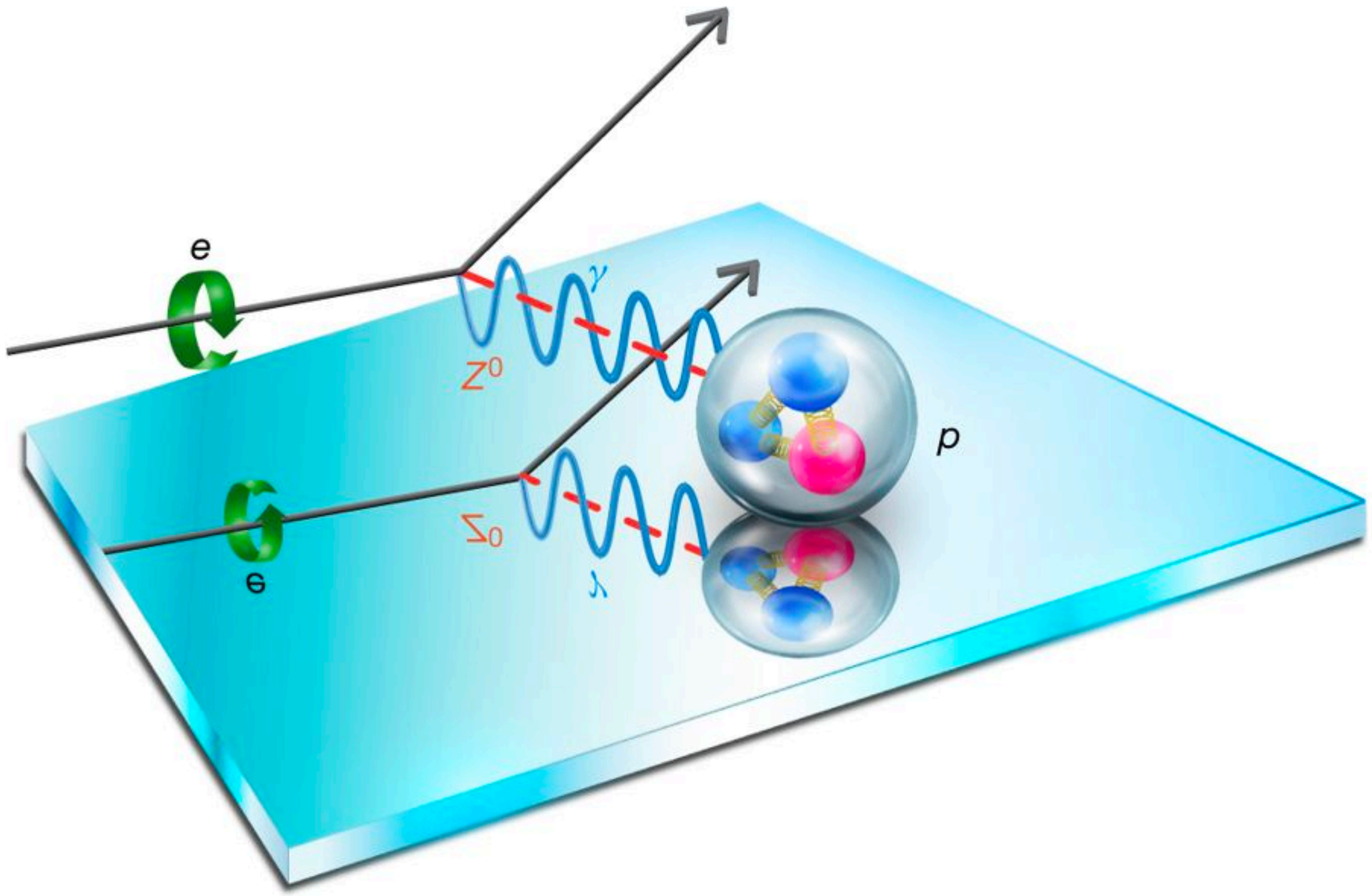
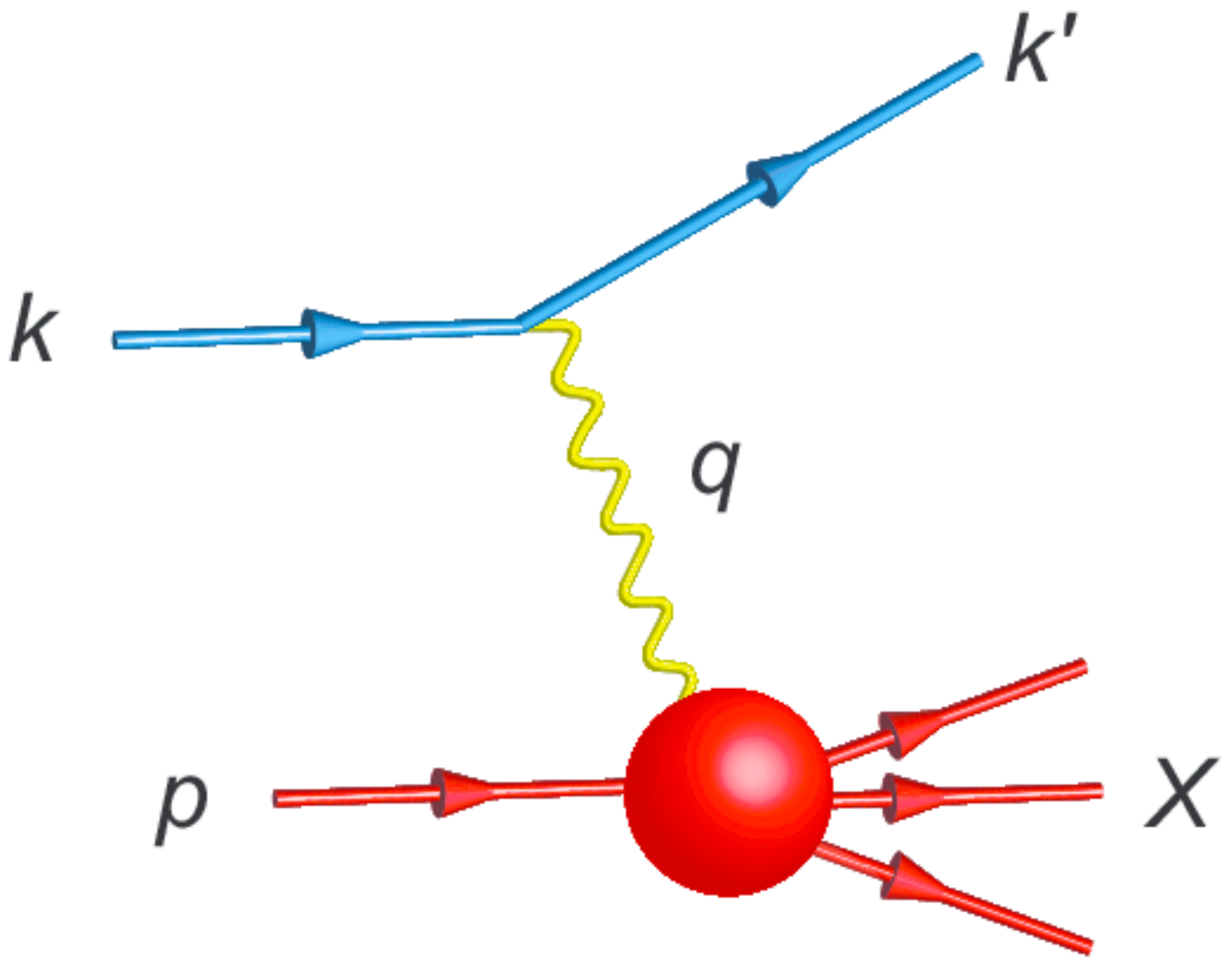
QUADRATIC LEVEL FULL ELECTROWEAK LEPTONIC CORRECTIONS WITH COVARIANT APPROACH

Winter Nuclear & Particle Physics Conference

WNPPC 2024





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- Tree level, one loop level and quadratic level leptonic tensor calculations
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- Results
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MOTIVATION

- The theory of Standard Model (SM) → unifies Electromagnetic, Weak and Strong interactions → can make predictions that match experiments to one part in ten billion.

