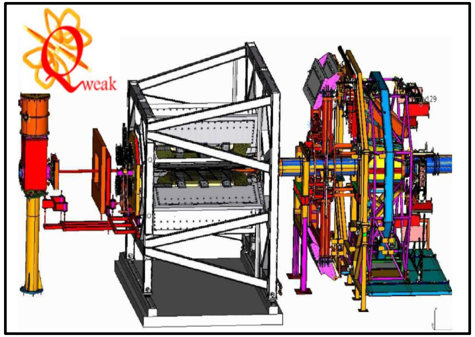
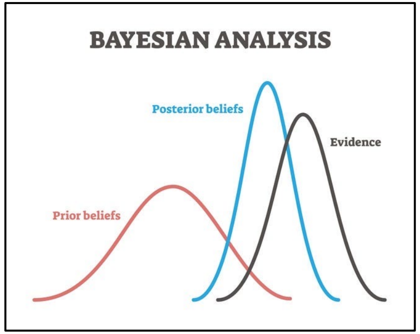
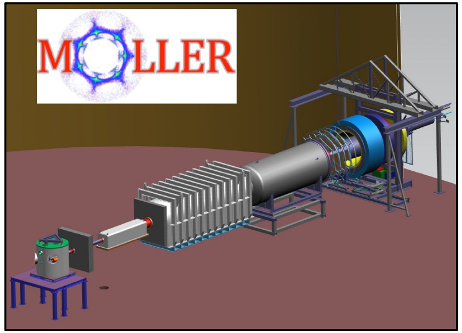
Enhancing Measurement Precision in PVES Experiments:
The Impact of Bayesian Analysis on the Results of the Qweak and MOLLER Experiments







Elham Gorgannejad

Dr. Wouter Deconinck

Winter Nuclear and Particle Physics Conference, 2024

University of Manitoba

MOLLER

Parity Violating Electron Scattering (PVES) Experiment

SLAC

https://www.cnet.com/pictures/slac-a-2-mile-particle-accelerator-next-to-stanford/

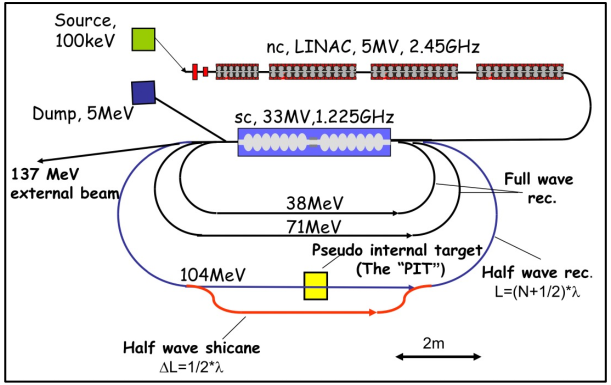
MIT–Bates

https://bateslab.mit.edu

Jefferson Lab

https://en.wikipedia.org/wiki/Thomas_Jefferson_National_Accelerator_Facility

Mainz



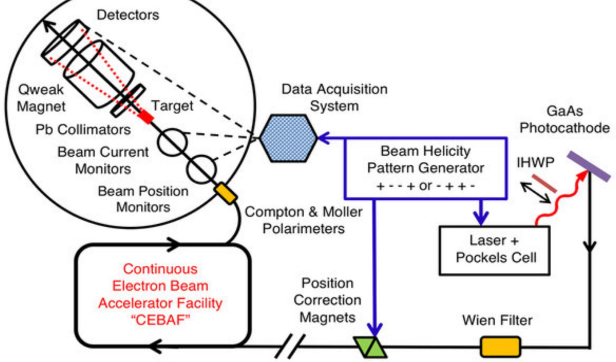
MESA accelerator layout
https://www.mesa.uni-mainz.de/eng/

Parity Violating Asymmetry

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

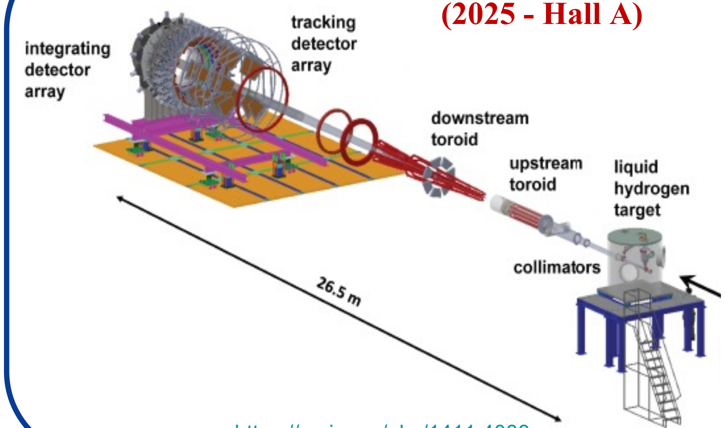
- ❖ A_{PV} arises from the interference of the weak and electromagnetic amplitudes
- ❖ Values of A_{PV} in the range from 10^{-4} to 10^{-8} can be measured with good accuracy

Qweak Experiment (Completed - Hall C)



https://arxiv.org/abs/1202.1255

MOLLER Experiment (2025 - Hall A)



https://arxiv.org/abs/1411.4088

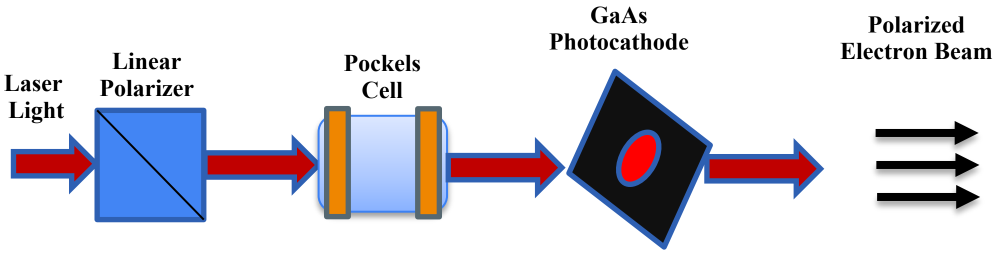
1.13 GeV Longitudinally polarized Electron Beam
Measure A_{PV} in Electron-Proton Scattering

11 GeV Longitudinally Polarized Electron Beam
Measure A_{PV} in Electron-Electron Scattering

Note: 3.35 GeV for an ancillary measurement (Pion Production)

Polarized Electron Source

- Electron polarization is determined by the polarization of the incident laser light
- Polarization of the laser light is controlled by the polarity of the voltage across a Pockels cell
- Pockels cell determines the sign of the longitudinal polarization of the emitted electron bunch
- Injector to provide Longitudinally polarized electrons
- Magnets in the arcs bend the beam from one Linac arm to the other
- Liquid Helium for ultra-low-temperature



