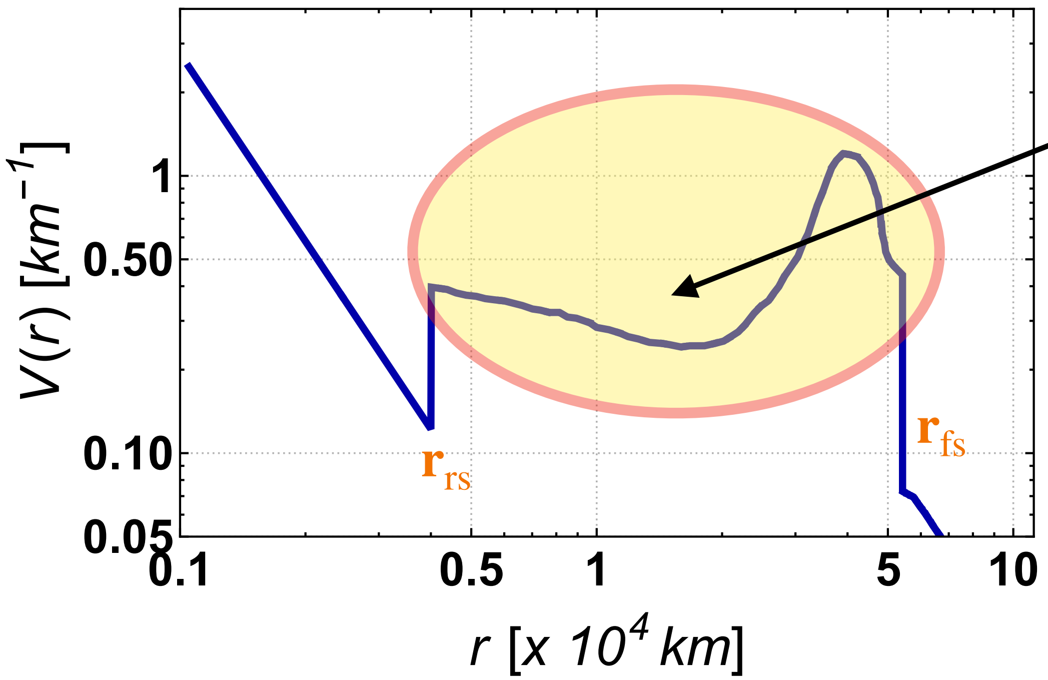
$$E(k) = (\alpha - 1) \left(\frac{k_*}{k}\right)^{\alpha}$$

Wavenumber cutoff $2\pi/\lambda_*$

Longest non-zero turbulence
wavelength (driving scale)

$$\lambda_* = 2(r_{\rm fs} - r_{\rm rs})$$

Generating turbulence

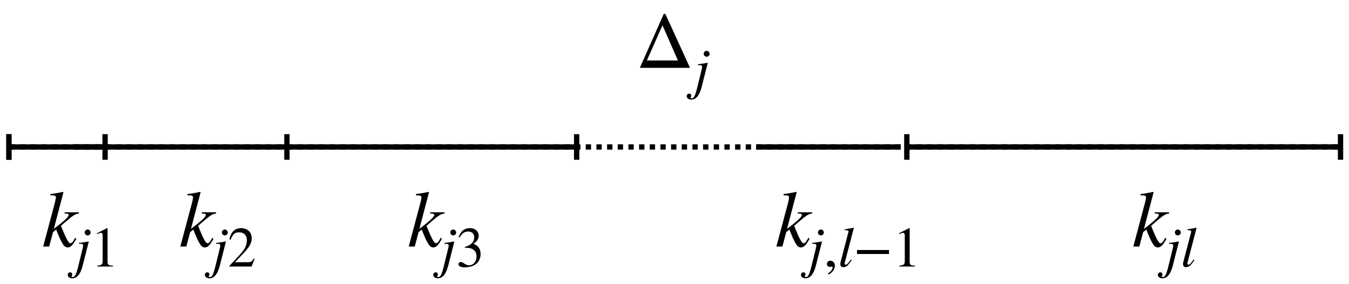


Turbulence develops

$$V(r) = \big(1 + F(r)\big)\, V_0(r)$$

$$F(r) = \begin{cases} C_* \tanh\left(\dfrac{r - r_{rs}}{\lambda}\right)\tanh\left(\dfrac{r_{fs} - r}{\lambda}\right) \times F_{rand}(r),\, r_{rs} \le r \le r_{fs}\,, \\ 0, \text{ elsewhere} \end{cases}$$

k-space



Selection of k's according to $$p_j(k) = \begin{cases} 2\ E(k)/\sigma_j^2, \text{ for } k \in \Delta_j\,, \\ 0, \text{ for } k \notin \Delta_j\,, \end{cases}$$

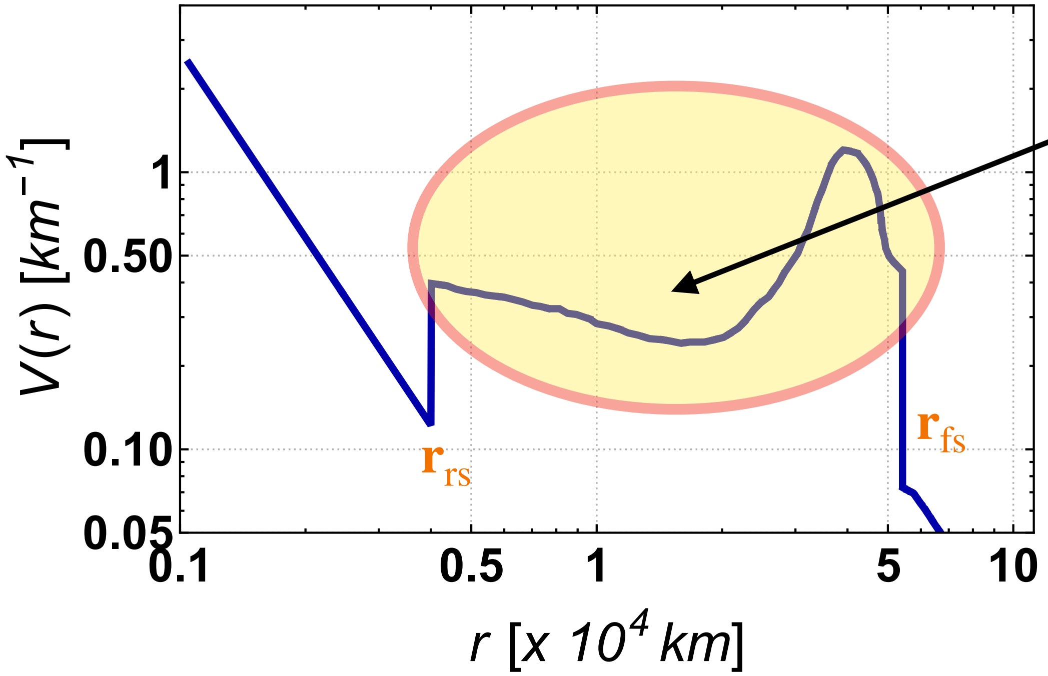
$$E(k) = (\alpha - 1)\left(\frac{k_*}{k}\right)^{\alpha}$$ Power spectrum

Real-valued homogenous Gaussian scalar random field

$$F_{rand}(r) = \sum_{j=1}^{n} \frac{\sigma_j}{\sqrt{n_0}} \sum_{l=1}^{n_0} \Big(\xi_{jl} \cos\big(2\pi k_{jl} r\big) + \eta_{jl} \sin\big(2\pi k_{jl} r\big)\Big)$$

Gaussian random variables

Generating turbulence



Turbulence develops

Stratified sampling helps in working with comparatively low number of modes $\sim \mathcal{O}(100)$

Amplitude of turbulence

$$F(r) = \begin{cases} C_* \tanh\left(\frac{r - r_{\mathrm{rs}}}{\lambda}\right)\tanh\left(\frac{r_{\mathrm{fs}} - r}{\lambda}\right) \times F_{\mathrm{rand}}(r), r_{\mathrm{rs}} \le r \le r_{\mathrm{fs}}\,, \\ 0, \text{ elsewhere} \end{cases}$$