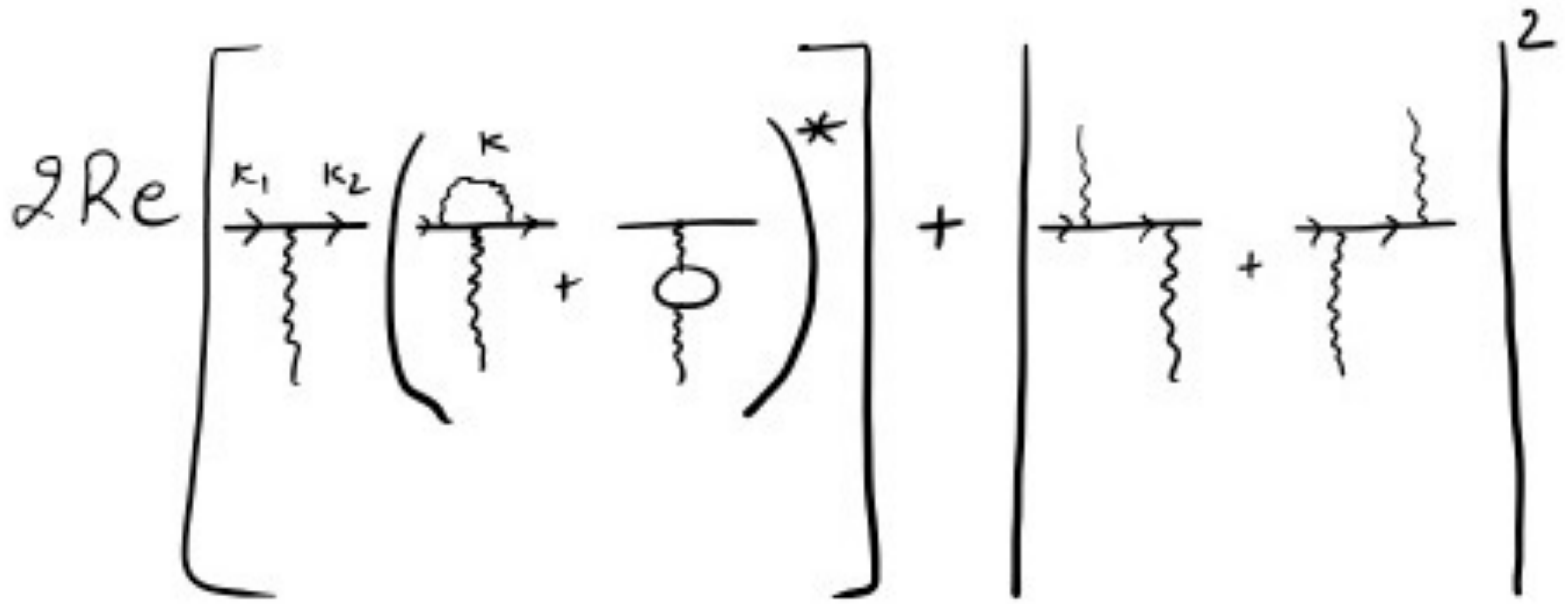
- SM limitations → don't include gravity, dark matter/dark energy existence, hierarchies of scale related to Higgs boson etc.

- Theoretical door open for Beyond the SM (BSM) physics to be observed at TeV scale → **but** till date no concrete evidence of BSM at 13 TeV centre-of-mass energy at LHC.

- **Low energy precision physics** becomes important → provides a way to reach mass scales not directly accessible at existing high-energy colliders.

- We are doing precision physics with full electroweak Parity Violating (PV) Asymmetry → achieve by calculating the higher order corrections up to quadratic level (NNLO α^4) using Covariant/ leptonic tensor approach.

LEPTONIC TENSOR AND INTRODUCTION TO COVARIANT APPROACH



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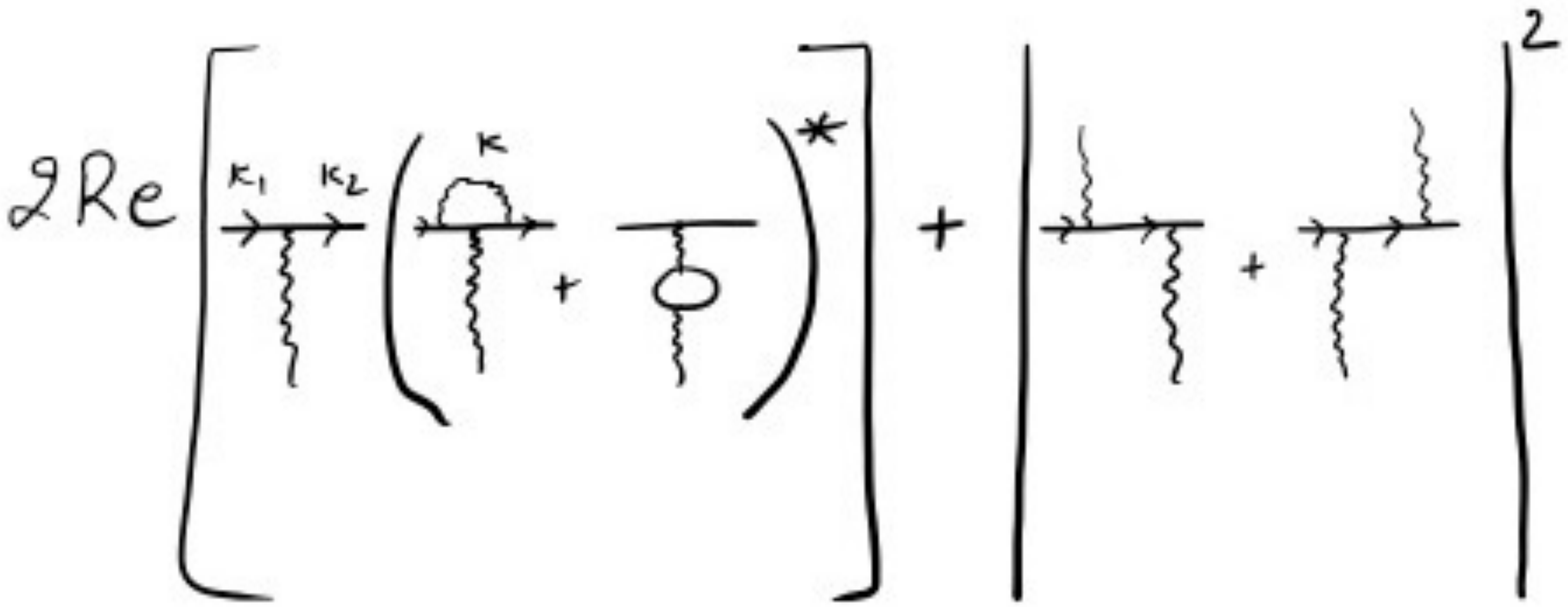

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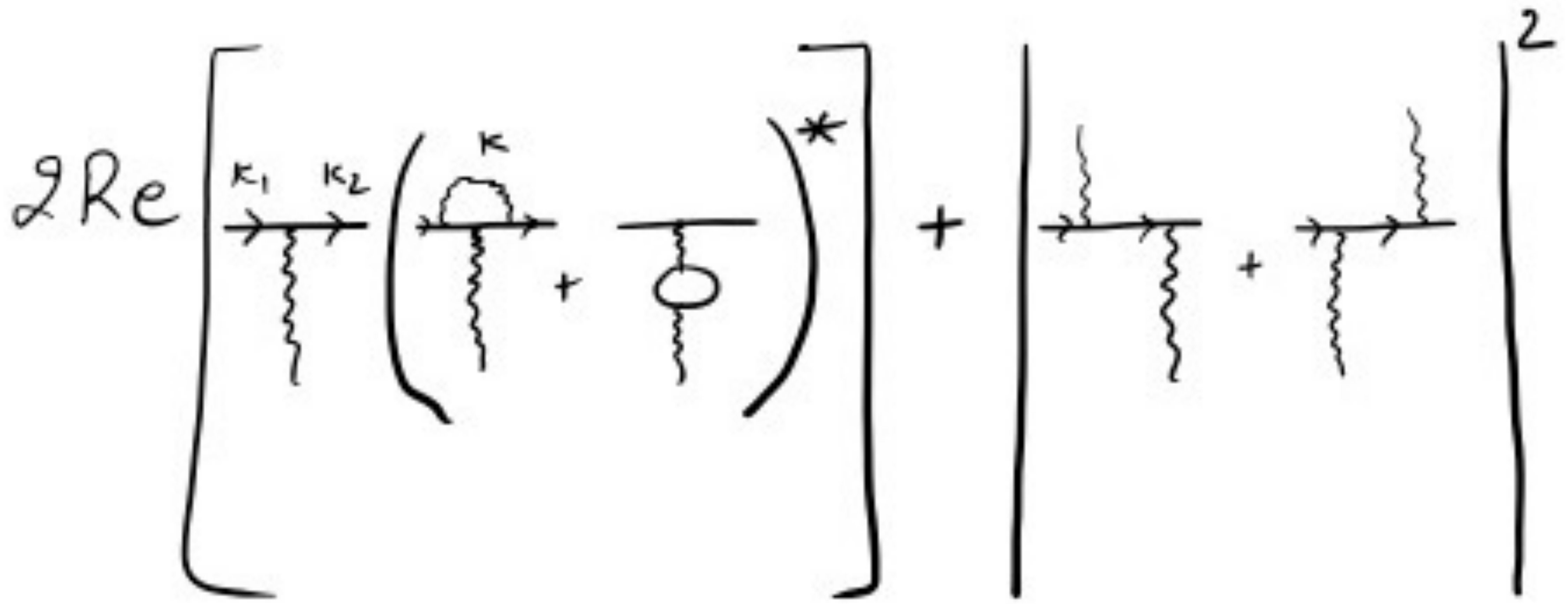
- Recently used by Afanasev et al. (Phys. Rev. D **66**) to calculate QED radiative corrections in processes of exclusive Pion electroproduction.