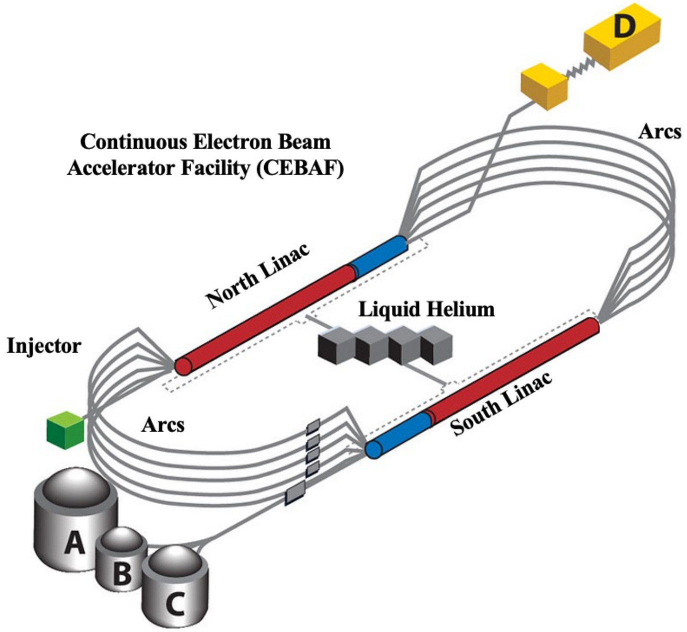
Laser Light

Linear Polarizer

Pockels Cell

GaAs Photocathode

Polarized Electron Beam

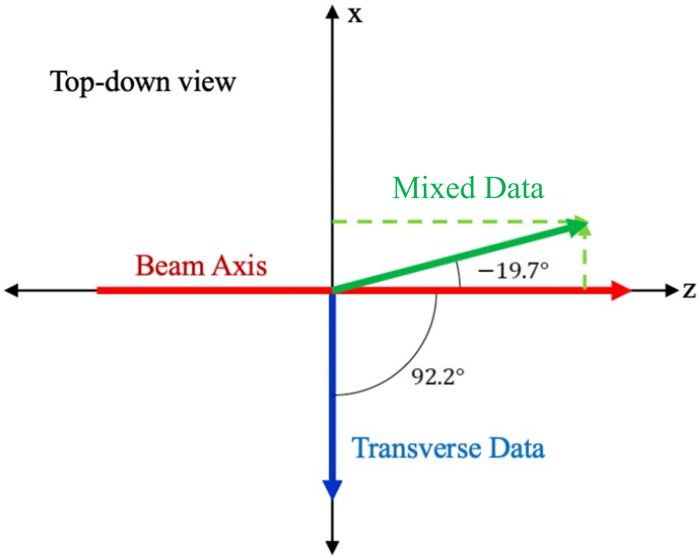


Continuous Electron Beam Accelerator Facility (CEBAF)

Mixed Data vs Transverse Data

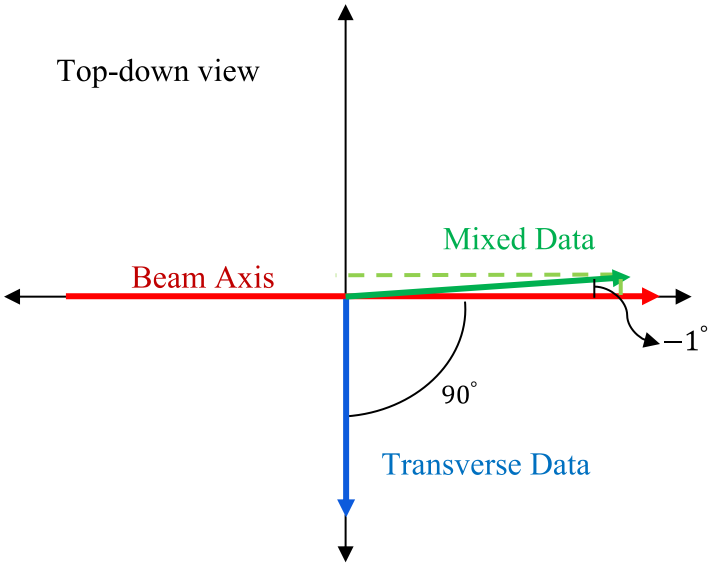
Qweak Experiment (Completed)

Ancillary Measurement



$\theta_{P(L)} = -19.7^{\circ} \pm 1.9^{\circ}$ (108 hours of data-taking)
$\theta_{P(L)} = 92.2^{\circ} \pm 1.9^{\circ}$ (4.3 hours of data-taking)

MOLLER Experiment (2025)

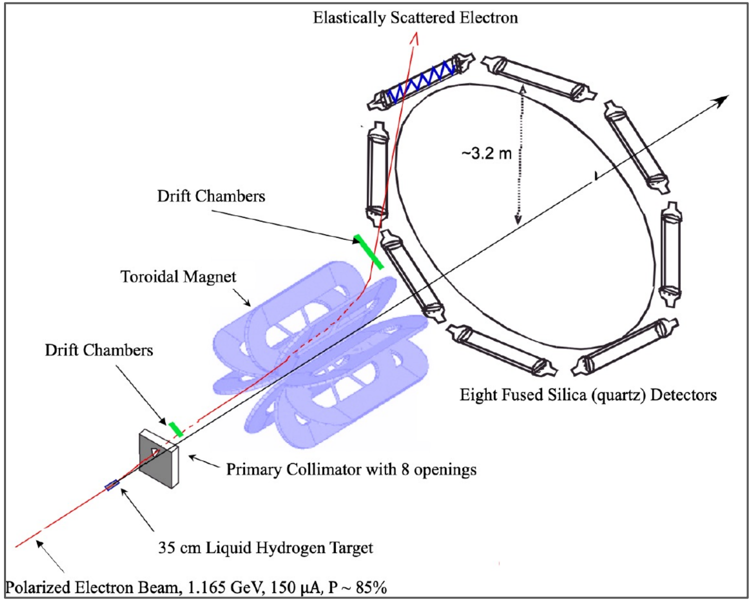


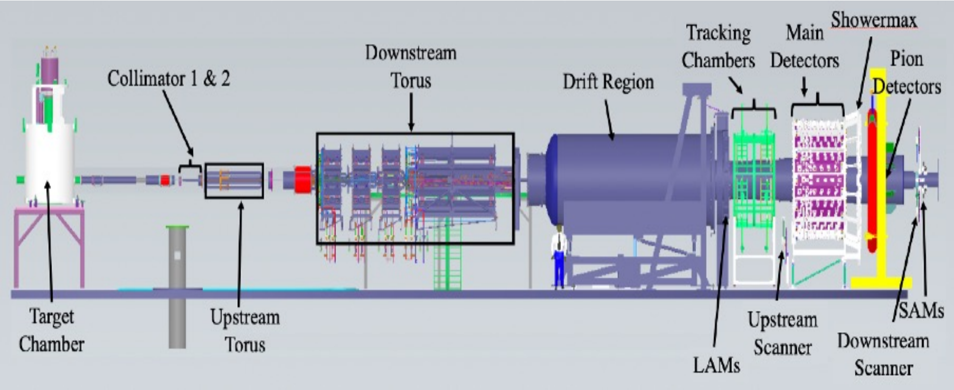
$\theta_{P(L)} = 0^{\circ} \pm 1^{\circ}$ (7430 hours of data-taking (90%))
$\theta_{P(L)} = 90^{\circ} \pm 1^{\circ}$ (826 hours of data-taking (10%))