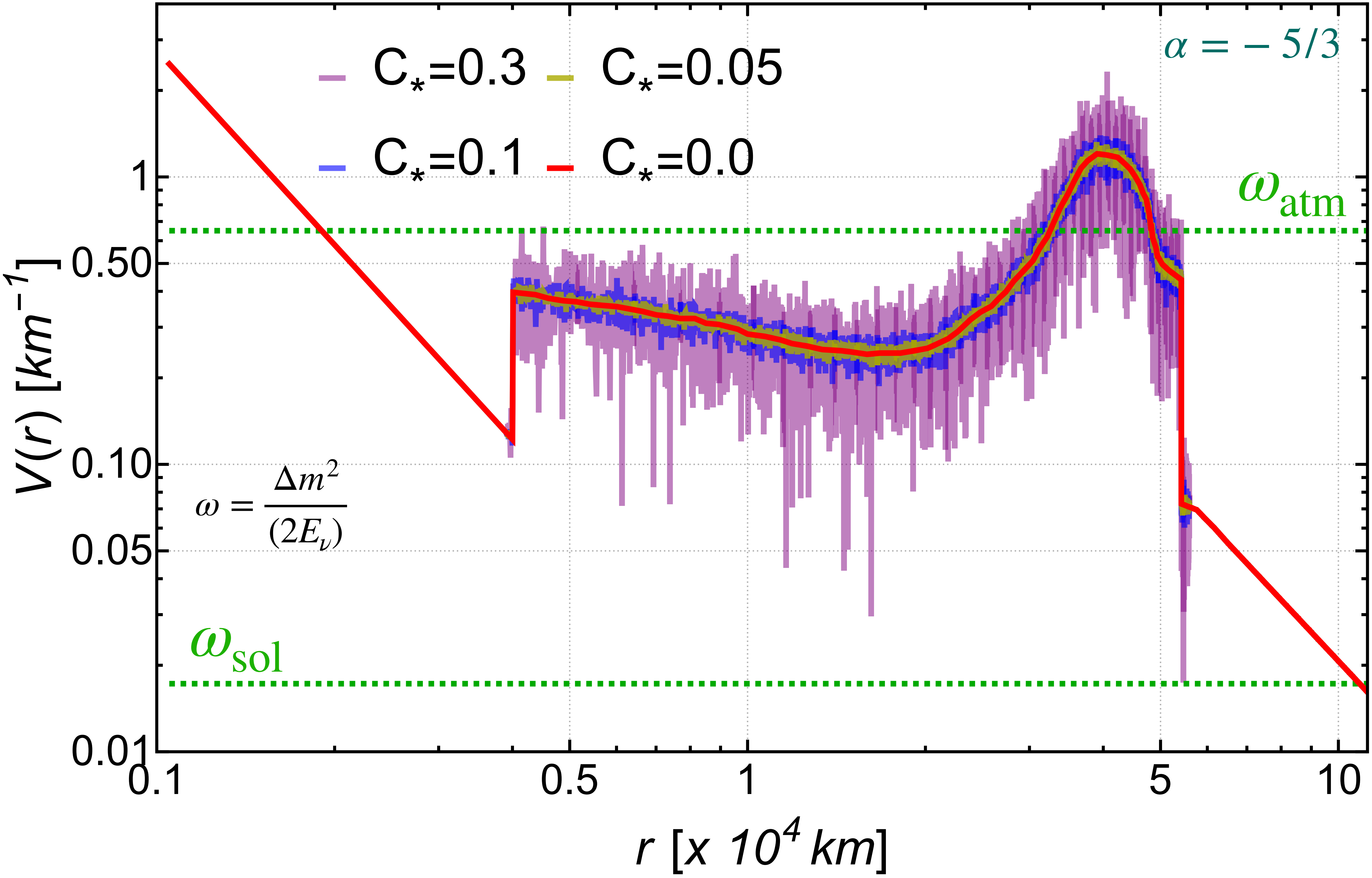
$$E(k) = (\alpha - 1)\left(\frac{k_*}{k}\right)^{\alpha}$$

Spectral Index

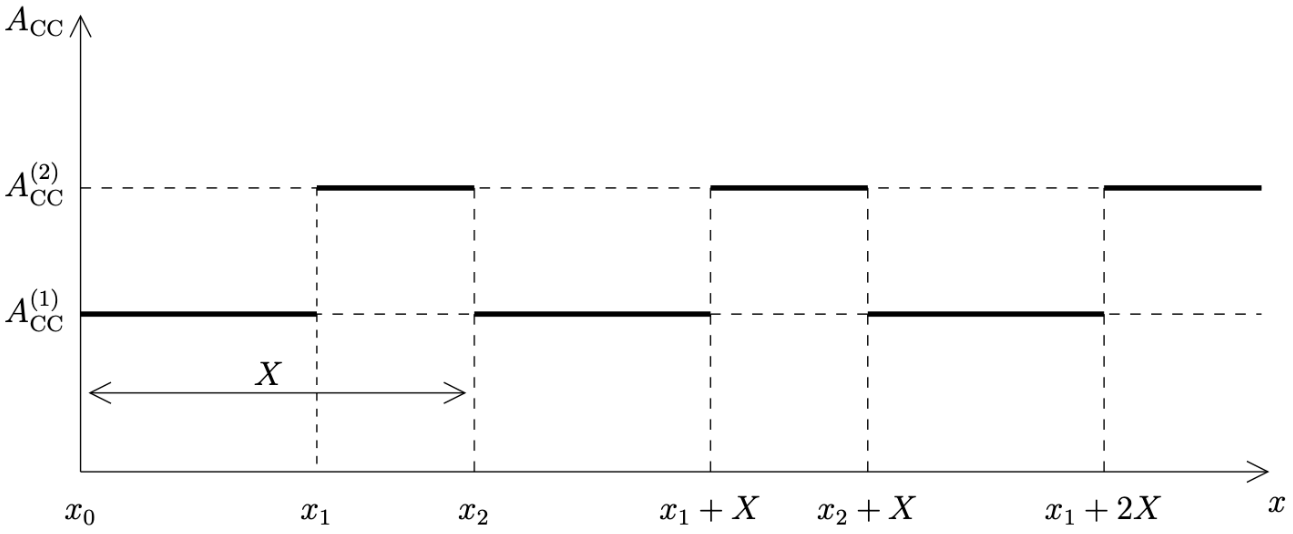
Kolmogorov, $\alpha = -5/3$ **??**

$$F_{\mathrm{rand}}(r) = \sum_{j=1}^{n} \frac{\sigma_j}{\sqrt{n_0}} \sum_{l=1}^{n_0} \left(\xi_{jl}\cos\left(2\pi k_{jl} r\right) + \eta_{jl}\sin\left(2\pi k_{jl} r\right)\right)$$

Density profile and Turbulence



Evolution: Slab Approximation



$$H_{\mathrm{vac}} = \mathrm{diag}(\Delta m^2/(2E), -\Delta m^2/(2E))$$

$$\mathrm{i}\frac{d}{dt}|\nu_\alpha\rangle = \left[U H_{\mathrm{vac}} U^\dagger + V(r)\right]|\nu_\alpha\rangle$$

$$\Psi_e(x_n) = \left[U_M\ \mathcal{U}_M(x_n - x_{n-1})\ U_M^\dagger\right]_{(n)} \left[U_M\ \mathcal{U}_M(x_{n-1} - x_{n-2})\ U_M^\dagger\right]_{(n-1)} \ldots \left[U_M\ \mathcal{U}_M(x_1 - x_0)\ U_M^\dagger\right]_{(1)} \Psi_e(x_0)$$

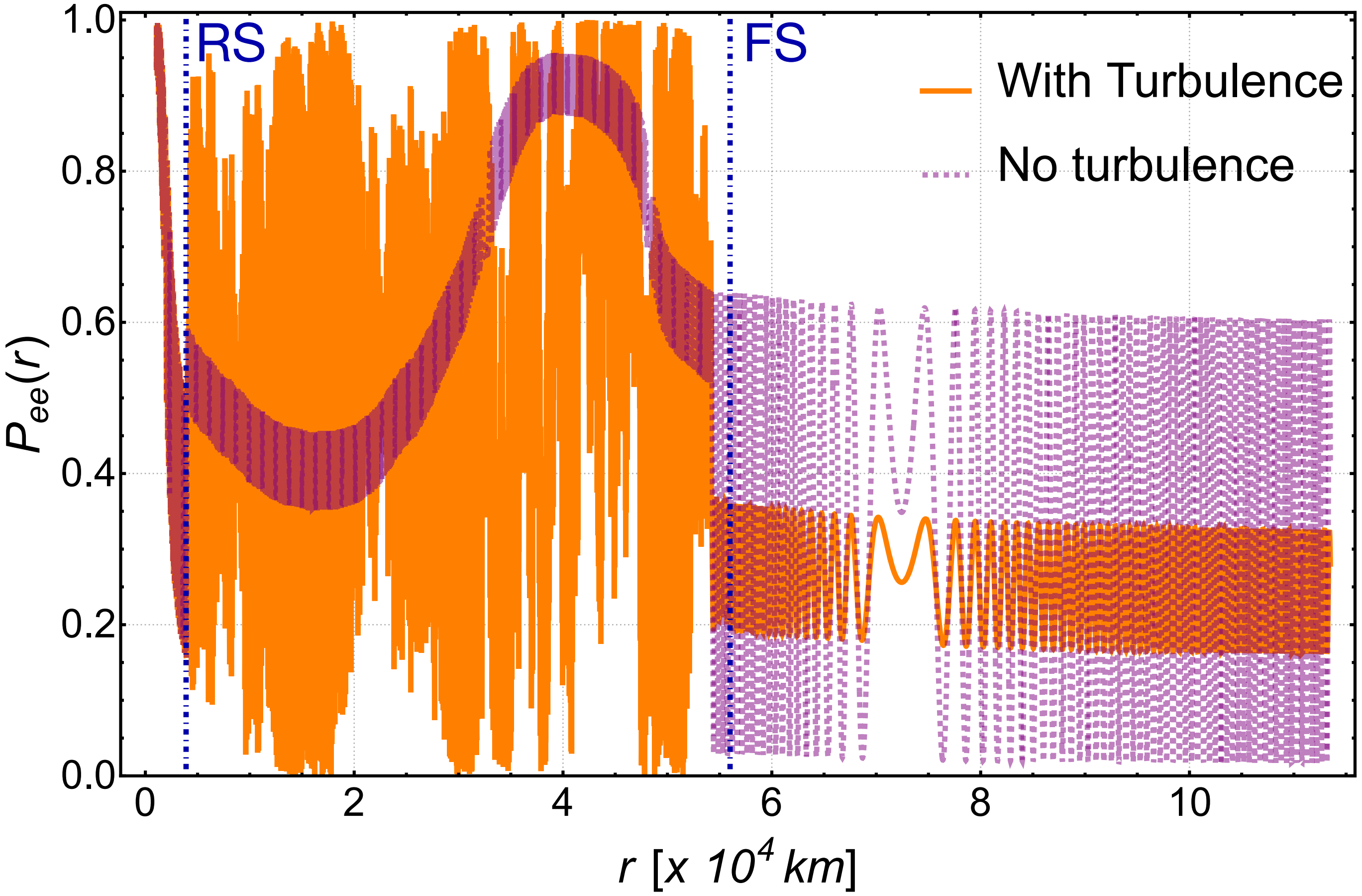
$$\Psi_e = \langle x|\nu_e\rangle = \begin{pmatrix} \psi_{ee} \\ \psi_{ex} \end{pmatrix}$$