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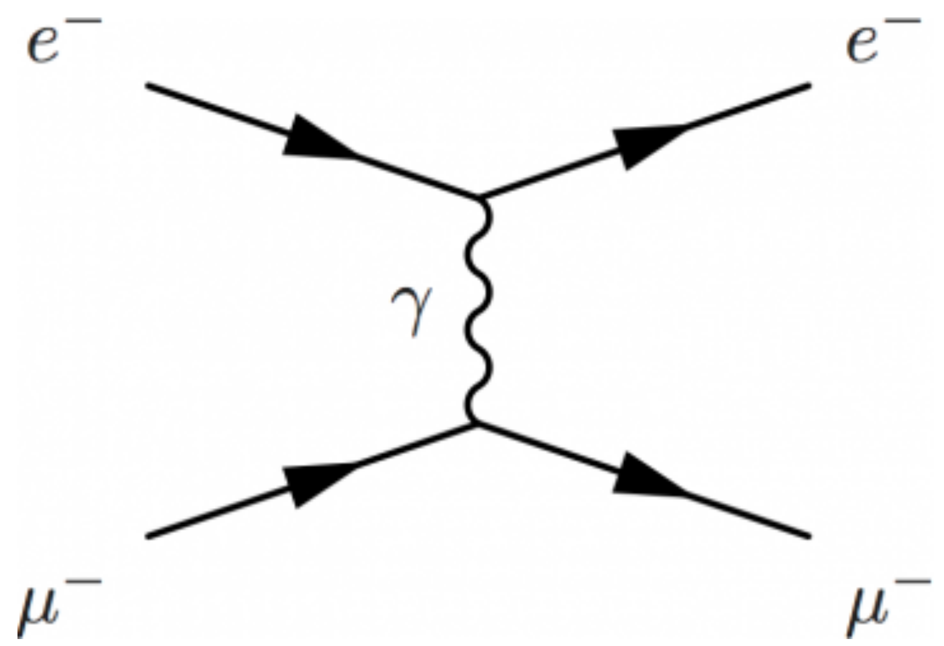

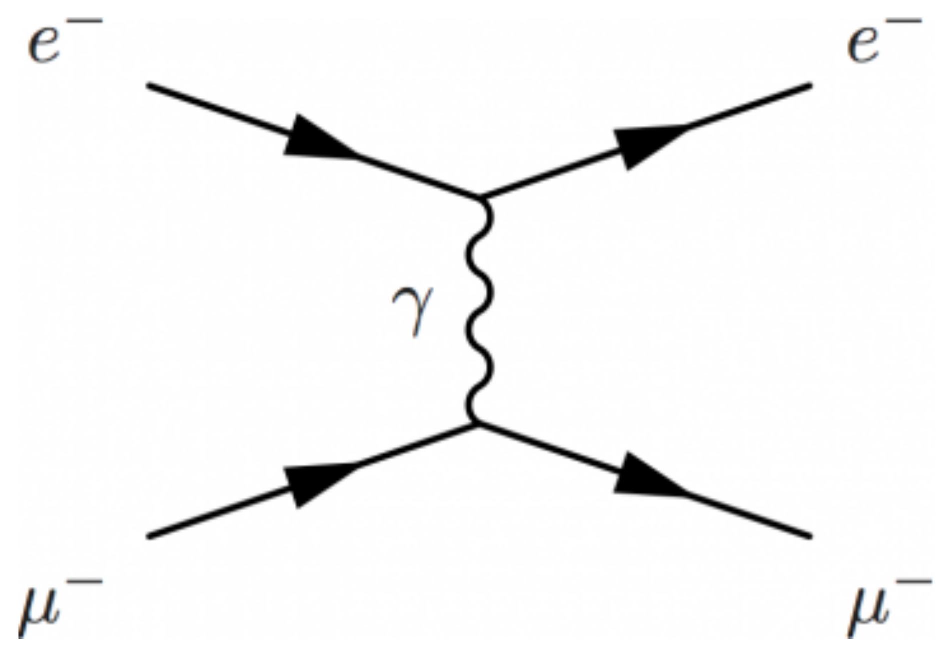

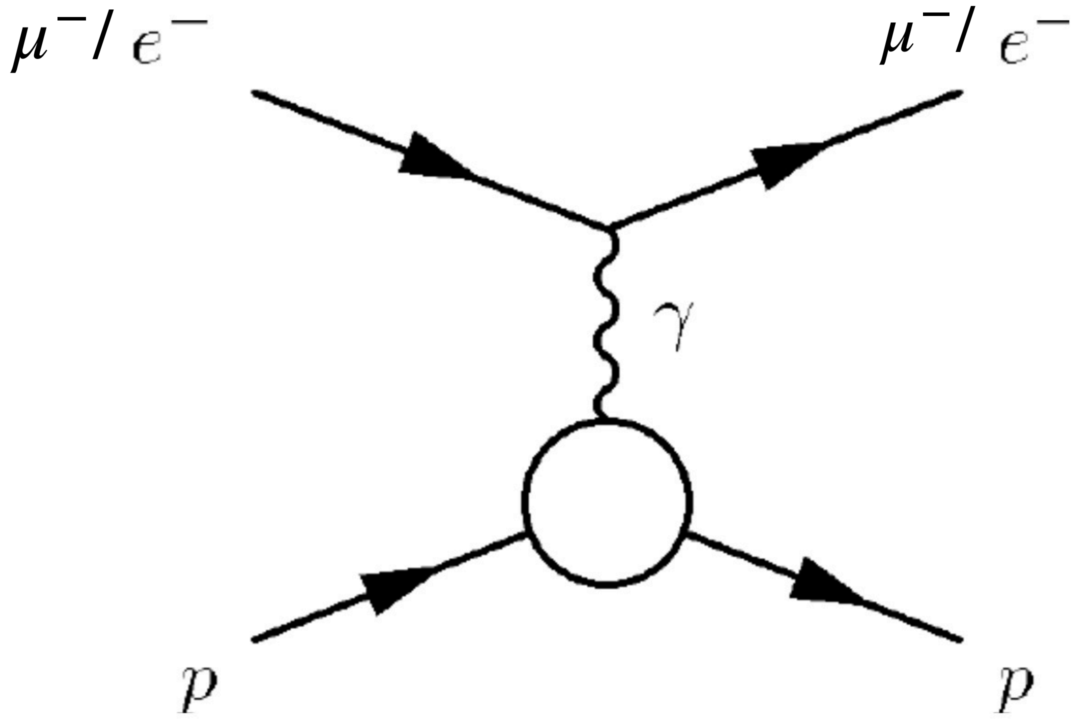

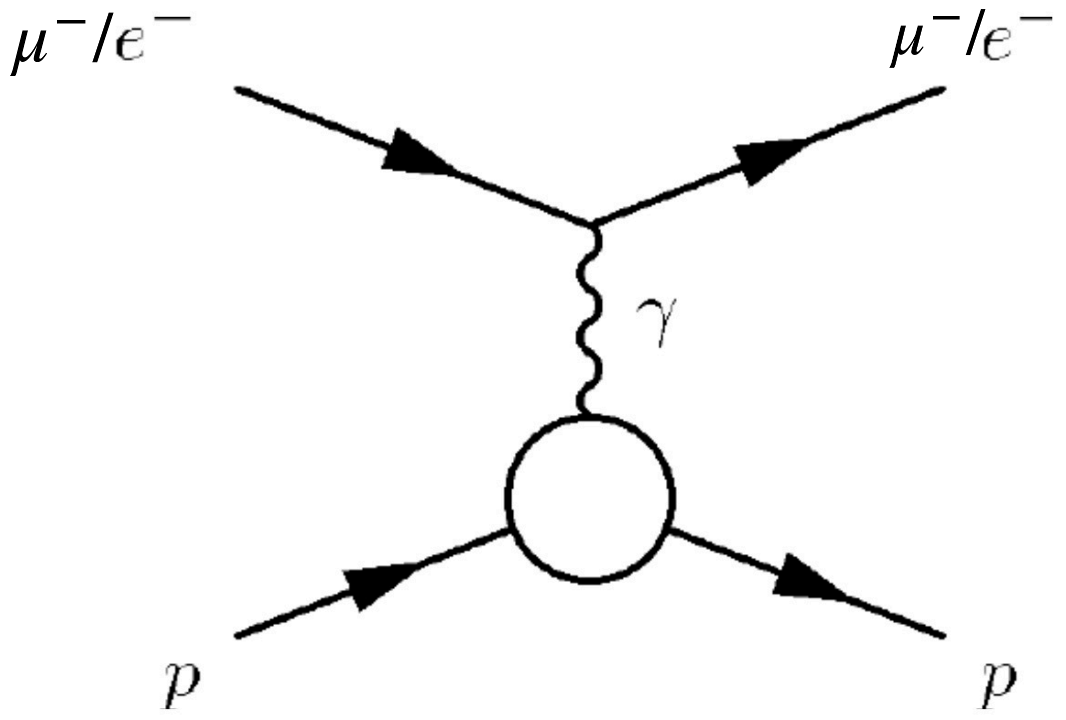

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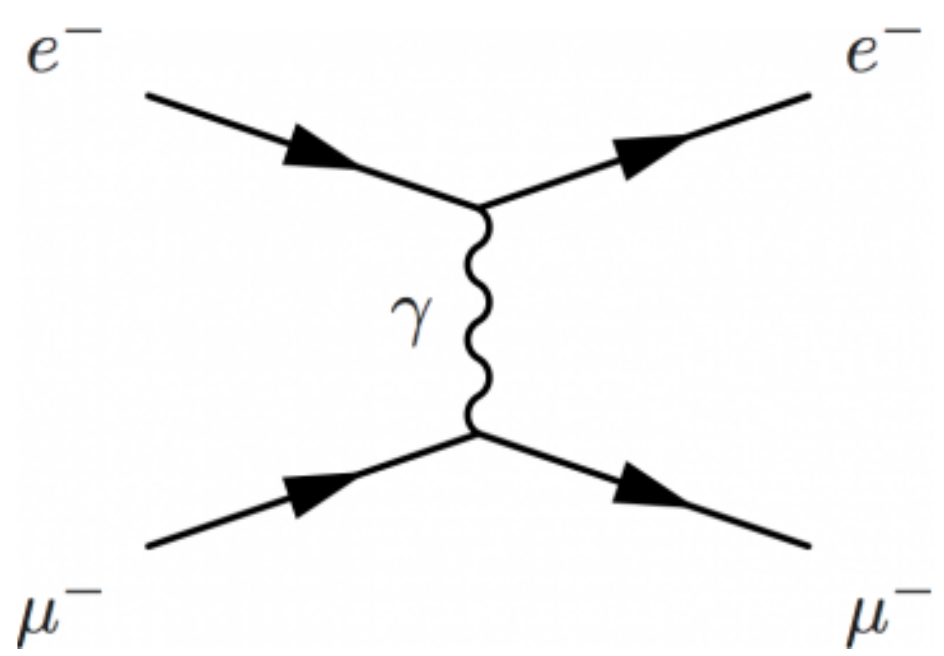