Geometries of the Experiments

Qweak Experiment (Completed)

MOLLER Experiment (2025)

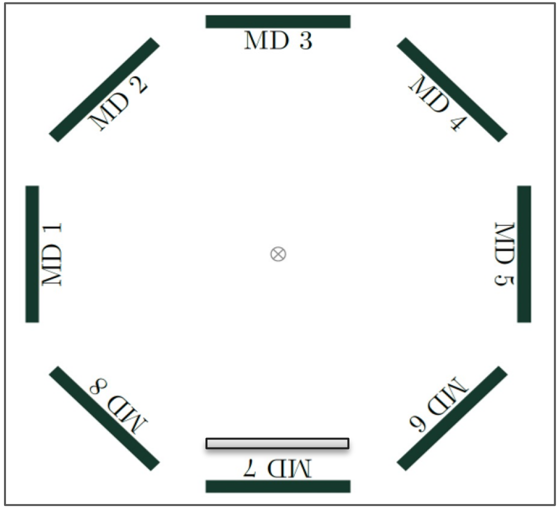




Geometries of the Experiments

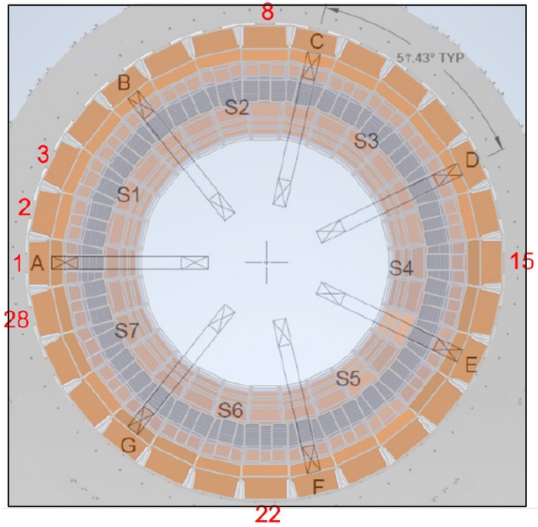
Qweak Experiment (Completed)

8 Detectors

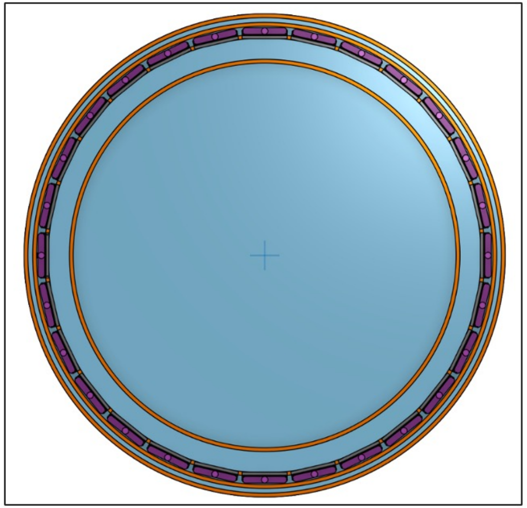


MOLLER Experiment (2025)

Main Detector, Ring(5), 84 Detectors



Pion Detector, 28 Detectors



Note: In both the Qweak and MOLLER experiments, charged pions are an important source of background noise. By tracking these pions, we can better understand and correct the background in our measurements.

Bayesian Analysis

Bayesian Analysis properties:

- Using probability statements
- Treating the parameters in a statistical model as random
- Using a prior distribution to quantify our knowledge about the parameter
- Using the conditional distribution of parameters, given the data to update our prior knowledge
- Update from the prior to the posterior via the Bayes theorem

$$\boxed{P(\theta|y) = \frac{P(y|\theta)P(\theta)}{P(y)}}$$

Bayes' rule

- $P(\theta|y)$ = Probability of the model parameters (θ) conditional on the data (y) = Posterior distribution
- $P(y|\theta)$ = Probability of the data (y) given the model parameters (θ) = Likelihood function
- $P(\theta)$ = Probability of model parameters = Prior distribution
- $P(y)$ = Normalizing factor

Posterior distribution ~ Likelihood function * Prior distribution

Analysis Steps: Qweak Experiment

Experiment

Asymmetry measurements in the experiment, per detector, for 2 data sets ((8) mixed and (8) transverse measurements)

Analysis Inputs

Feed the measured asymmetries, dilution factors, and polarization angles in the model

Analysis Outputs

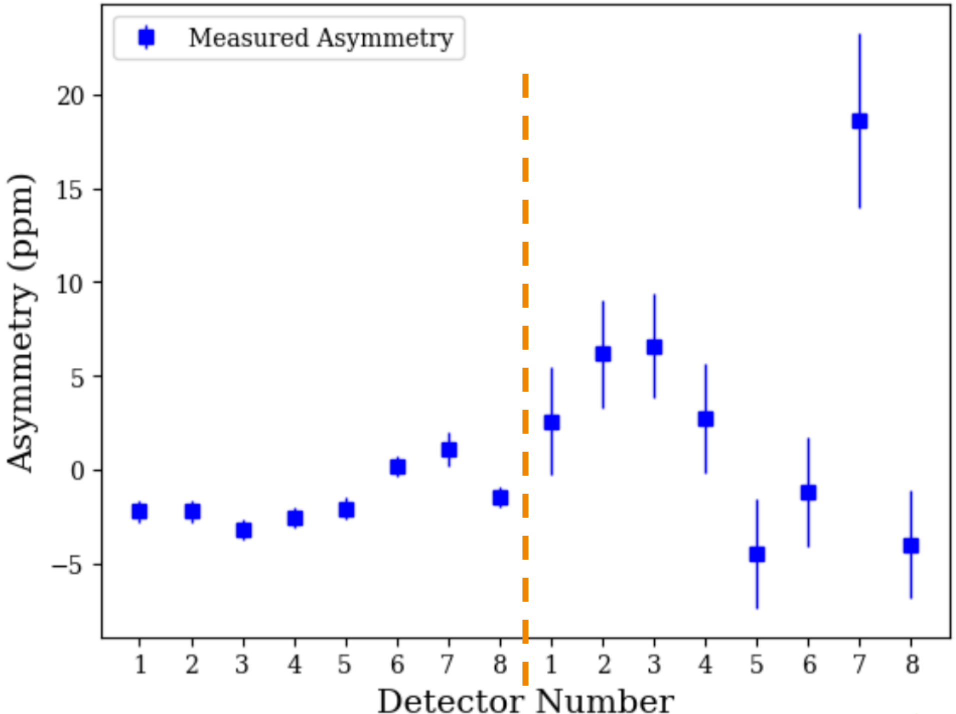
Extract Asymmetry components ($A_{eL}, A_{piL}, A_{eT}, A_{piT}$)

Fitting to the Model

Substitute Asymmetry components in the formula and regenerate the asymmetry values (Fitted Asymmetry)