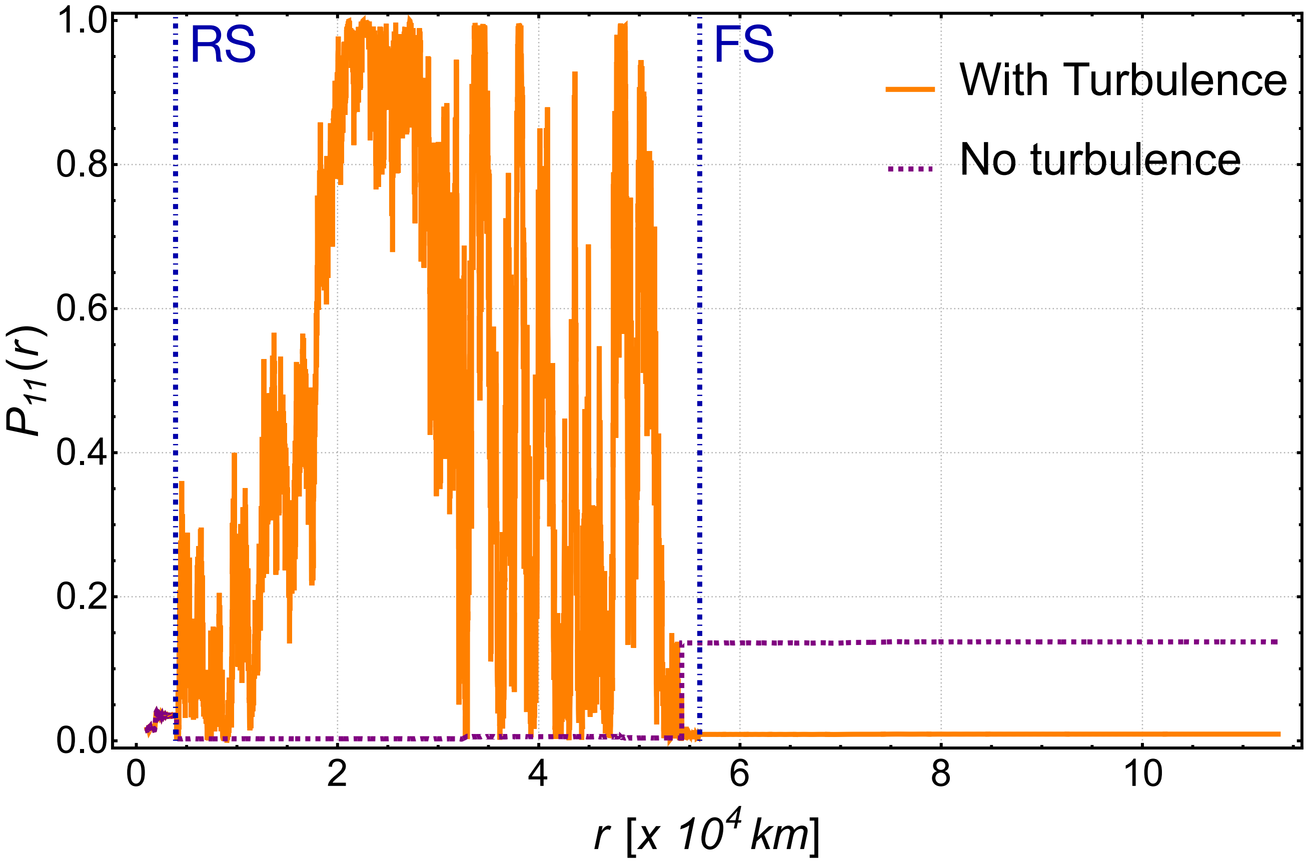
$$\mathcal{U}_M(\Delta x) = \mathrm{diag}\left(\exp\left(+i\Delta m_M^2 \Delta x/4E\right), exp\left(-i\Delta m_M^2 \Delta x/4E\right)\right)$$

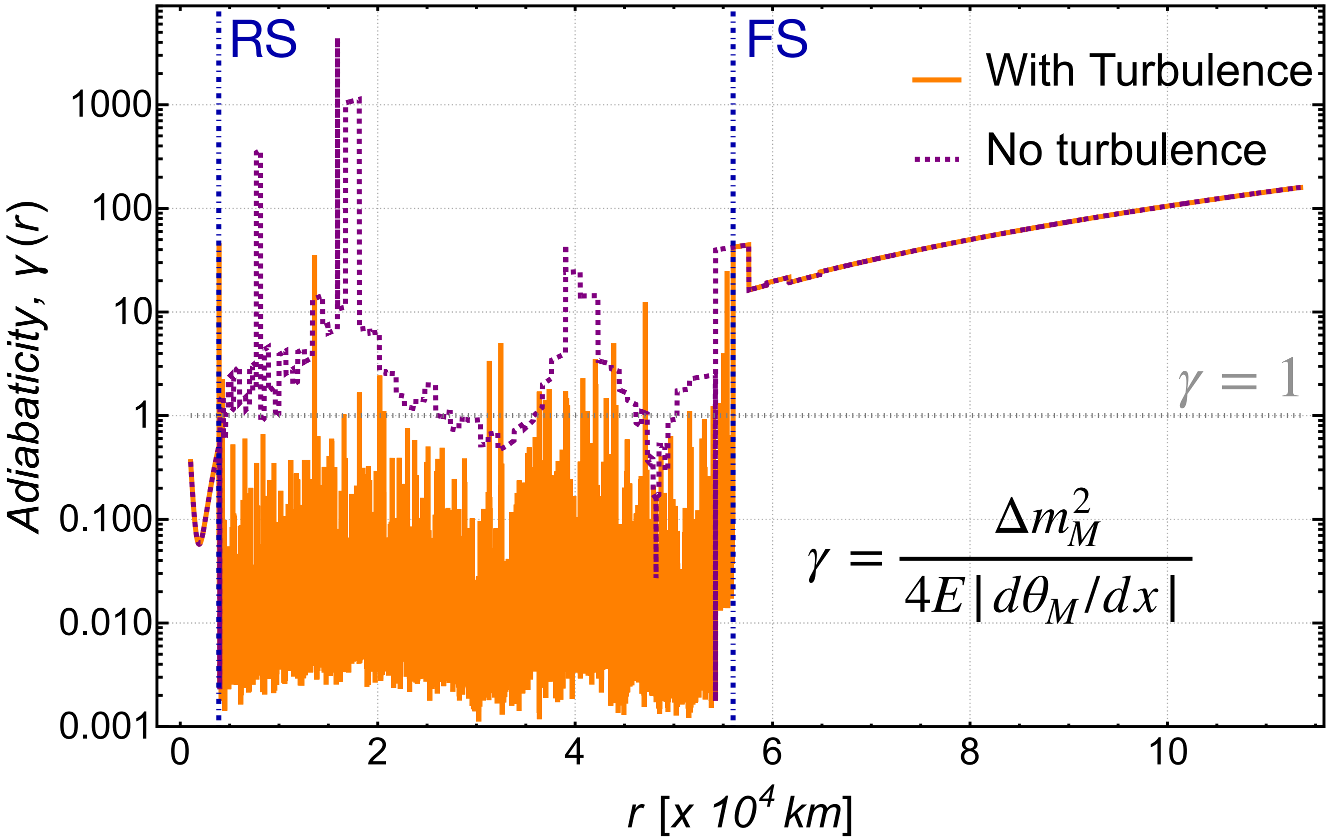
Parallelized Python code….