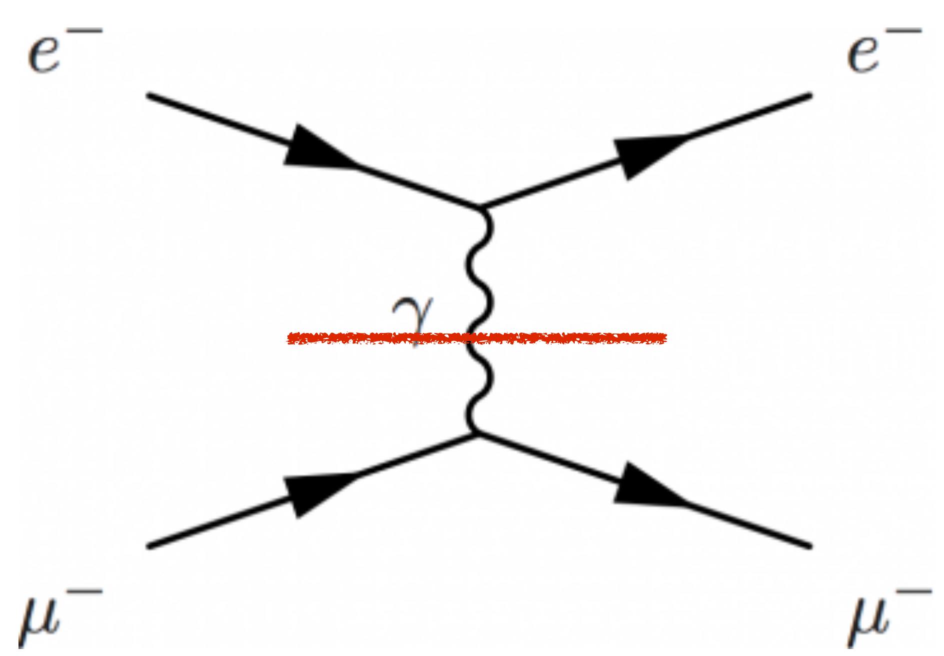

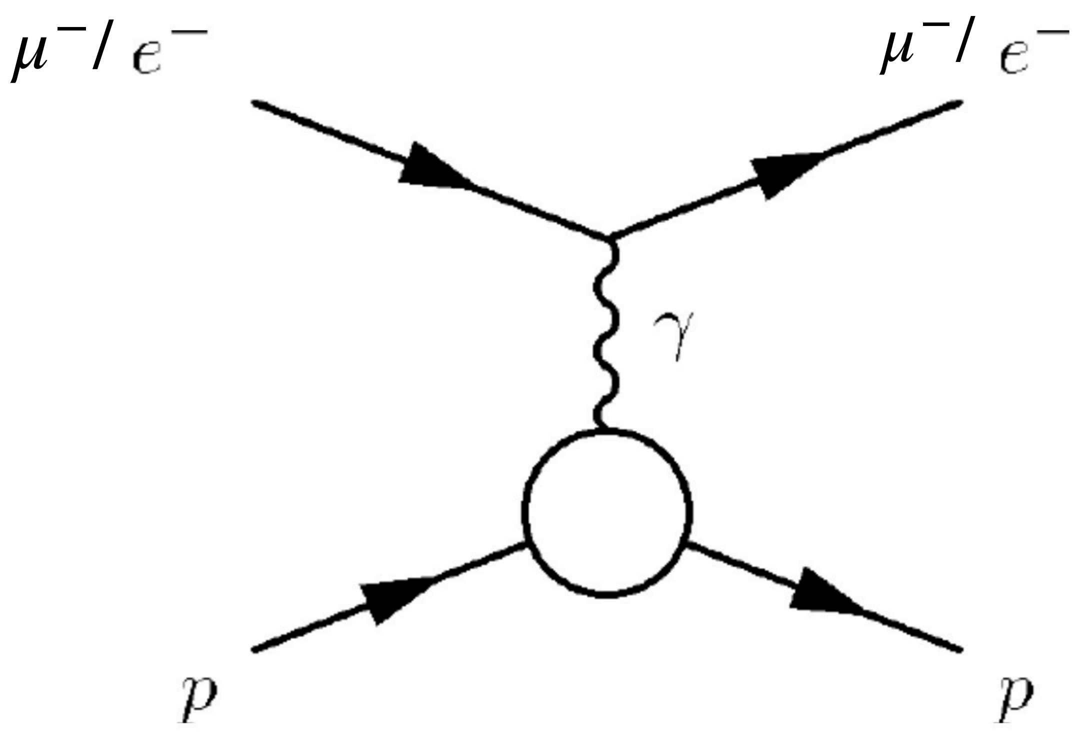

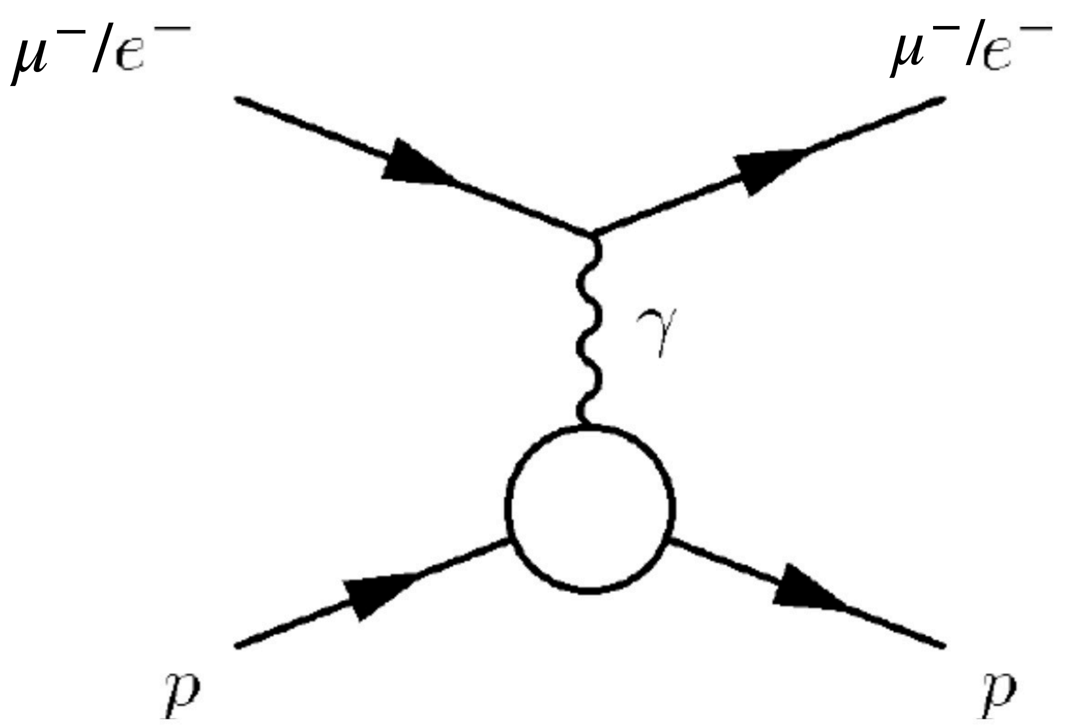

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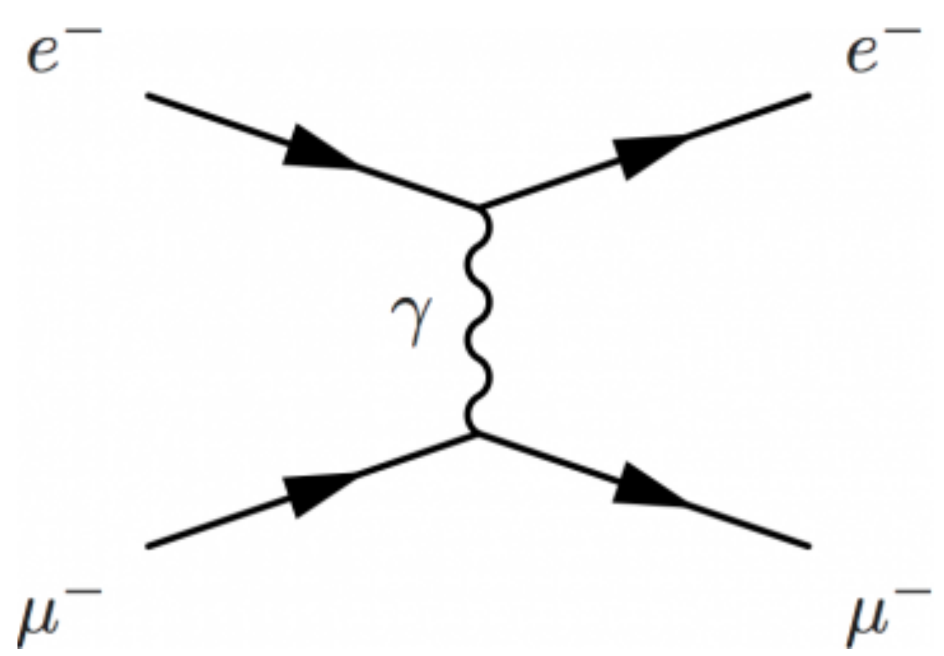