Analysis Steps: Qweak Experiment

Experiment



Mixed Data Set

Transverse Data Set

Analysis Steps: Qweak Experiment

Experiment

Asymmetry measurements in the experiment, per detector, for 2 data sets ((8) mixed and (8) transverse measurements)

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Fitting to the Model

Substitute Asymmetry components in the formula and regenerate the asymmetry values (Fitted Asymmetry)

Analysis Steps: Qweak Experiment

Analysis Inputs

$$\mathrm{A}^{ij}_{measured} = (1 - f_{pi}) \times [A_{eL} \times \cos(\theta_P^j) + A_{eT} \times \sin(\theta_P^j) \times sin\phi^i] + f_{pi} \times [A_{piL} \times \cos(\theta_P^j) + A_{piT} \times \sin(\theta_P^j) \times sin\phi^i]\}$$

Analysis Outputs

Analysis Steps: Qweak Experiment

Experiment

Asymmetry measurements in the experiment, per detector, for 2 data sets ((8) mixed and (8) transverse measurements)

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Substitute Asymmetry components in the formula and regenerate the asymmetry values (Fitted Asymmetry)

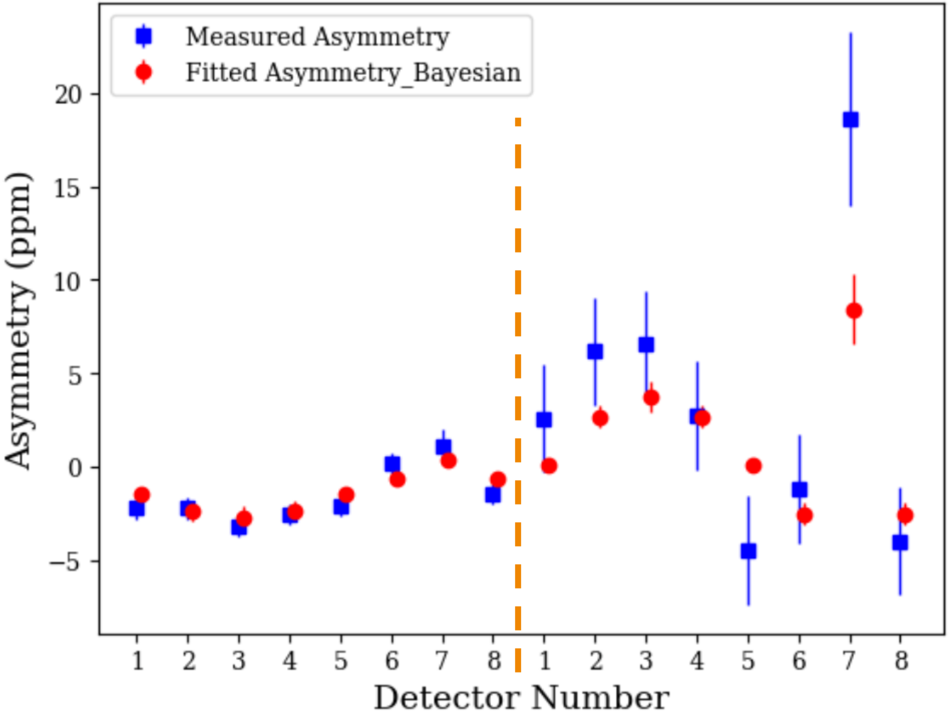
Analysis Steps: Qweak Experiment

Fitting to the Model

$$\mathbf{A}^{ij}_{Fitted} = (1 - f_{pi}) \times [A_{eL} \times \cos(\theta_P^j) + A_{eT} \times \sin(\theta_P^j) \times sin\phi^i] +$$

$$f_{pi} \times [A_{piL} \times \cos(\theta_P^j) + A_{piT} \times \sin(\theta_P^j) \times sin\phi^i]\}$$

Analysis Steps: Qweak Experiment



Mixed Data Set

Transverse Data Set

Qweak Experiment Analysis: Bayesian vs Frequentist

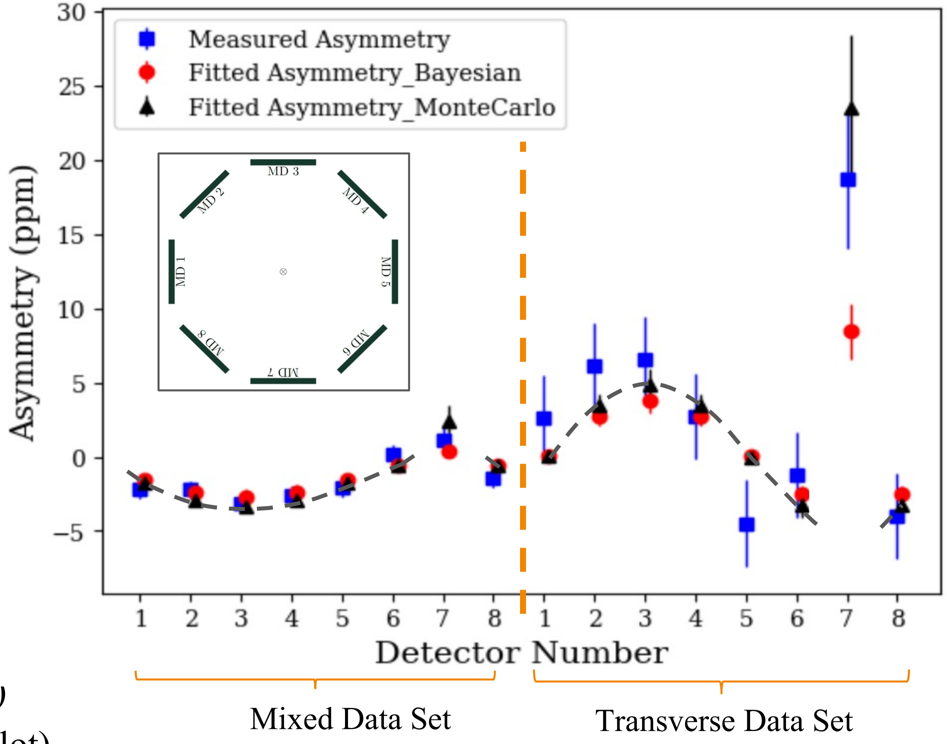
Method (1)

Monte Carlo Minimization

https://scholarworks.wm.edu/etd/1550153997/
https://arxiv.org/abs/1910.14591

Method (2)

Bayesian Analysis



$\theta_{P(L)} = -19.7^\circ \pm 1.9^\circ$ (The first 8 values in the plot)

$\theta_{P(L)} = 92.2^\circ \pm 1.9^\circ$ (The second 8 values in the plot)

Analysis Steps: MOLLER Experiment

Simulations

Mock asymmetry in the experiment, per detector, for 2 data sets
(28 mixed and transverse measurements for the pion detector and 84 for the main detector)

Analysis Inputs

Feed mock asymmetries, dilution factors, and polarization angles in the model

Analysis Outputs

Extract asymmetry components ($A_{eL}, A_{piL}, A_{eT}, A_{piT}$)