C. Giunti and C.W. Kim, Fundamentals of Neutrino Physics and Astrophysics (2007)

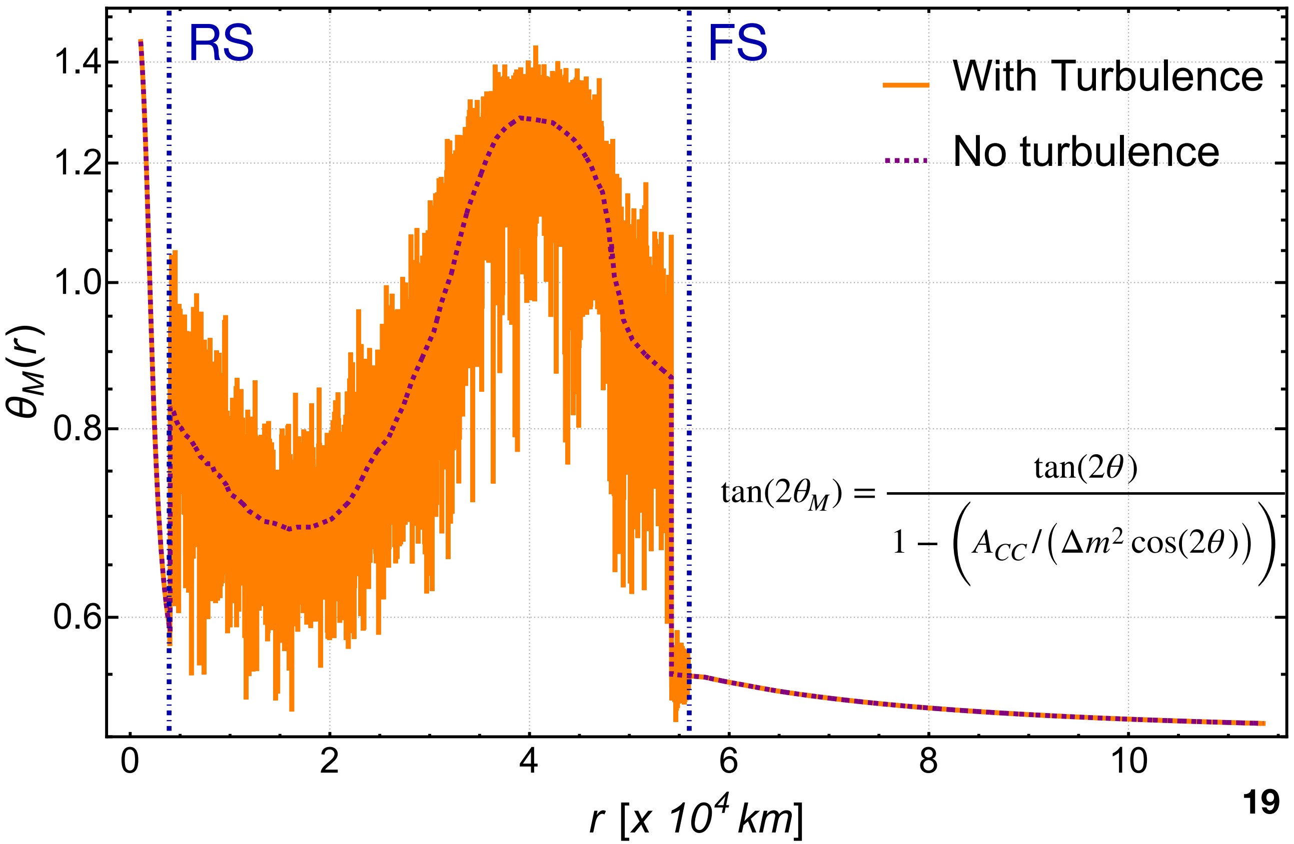
Evolution of Probabilities: With and Without Turbulence





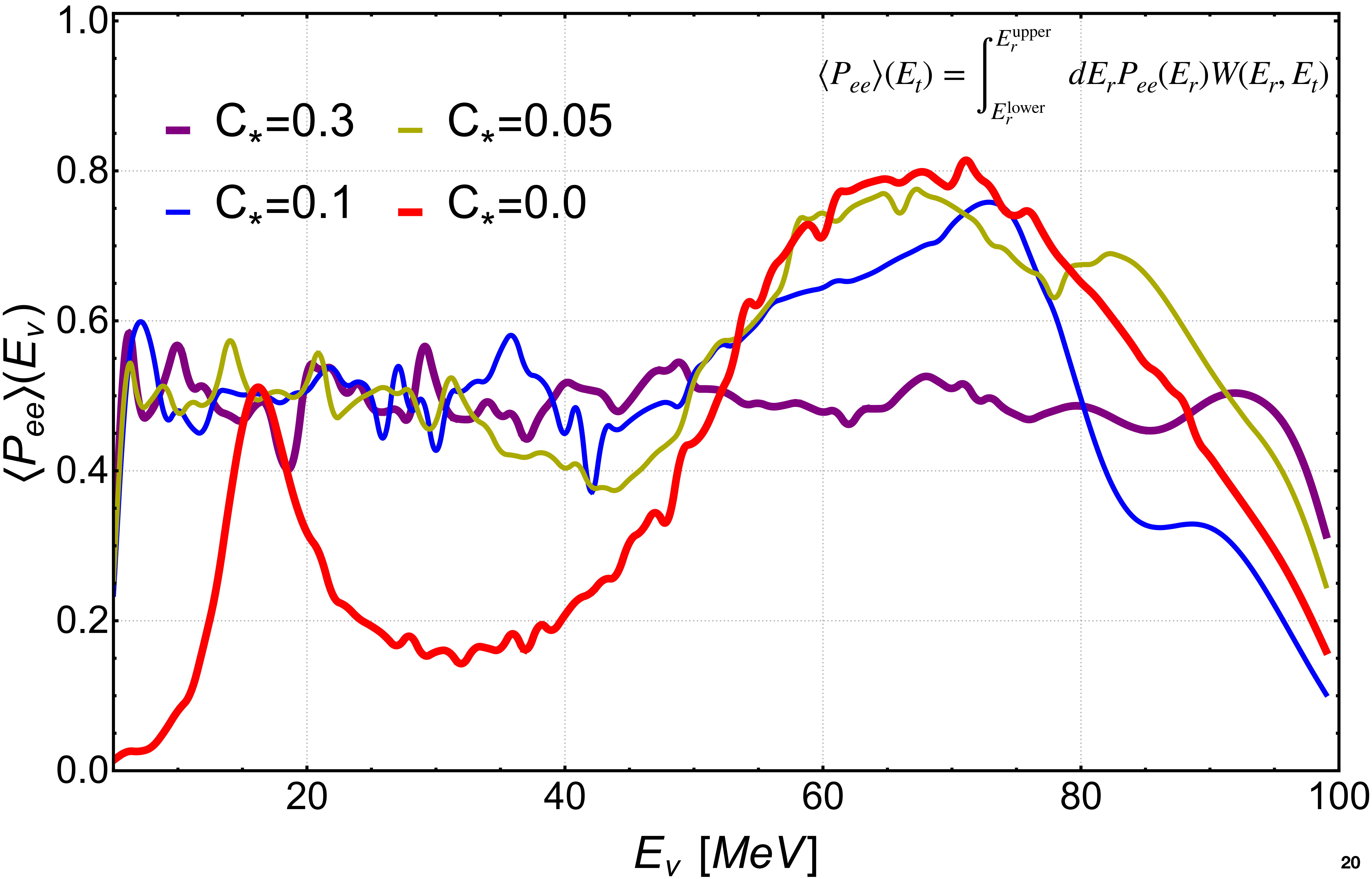


$$\gamma = \frac{\Delta m_M^2}{4E\,|d\theta_M/dx|}$$



$$\tan(2\theta_M) = \frac{\tan(2\theta)}{1 - \left(A_{CC}/\left(\Delta m^2 \cos(2\theta)\right)\right)}$$

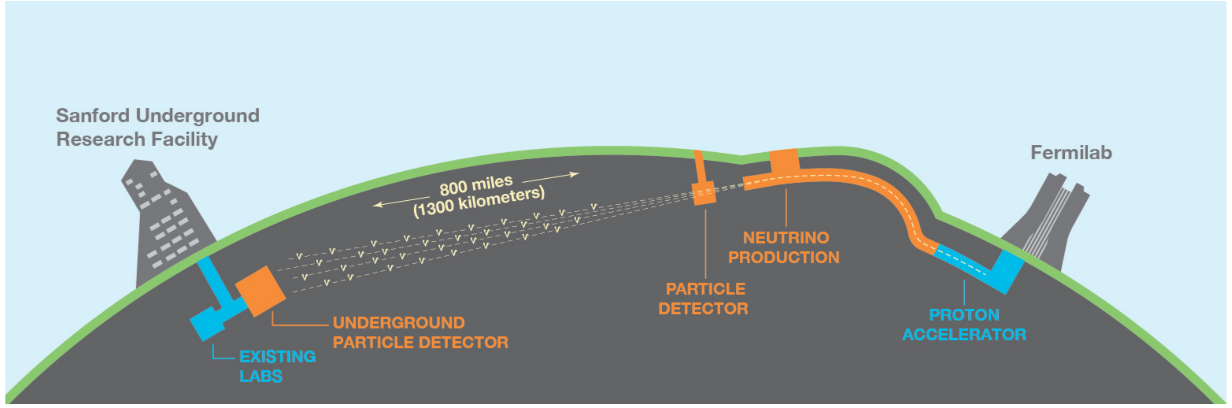
The Double-dip: Effects of Turbulence



$$\langle P_{ee}\rangle(E_t) = \int_{E_r^{\mathrm{lower}}}^{E_r^{\mathrm{upper}}} dE_r P_{ee}(E_r) W(E_r, E_t)$$