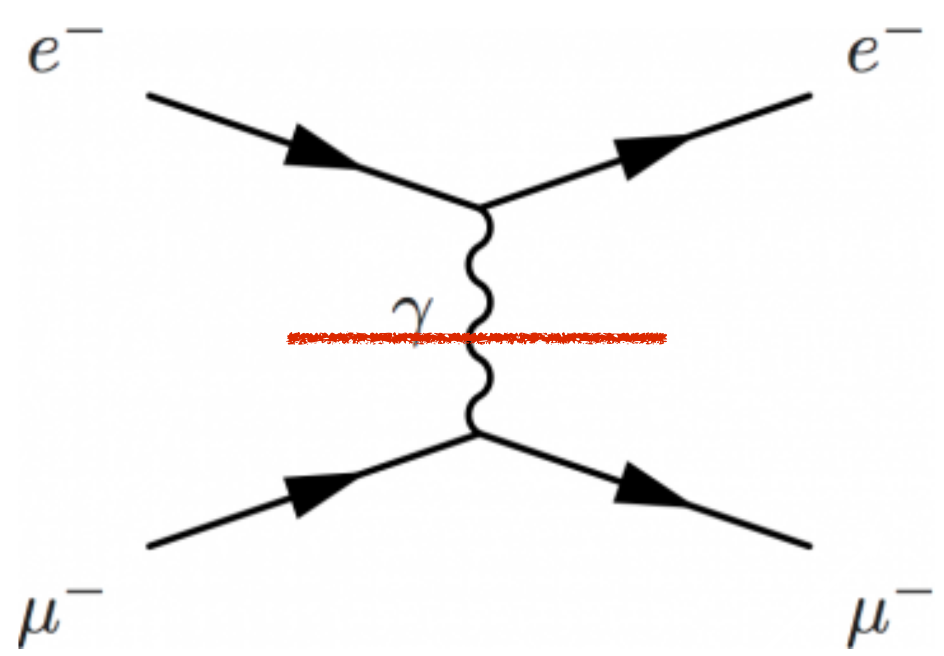

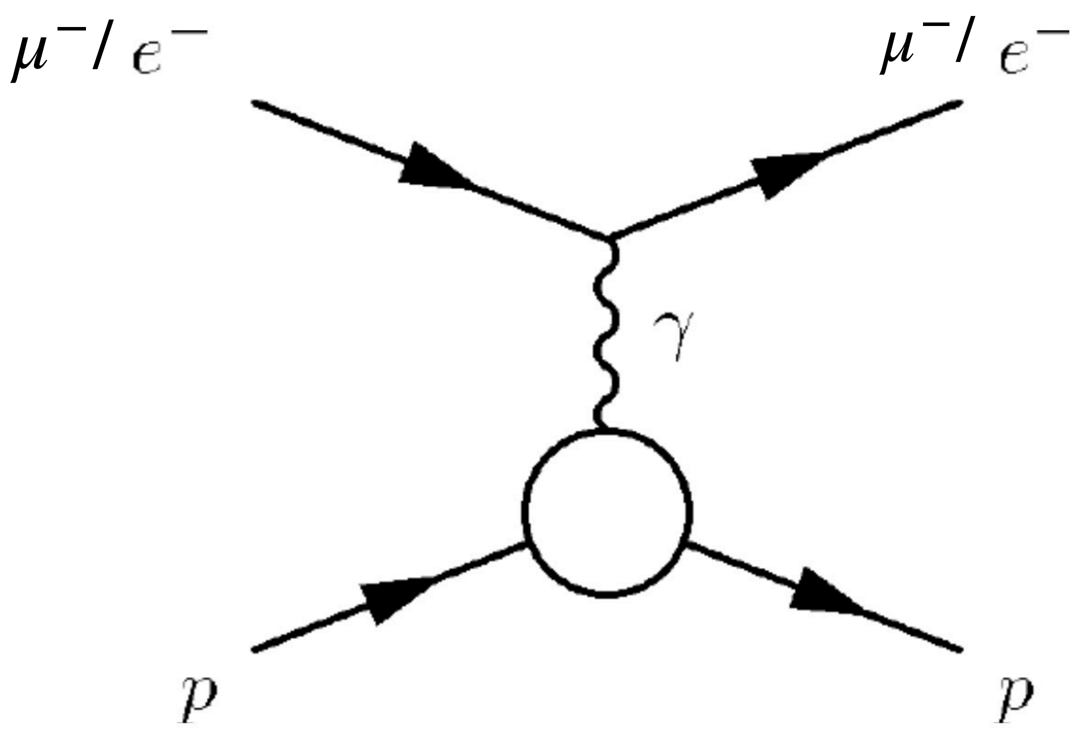

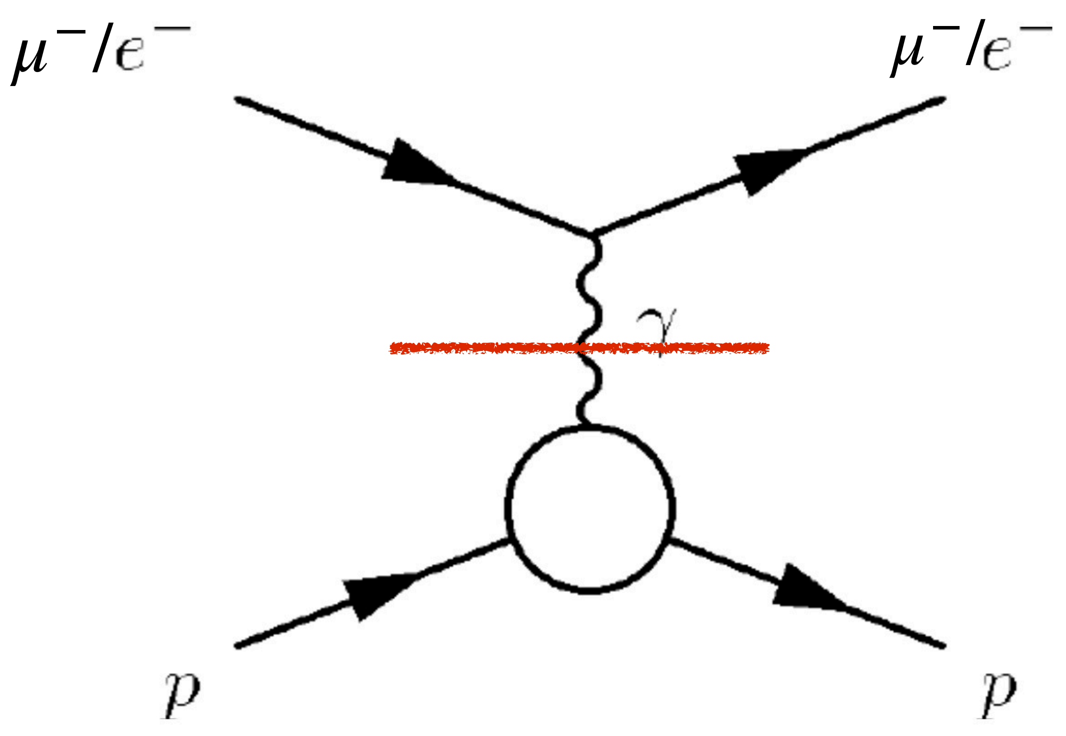

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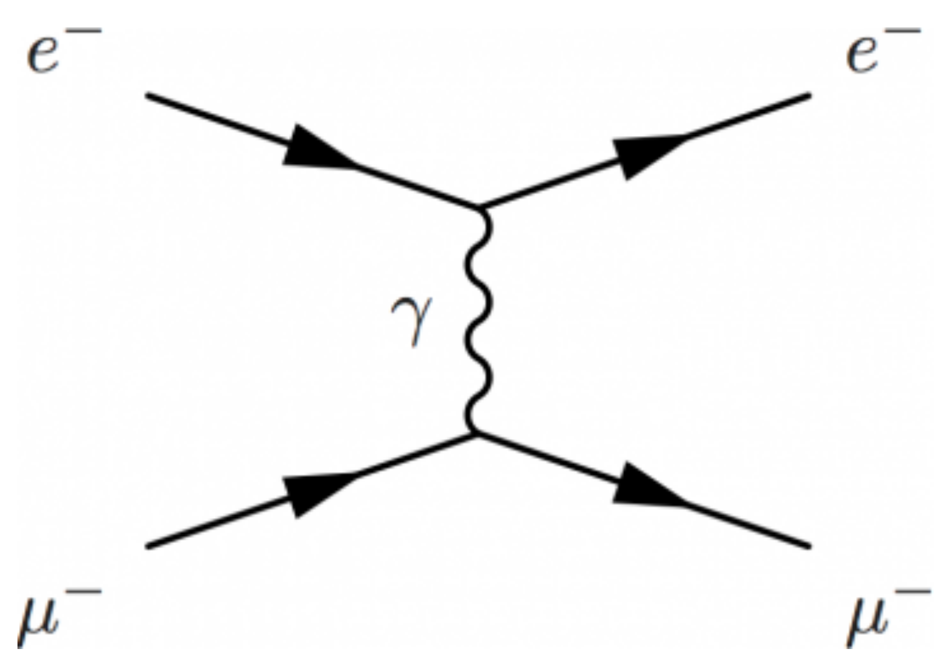