Fitting to the Model

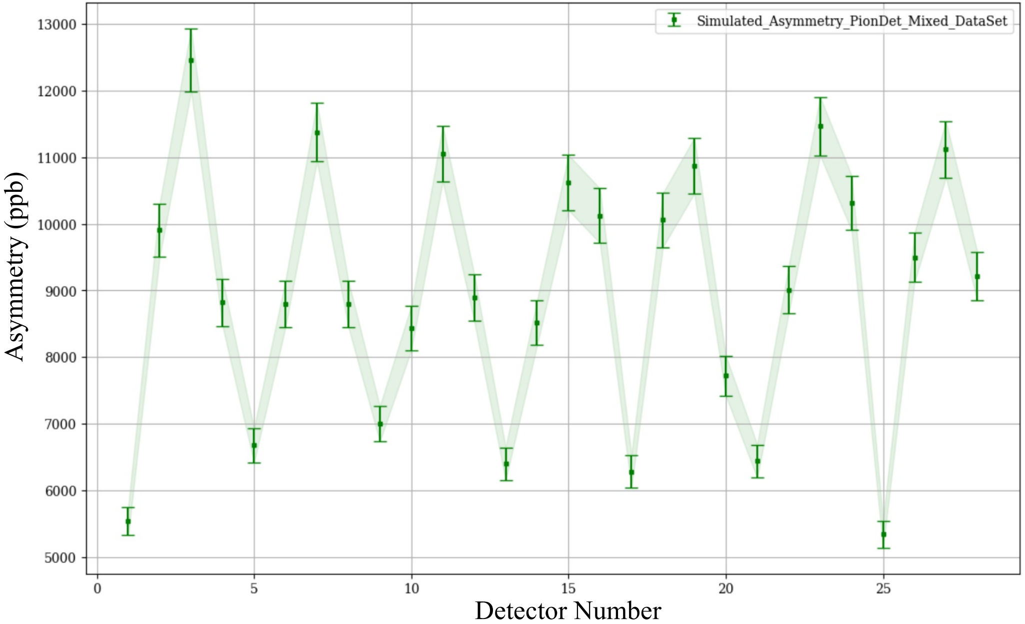
Substitute asymmetry components in the formula and regenerate the asymmetry values (Fitted Asymmetry)

Analysis Steps: MOLLER Experiment

Simulations

$$A^{ij}_{Simulated} = (1 - f_{pi}) \times [A_{eL} \times \cos(\theta_P^j) + A_{eT} \times C_e \times \sin(\theta_P^j)] + f_{pi} \times [A_{piL} \times \cos(\theta_P^j) + A_{piT} \times C_{pi} \times \sin(\theta_P^j)]\}$$

Simulated Asymmetry (Pion Detector-Mixed DataSet)



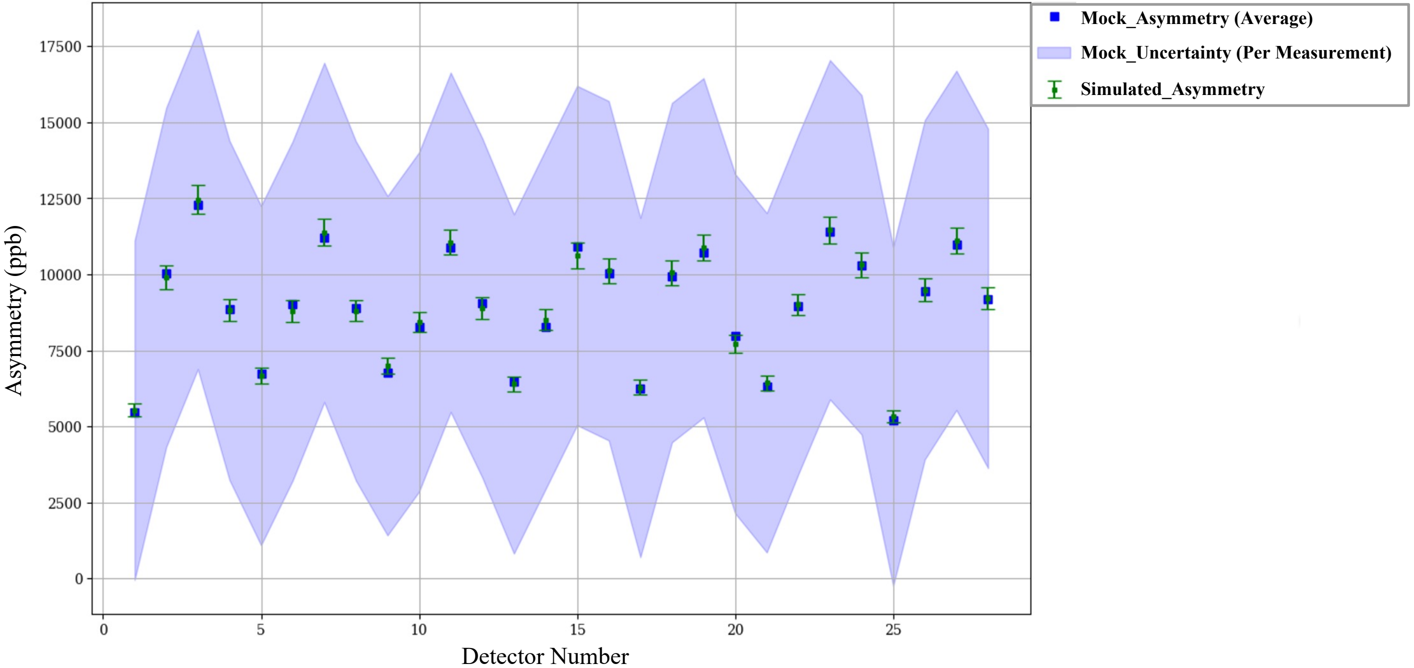
Analysis Steps: MOLLER Experiment

Simulations

$$A^{ij}_{MocK} = A^{ij}_{Simulated} \pm Measured\ Uncertainty$$

$$Measured\ Uncertainty = \frac{1}{\sqrt{rate \times time}}$$

Simulated Asymmetry vs Mock Asymmetry (Pion Detector-Mixed DataSet)



Analysis Steps: MOLLER Experiment

Simulations

Mock asymmetry in the experiment, per detector, for 2 data sets
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Feed mock asymmetries, dilution factors, and polarization angles in the model

Analysis Outputs

Extract asymmetry components $(A_{eL}, A_{piL}, A_{eT}, A_{piT})$

Fitting to the Model

Substitute asymmetry components in the formula and regenerate the asymmetry values (Fitted Asymmetry)

Analysis Steps: MOLLER Experiment

Analysis Inputs

$$\mathrm{A}_{Mock}^{ij} = (1 - f_{pi}) \times [A_{eL} \times \cos(\theta_P^j) + A_{eT} \times C_e \times \sin(\theta_P^j)] + f_{pi} \times [A_{piL} \times \cos(\theta_P^j) + A_{piT} \times C_{pi} \times \sin(\theta_P^j]\}$$

Analysis Outputs

Analysis Steps: MOLLER Experiment

Simulations

Mock asymmetry in the experiment, per detector, for 2 data sets
(28 mixed and transverse measurements for the pion detector and 84 for the main detector)