Upcoming neutrino detectors

DUNE



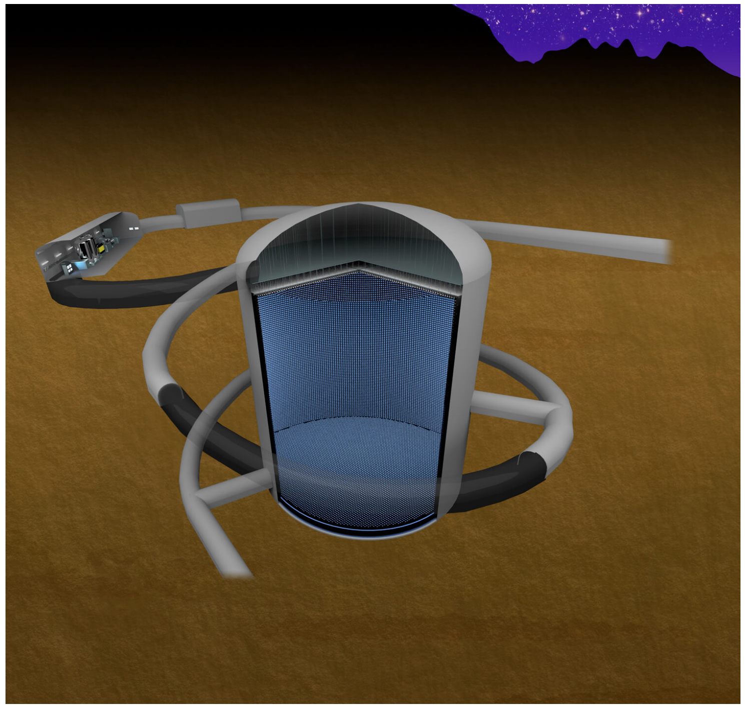
Liquid Argon time-projection chambers (LArTPC)

70 ton of ultra cold liquid argon

Charged current interaction: $\nu_e +^{40}\text{Ar} \rightarrow^{40}\text{K}^* + e^-$

DUNE collaboration JINST 15 (2020), arXiv: 2002.02967

Hyper-Kamiokande



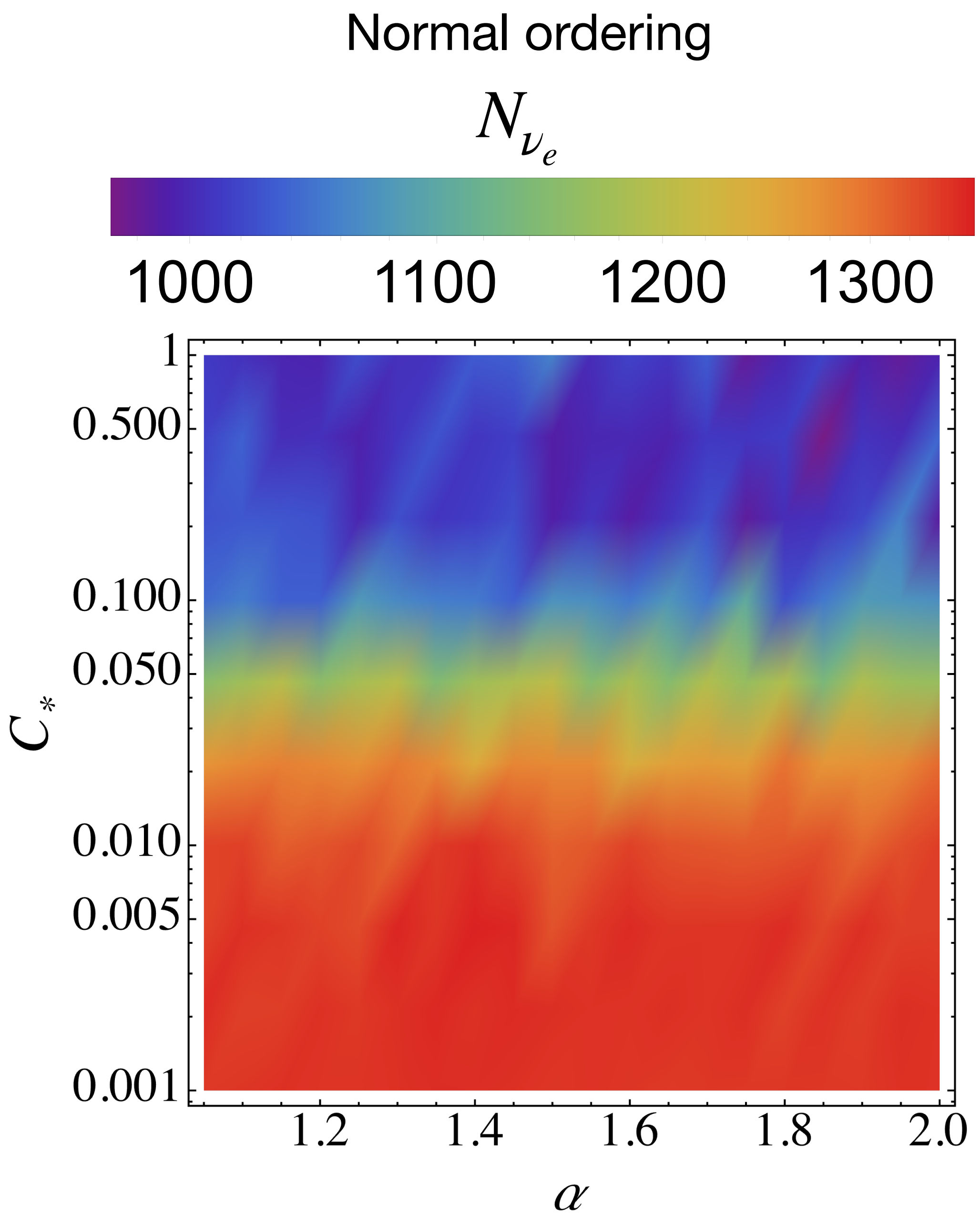
Water Cherenkov detector

187 kton of ultra pure water

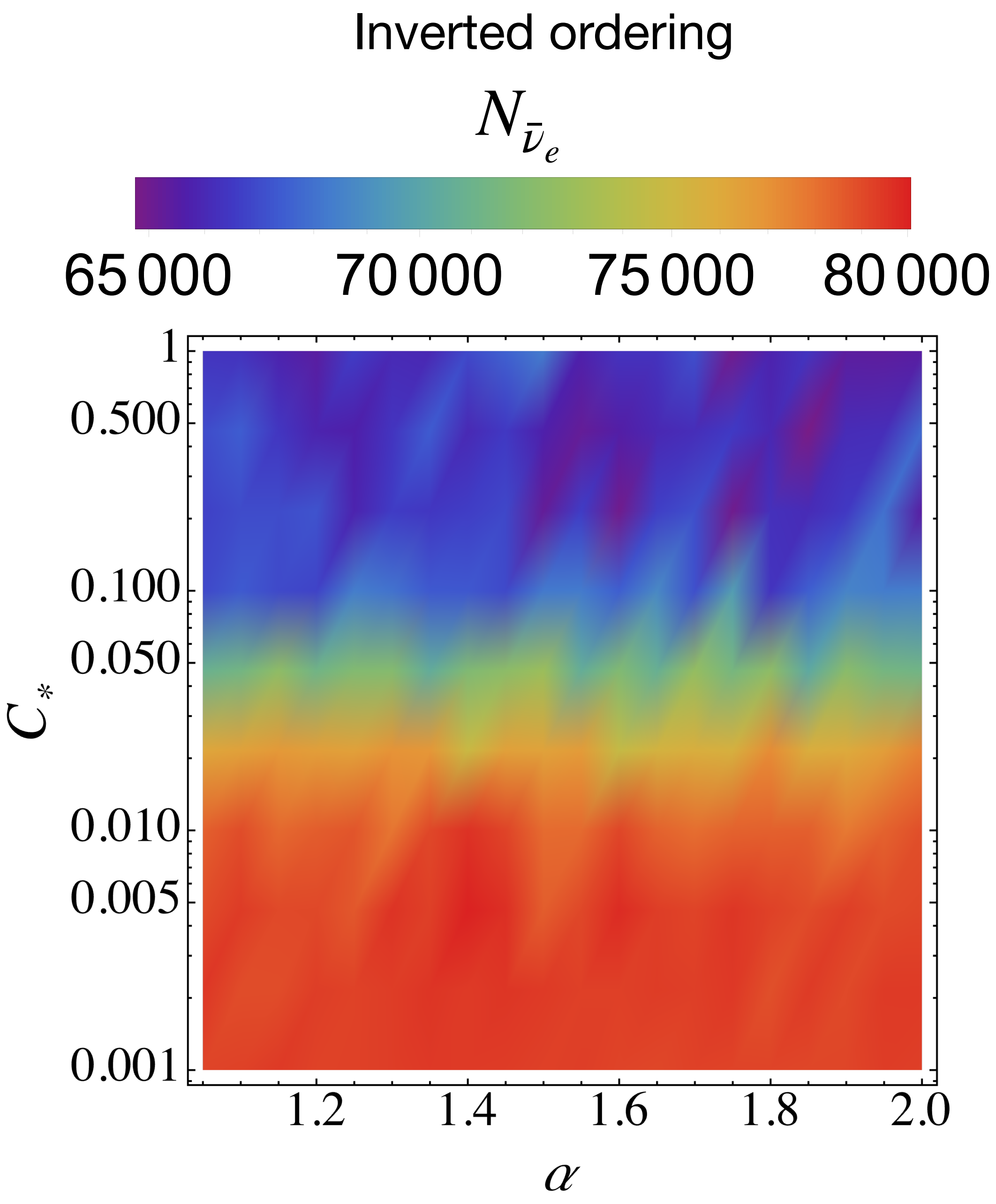
Inverse-beta decay (IBD): $\bar{\nu}_e + p \rightarrow e^+ + n$

https://www-sk.icrr.u-tokyo.ac.jp/en/hk/about/detector/

Event rates in DUNE and Hyper-Kamiokande (HK)