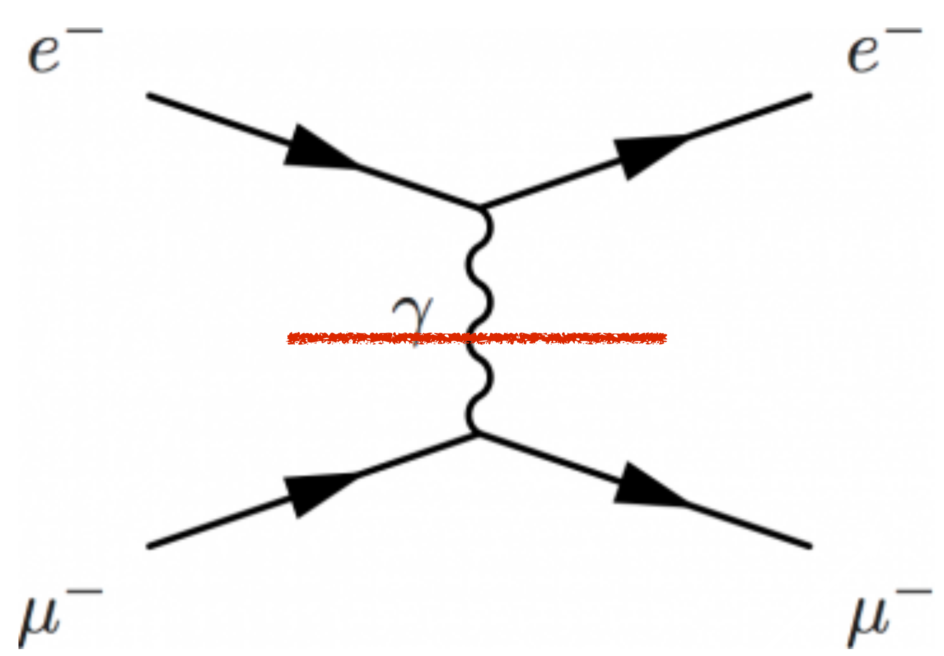

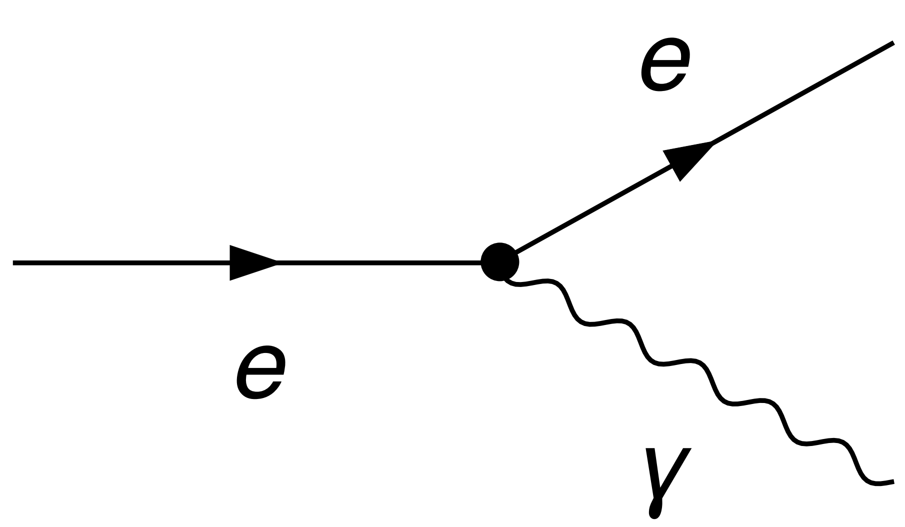

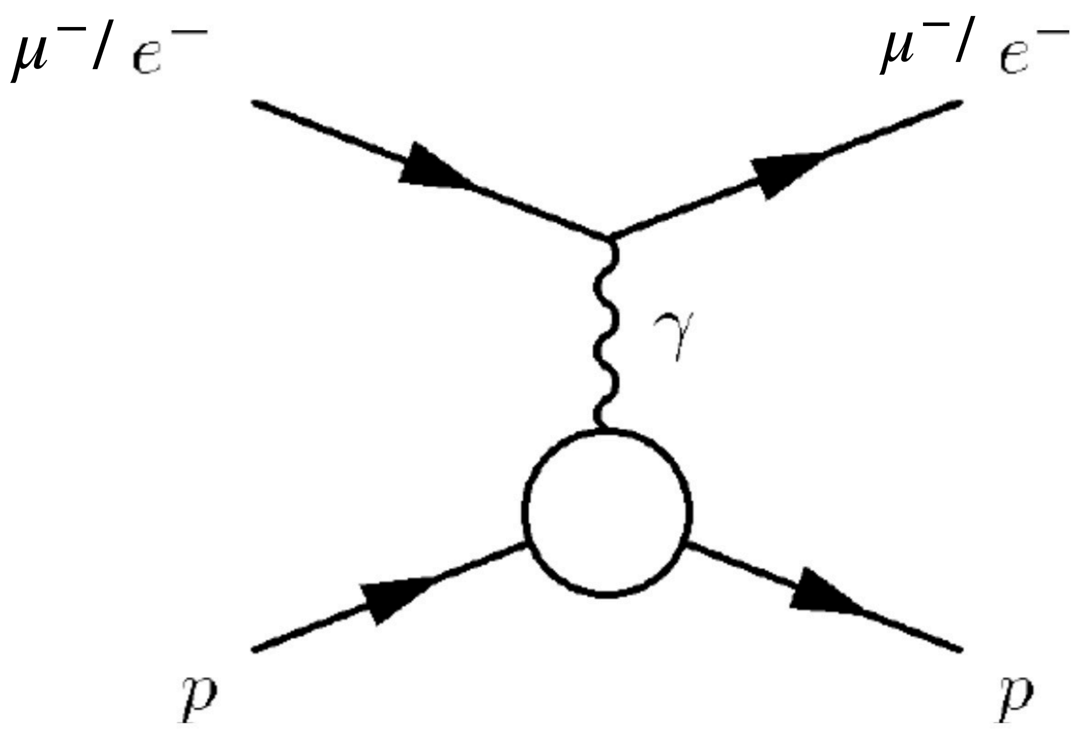

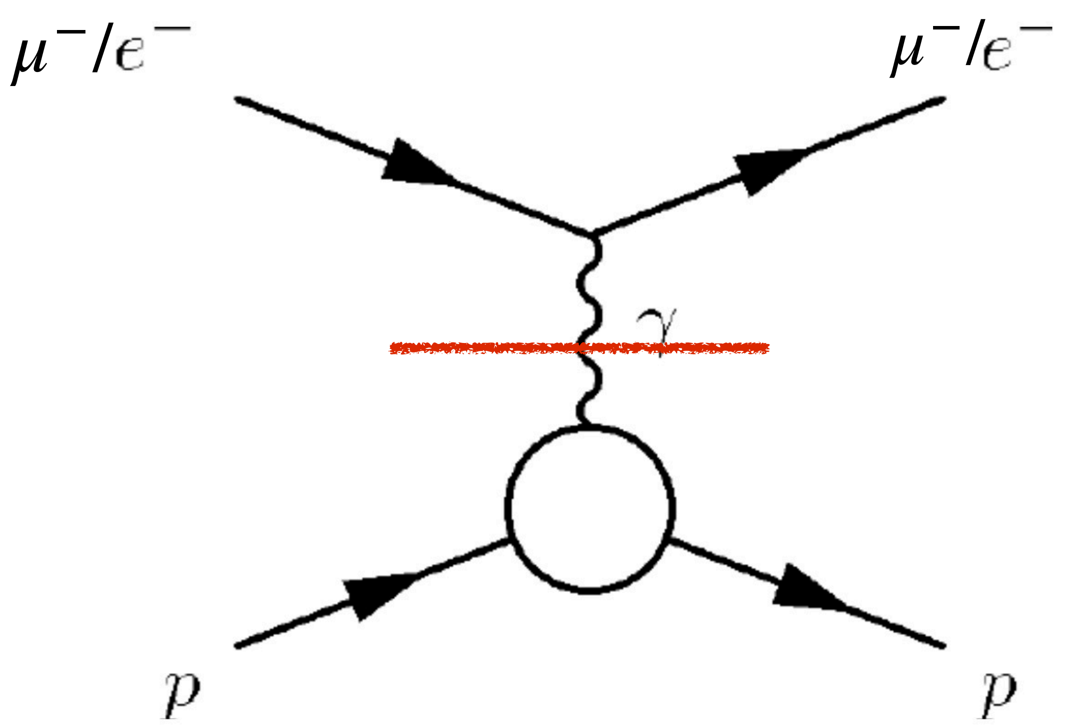

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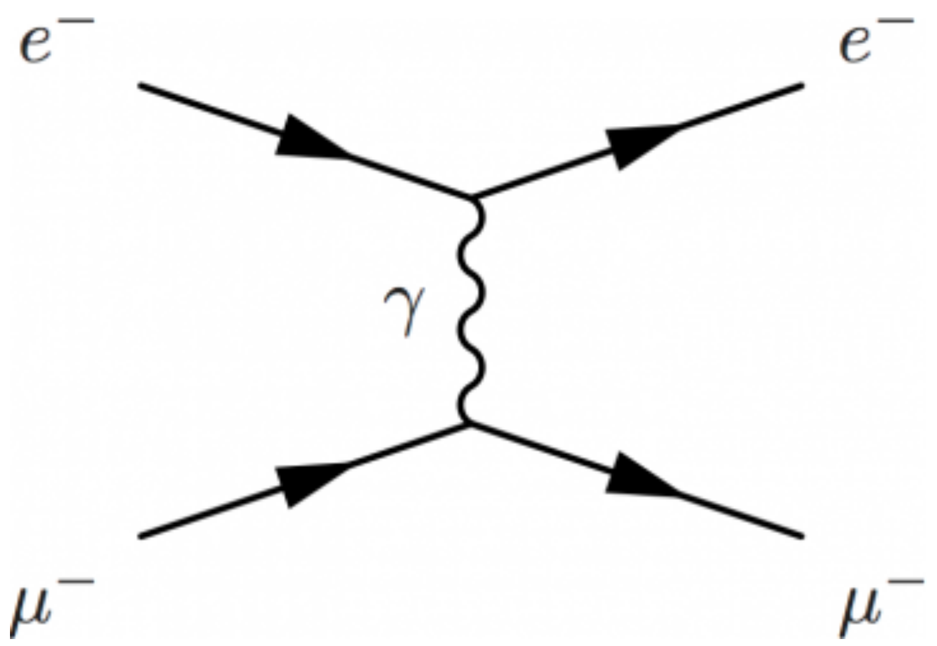