Analysis Inputs

Feed mock asymmetries, dilution factors, and polarization angles in the model

Analysis Outputs

Extract asymmetry components $(A_{eL}, A_{piL}, A_{eT}, A_{piT})$

Fitting to the Model

Substitute asymmetry components in the formula and regenerate the asymmetry values (Fitted Asymmetry)

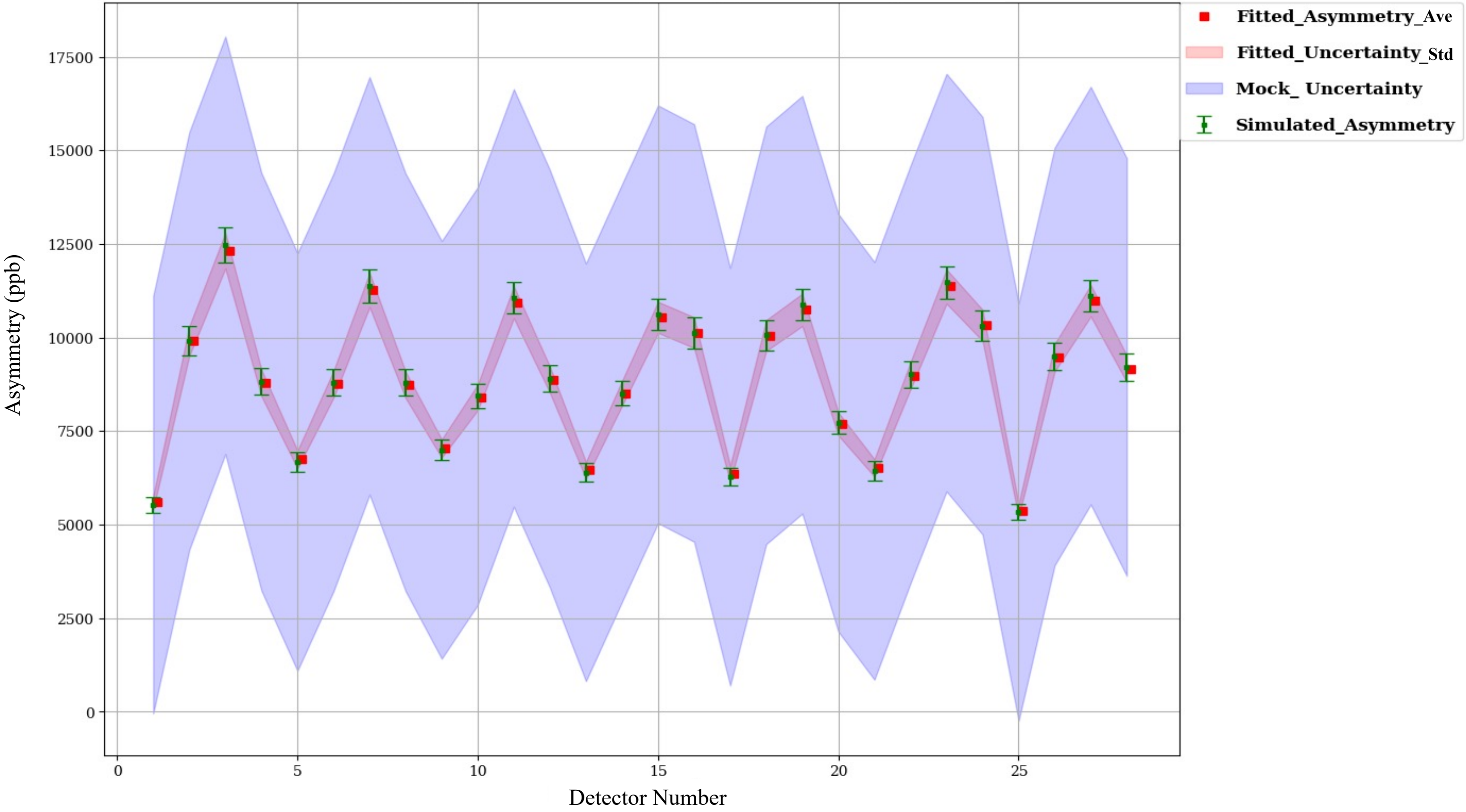
Analysis Steps: MOLLER Experiment

Fitting to the Model

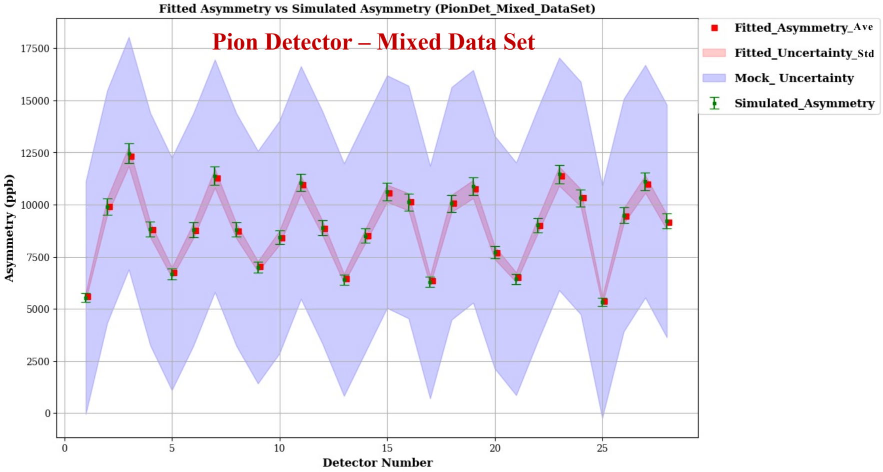
$$\mathrm{A}_{Fitted}^{ij} = (1 - f_{pi}) \times [A_{eL} \times \cos(\theta_P^j) + A_{eT} \times C_e \times \sin(\theta_P^j)] +$$
$$f_{pi} \times [A_{piL} \times \cos(\theta_P^j) + A_{piT} \times C_{pi} \times \sin(\theta_P^j)]\}$$

Bayesian Analysis: MOLLER Experiment

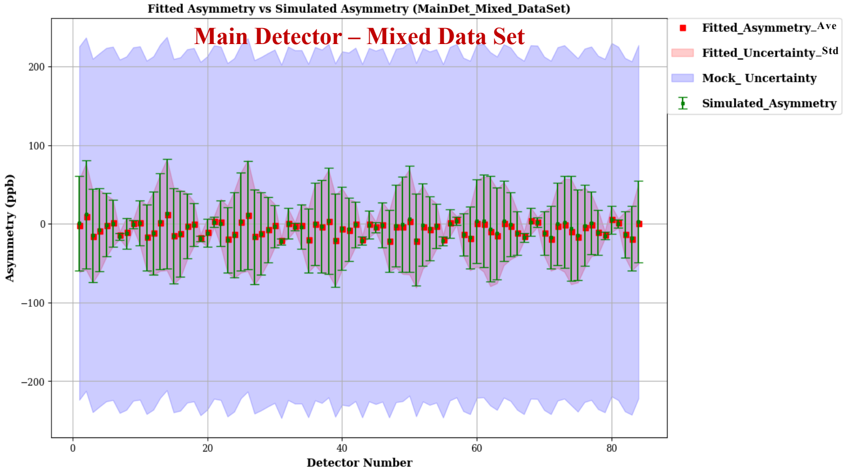
Fitted Asymmetry vs Simulated Asymmetry (Pion Detector-Mixed DataSet)