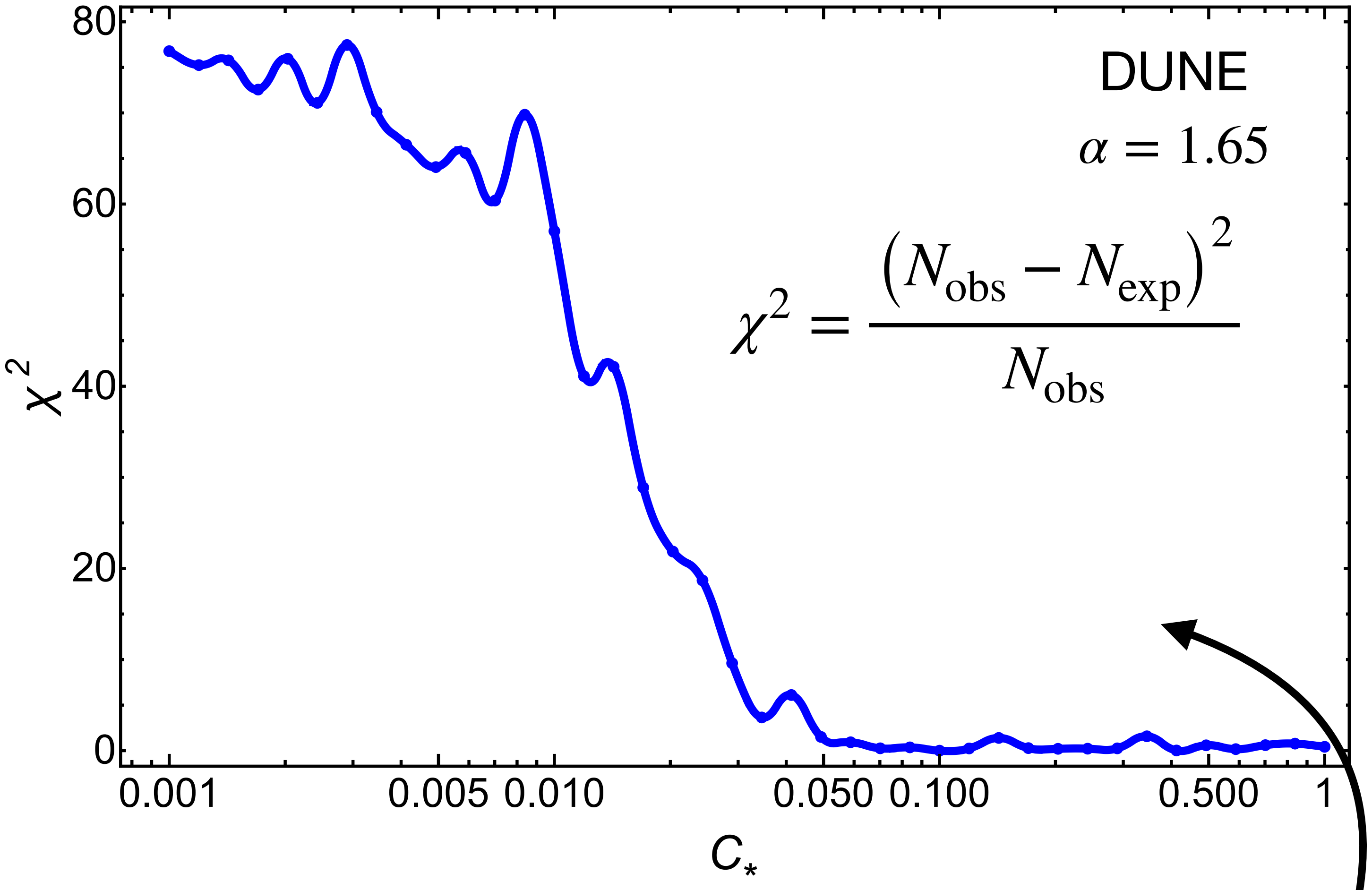
DUNE



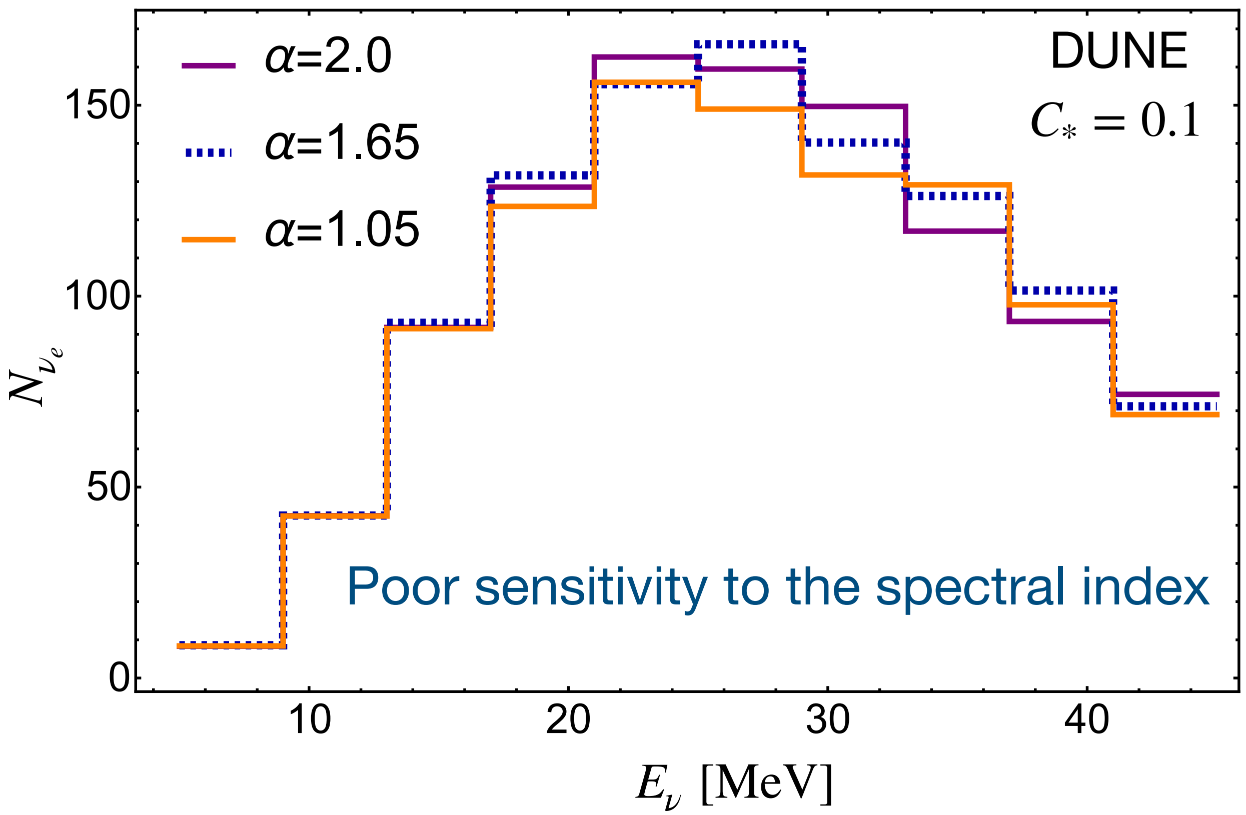
Hyper-Kamiokande

$$N_{\nu_\alpha} = \Delta t \frac{N_{\mathrm{tar}}}{4\pi R^2} \int dE_r \int dE_t \, \frac{dN^{\mathrm{earth}}_{\nu_\alpha}(E_t)}{dE_t} \sigma_{\nu_\alpha}(E_t) W(E_r, E_t)$$