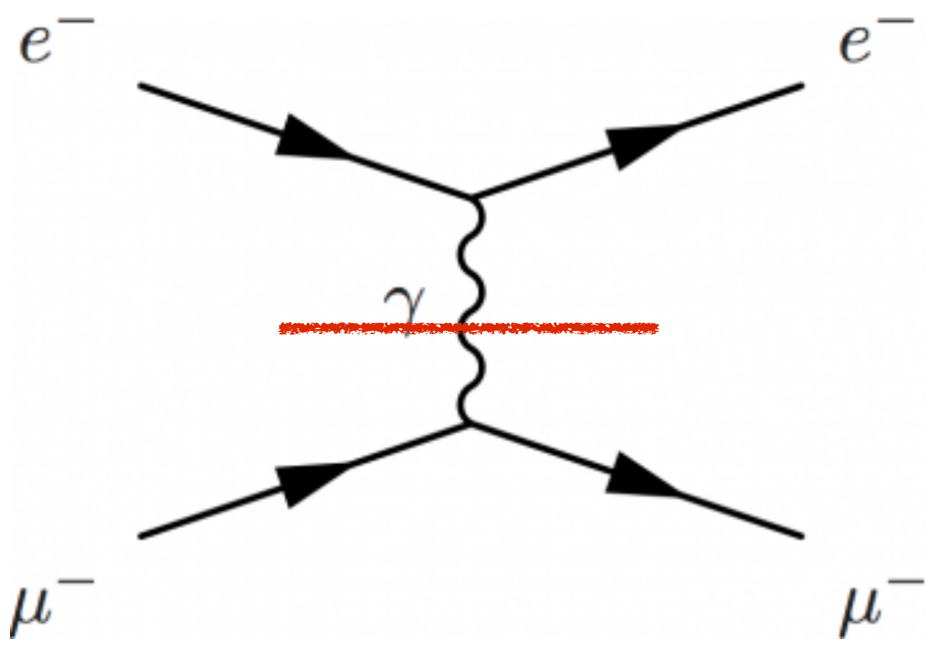

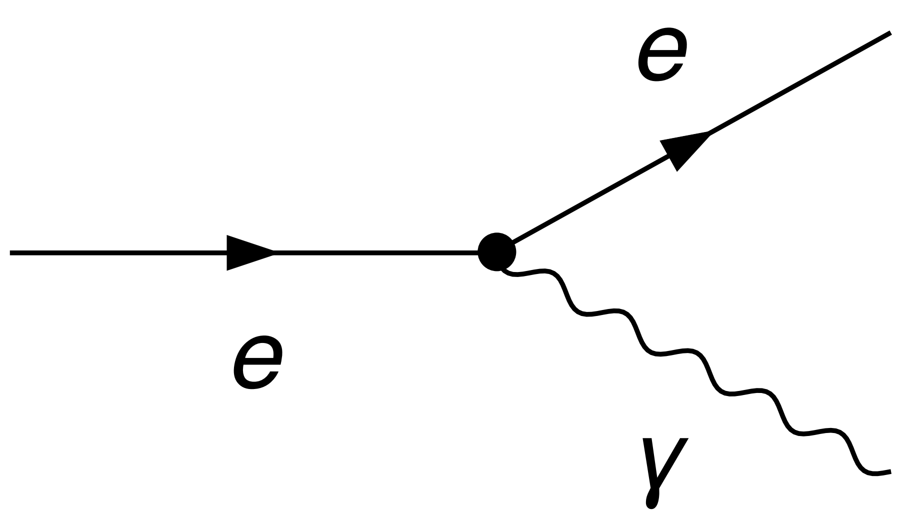

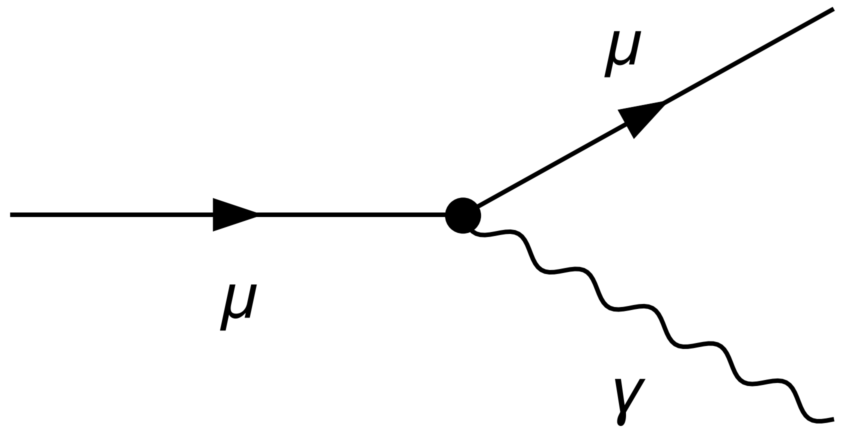

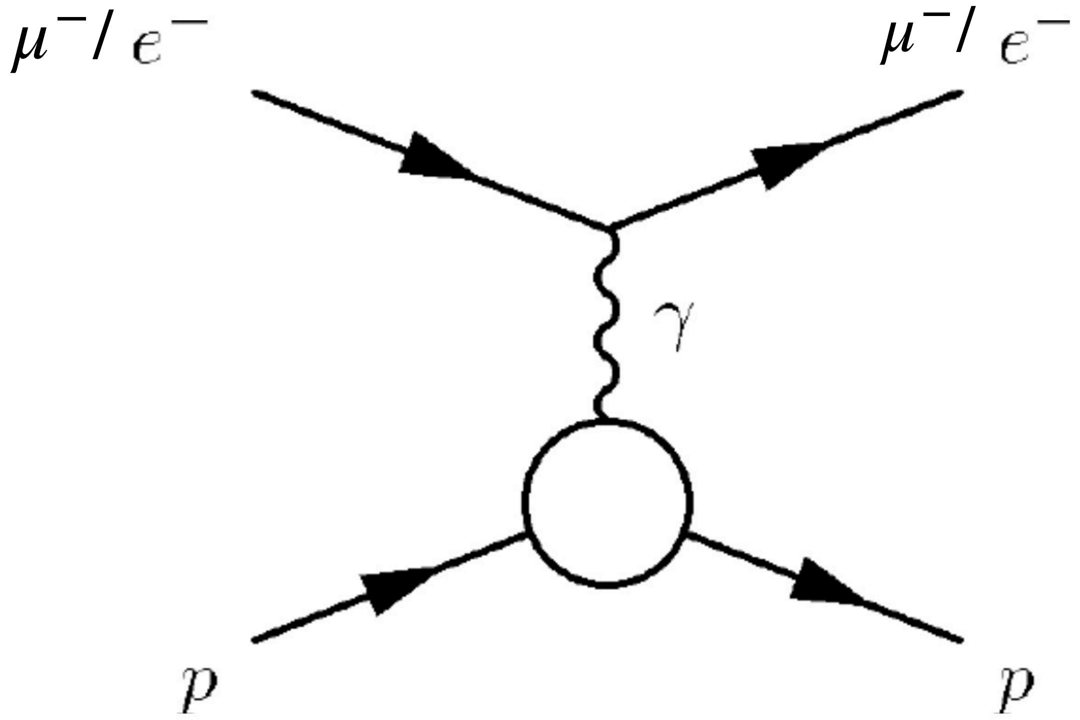

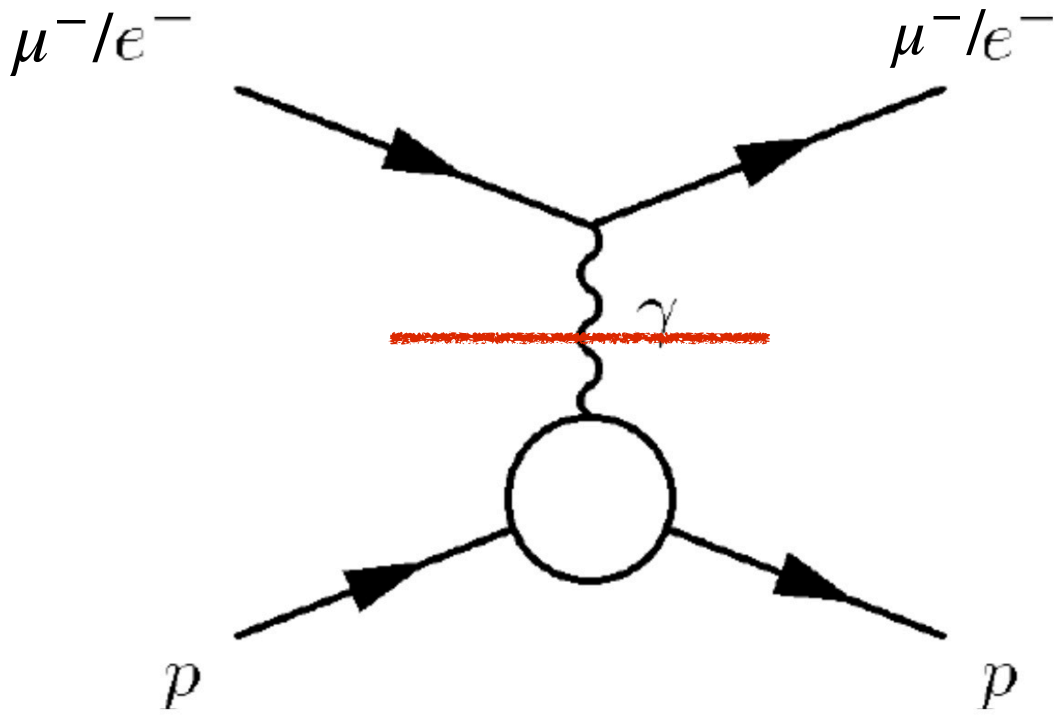