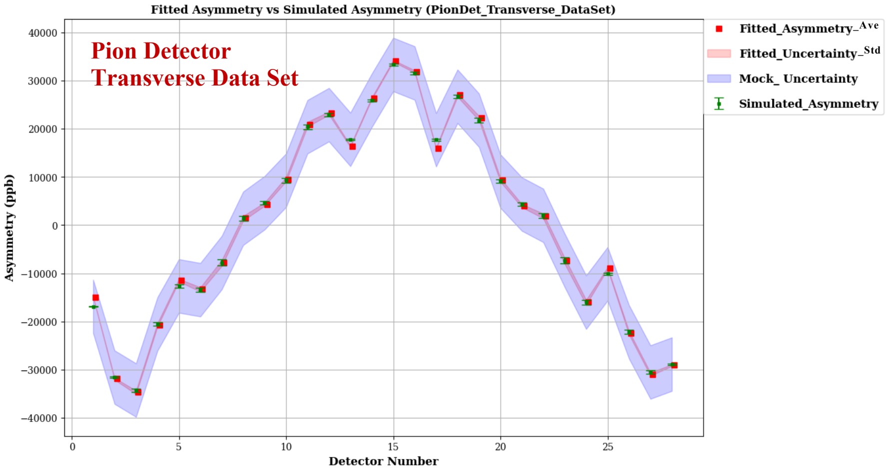
Bayesian Analysis: MOLLER Experiment


Fitted Asymmetry vs Simulated Asymmetry (PionDet_Mixed_DataSet)

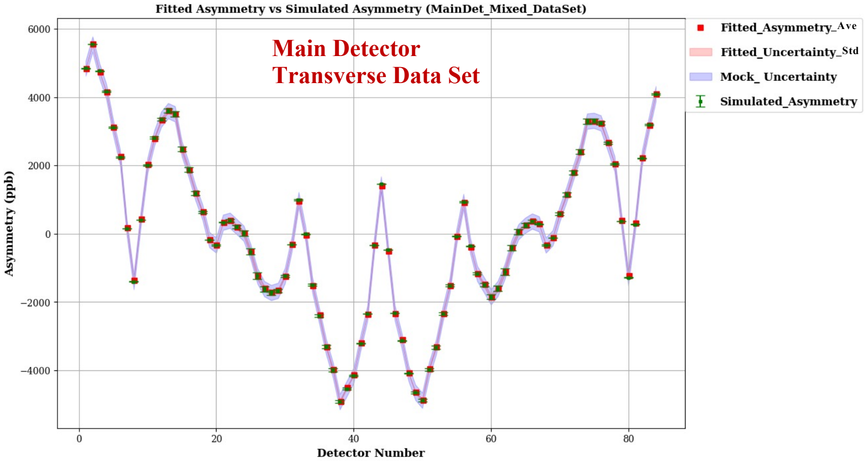
Pion Detector – Mixed Data Set


Fitted Asymmetry vs Simulated Asymmetry (MainDet_Mixed_DataSet)

Main Detector – Mixed Data Set


Fitted Asymmetry vs Simulated Asymmetry (PionDet_Transverse_DataSet)

Pion Detector
Transverse Data Set


Fitted Asymmetry vs Simulated Asymmetry (MainDet_Mixed_DataSet)

Main Detector
Transverse Data Set

Conclusion & Acknowledgement

Conclusion

- Bayesian analysis was introduced as an alternative to Frequentist methods for analyzing data from PVES experiments.
- The method has been applied using two types of inputs: real and mock.
- In the Qweak experiment, there is a good level of agreement between the fitted values and the measured values, except for detector 7.
- In the MOLLER experiment, Bayesian analysis is capable of correcting the values based on the model, even when the inputs are noisy.

Next steps

- Address the issue in the Qweak experiment
- Evaluate the method under various assumptions in the MOLLER experiment

Team Members:

- Elham Gorgannejad (U. Manitoba)
- Dr. Wouter Deconinck (U. Manitoba)
- Dr. David Armstrong (William & Mary)

Funding sources:

- Natural Sciences and Engineering Research Council of Canada (NSERC)
- Canada Foundations for Innovation (CFI)
- Canadian Institute of Nuclear Physics (CINP) graduate fellowship

NSERC CRSNG

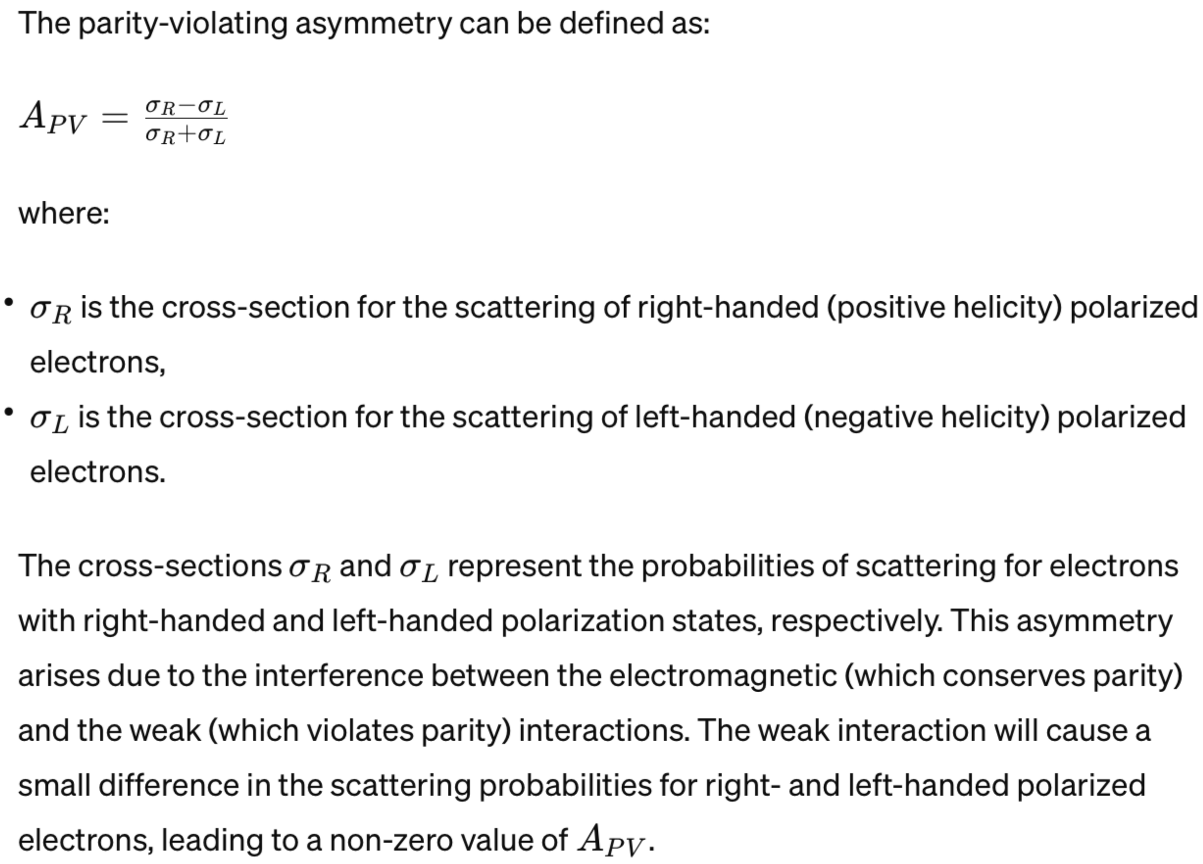
INNOVATION
Canada Foundation for Innovation
Fondation canadienne pour l'innovation

CINP ICPN

Thank you!