Constraining turbulence in DUNE

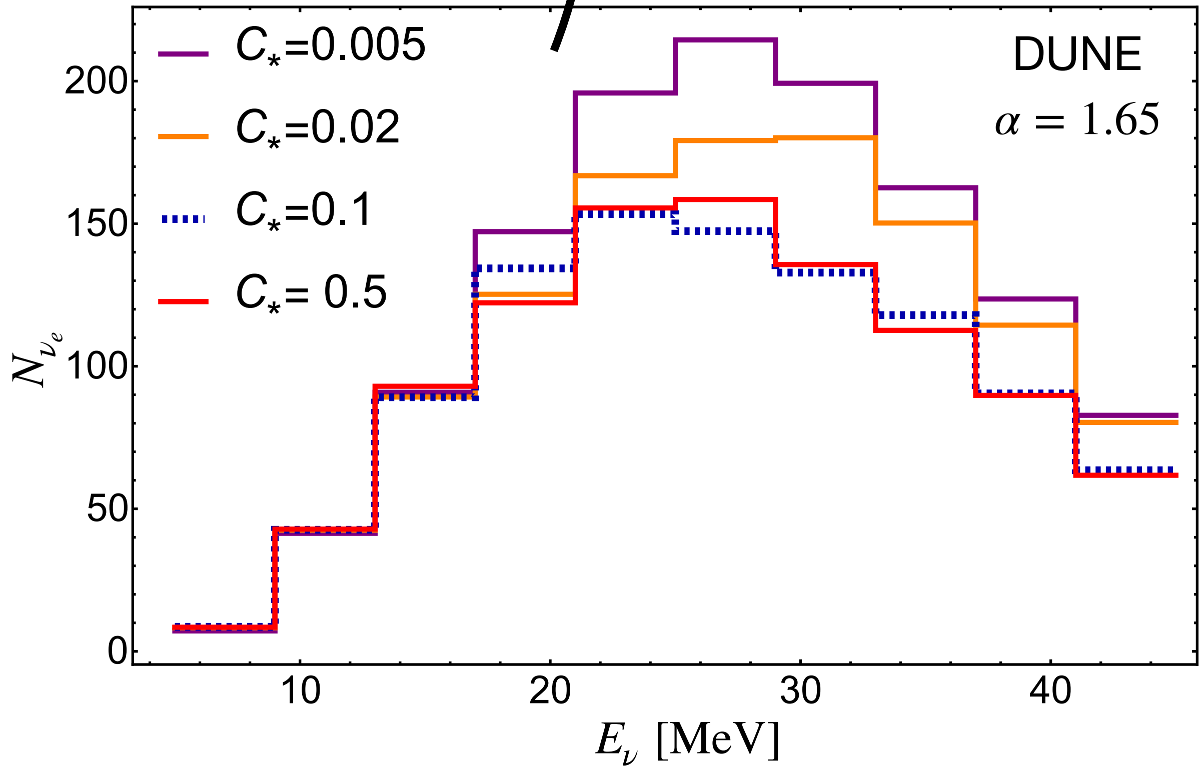


$$\chi^2 = \frac{\left(N_{\rm obs} - N_{\rm exp}\right)^2}{N_{\rm obs}}$$

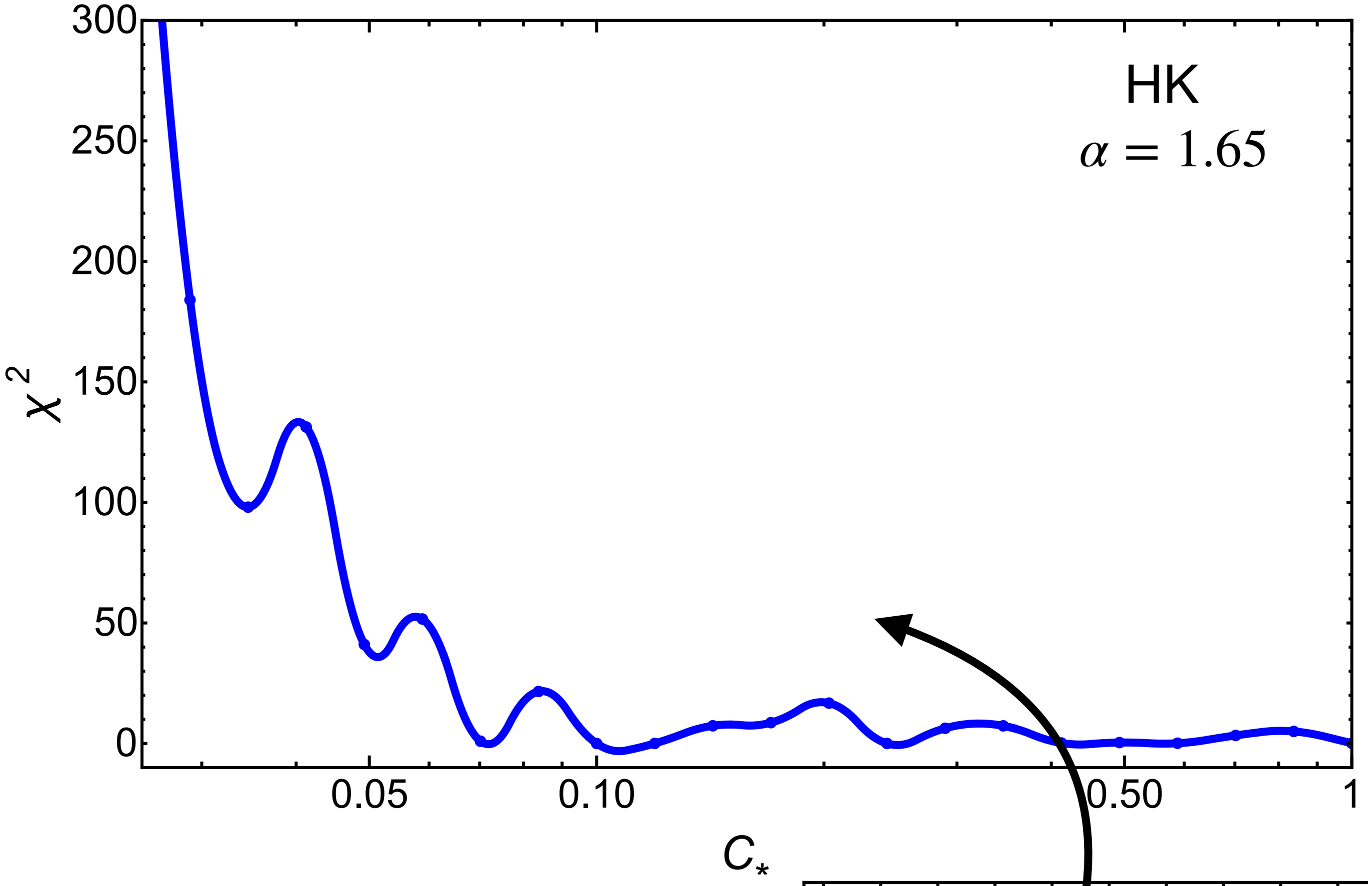


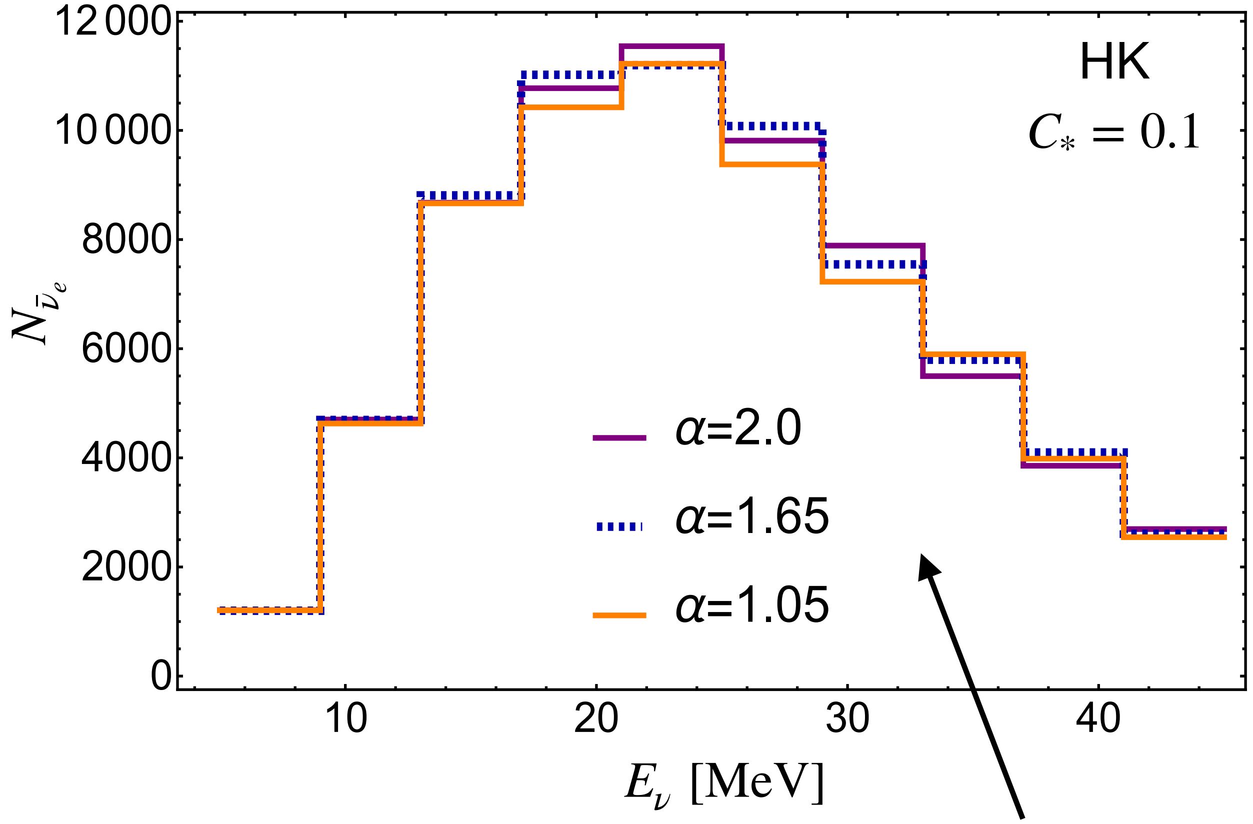
Poor sensitivity to the spectral index

Will be able to constrain the amplitude of turbulence



Constraining turbulence in Hyper-K





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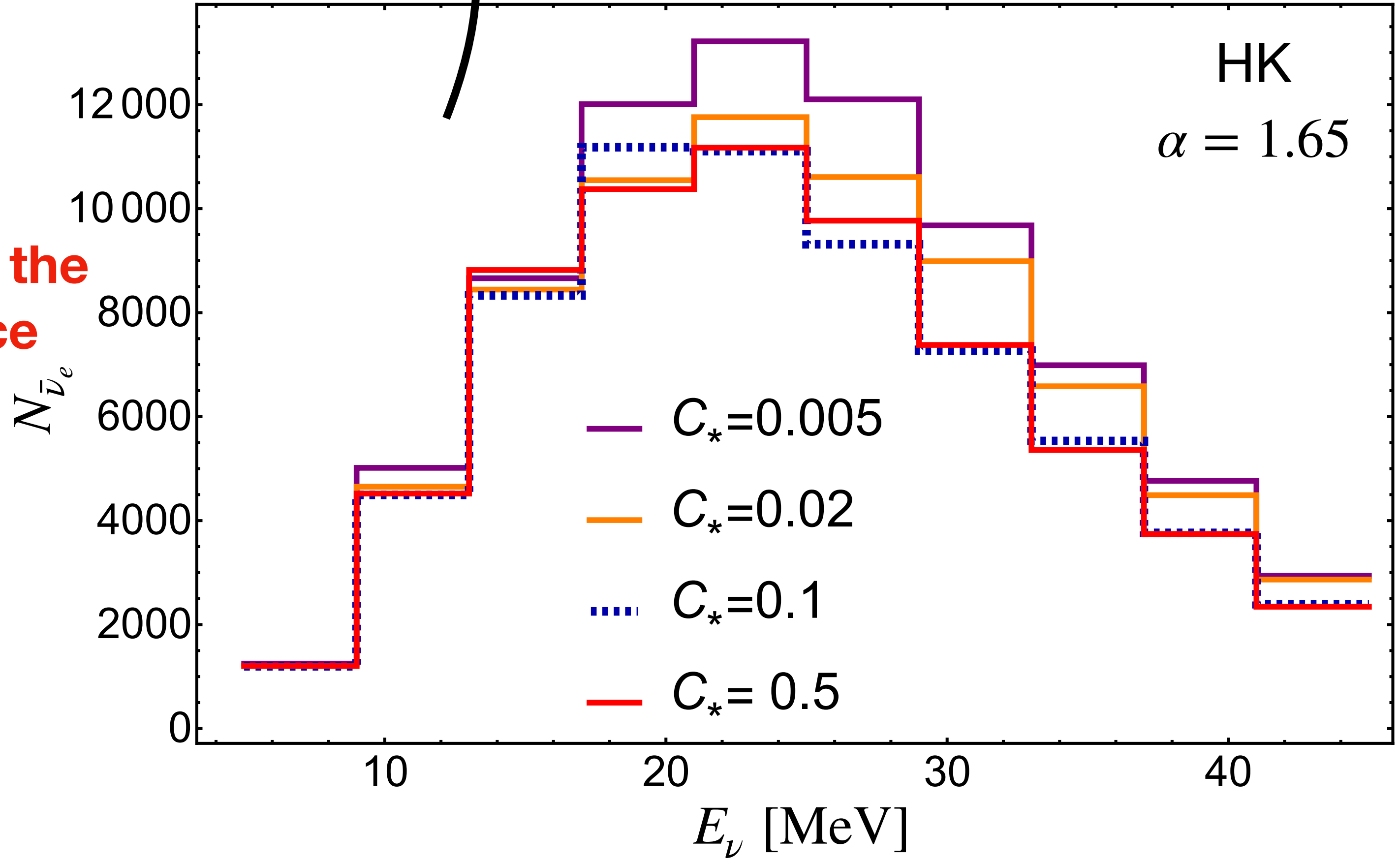


Will be able to constrain the amplitude of turbulence

Dessert: Takeaways

Upcoming neutrino detectors can constrain turbulence in supernovae

- **Neutrinos** can act as **probes of shock propagation** in CCSNe: only messengers with shock wave information from deep inside

- **Turbulence** can develop behind shocks due to various **instabilities**

- The **double-dip feature** associated with the FS and RS can be **washed out due to turbulence**

- **DUNE and Hyper-K** can help **constrain the amplitude of turbulence** for a galactic supernova, however **no significant information** is obtained regarding the **spectrum of turbulence.**

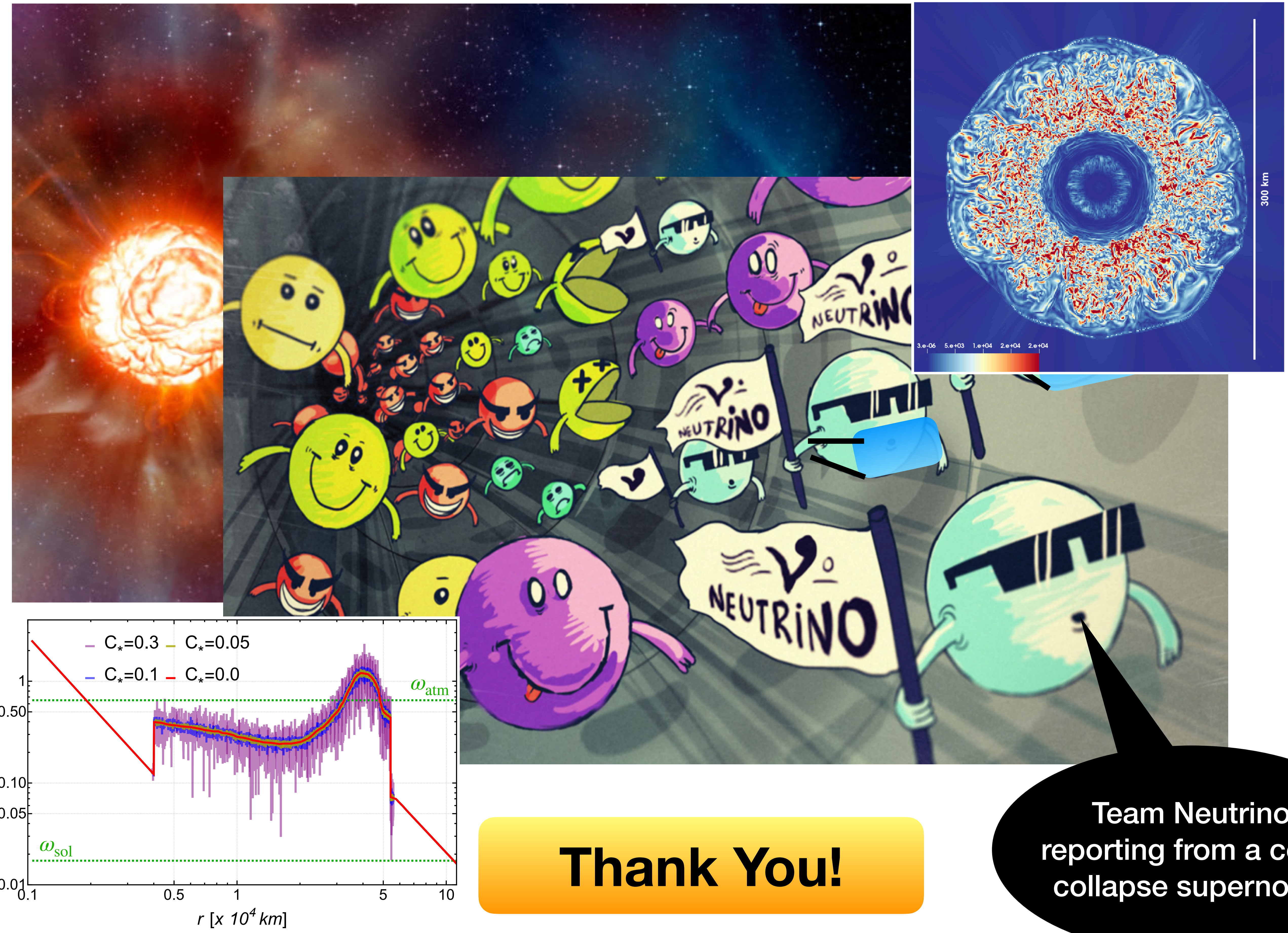
Questions and caveats

- More realistic: 3D turbulence (k_x, k_y, k_z), evolving matter density profile

- Large amplitude effects

- Collective oscillation effects: fast-flavor instabilities

- ….



Thank You!

Team Neutrino reporting from a core-collapse supernova!