Backup slides

Relationship between the Polarization and Parity Violating Asymmetry



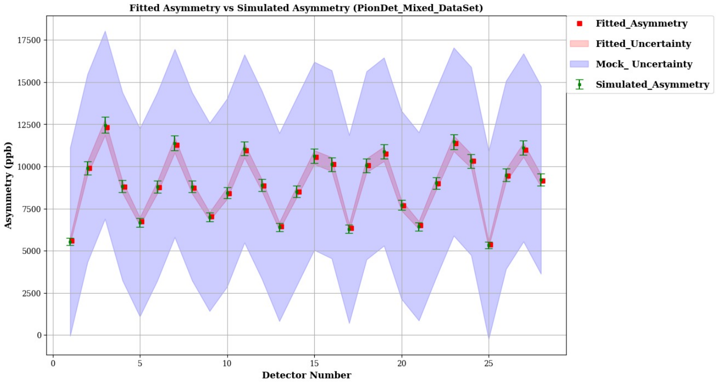
The parity-violating asymmetry can be defined as:

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

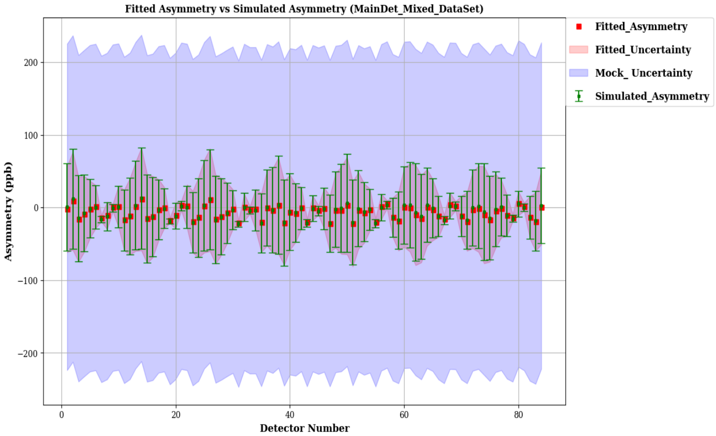
where:

- σ_R is the cross-section for the scattering of right-handed (positive helicity) polarized electrons,
- σ_L is the cross-section for the scattering of left-handed (negative helicity) polarized electrons.

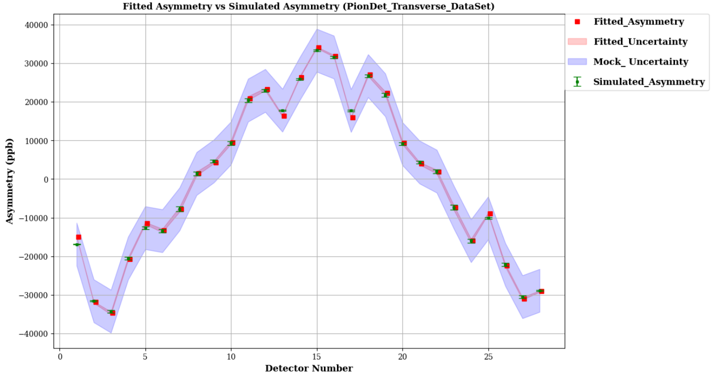
The cross-sections σ_R and σ_L represent the probabilities of scattering for electrons with right-handed and left-handed polarization states, respectively. This asymmetry arises due to the interference between the electromagnetic (which conserves parity) and the weak (which violates parity) interactions. The weak interaction will cause a small difference in the scattering probabilities for right- and left-handed polarized electrons, leading to a non-zero value of A_{PV}.



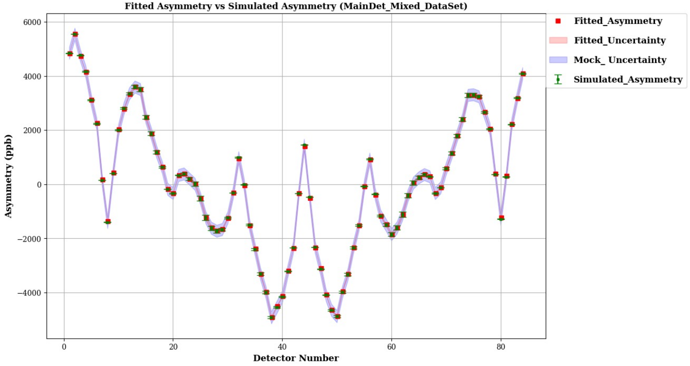
Longitudinal Pion asymmetry varies as a sine function of phi (detectors placement and the geometries or the polarization variation?!)



The electron longitudinal asymmetry is a constant value but, because of the impact of the pions, there is a sine wave variation



Transverse Pion asymmetry varies as a cosine function of phi in the azimuthal plane



This kind of variation is because there is a mixture of two electrons in opposite directions. There are different probabilities of the acceptance or rejection of each of the electrons in different detectors

In the presentation, do we need to include the detector displacement in the formula at all? Couldn’t assume it was embedded? In the case of the MOLLER experiment(remoll), it is embedded. Is this not the case for the Qweak’s simulations?

$$A^{ij}_{measured} = (1 - f_{pi}) \times [A_{eL} \times \cos(\theta^j_P) + C_e \times \sin(\theta^j_P)] +$$

$$f_{pi} \times [A_{piL} \times \cos(\theta^j_P) + C_{pi} \times \sin(\theta^j_P]\}$$

$$C_e = A_{eTV} \times \sin(\varphi^j_P) + A_{eTH} \times \cos(\varphi^j_P)$$

$$C_{pi} = A_{piTV} \times \sin(\varphi^j_P) + A_{piTH} \times \cos(\varphi^j_P)$$

MOLLER Experiment

If $\varphi^j_P = 0$

$$A^{ij}_{measured} = (1 - f_{pi}) \times [A_{eL} \times \cos(\theta^j_P) + A_{eT} \times \sin(\theta^j_P) \times sin\phi^i] +$$

$$f_{pi} \times [A_{piL} \times \cos(\theta^j_P) + A_{piT} \times \sin(\theta^j_P) \times sin\phi^i]\}$$

Qweak Experiment