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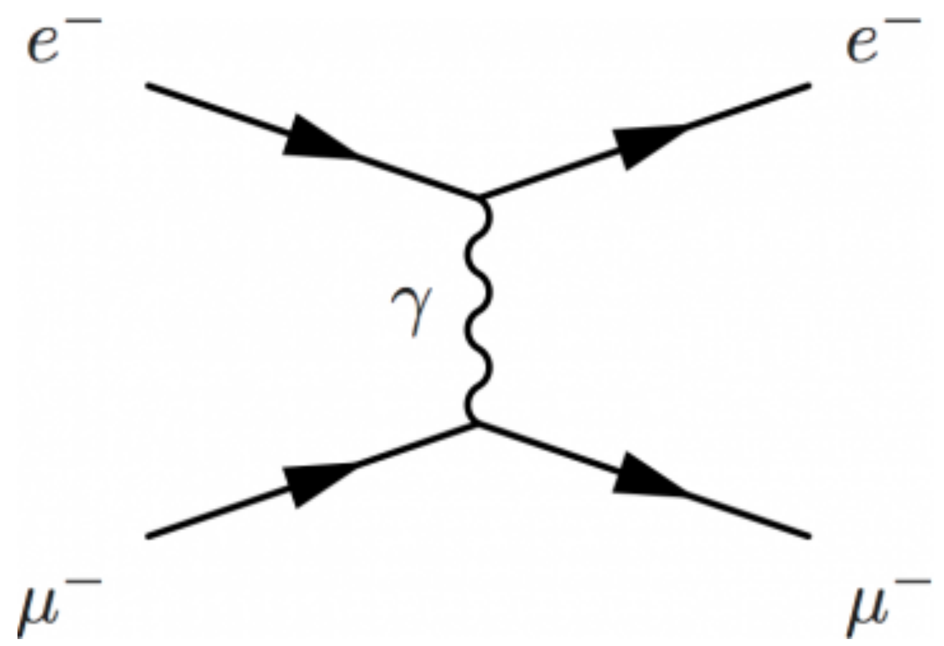

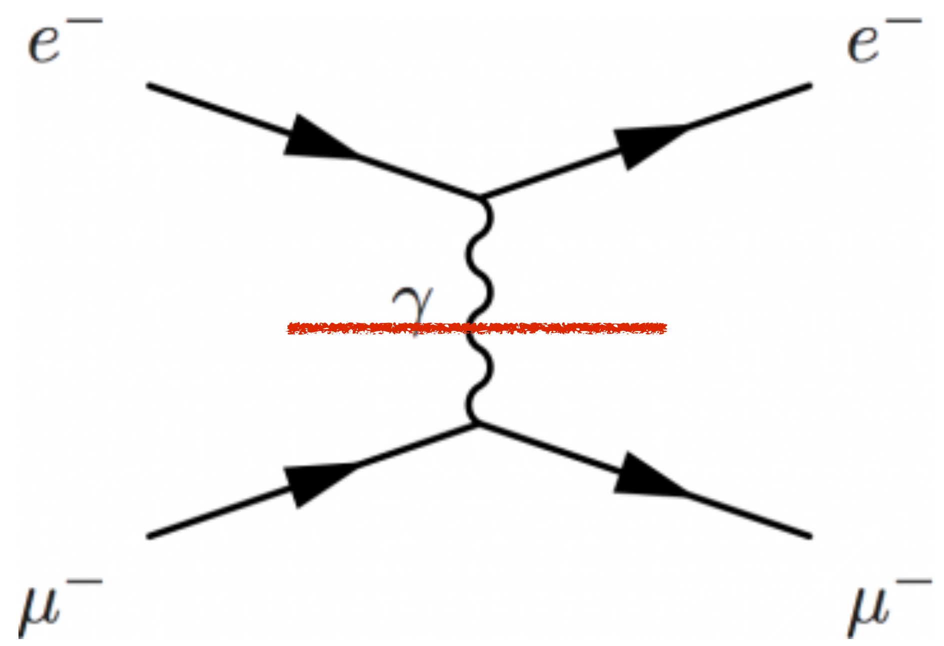

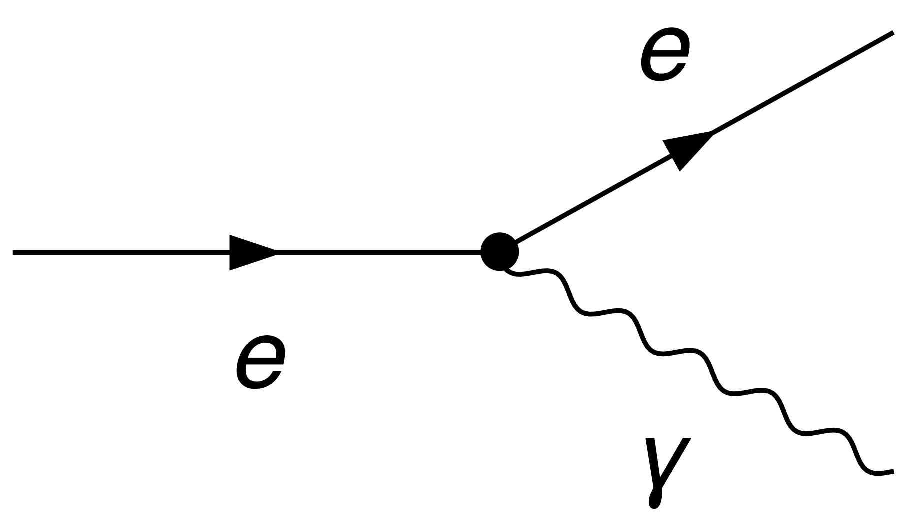

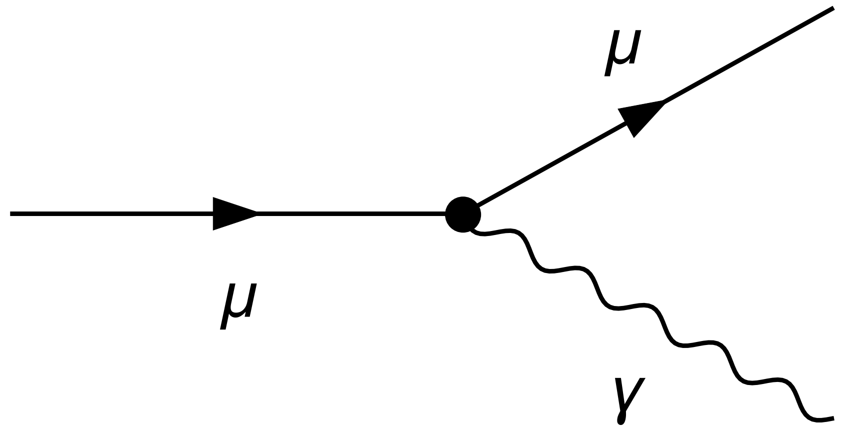

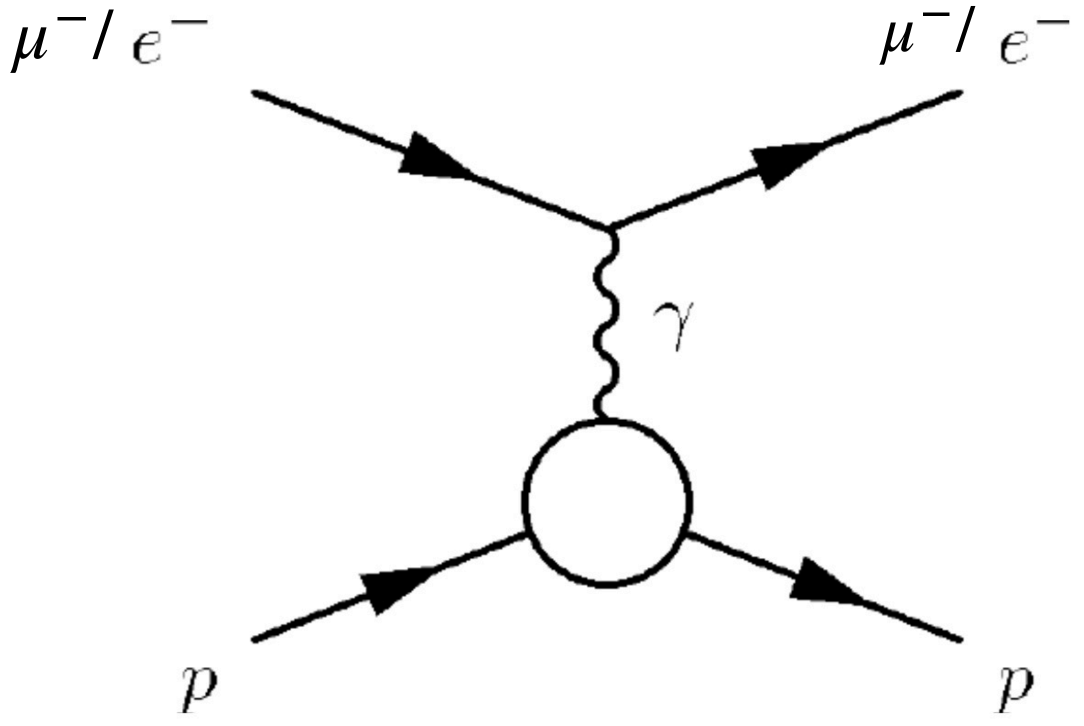

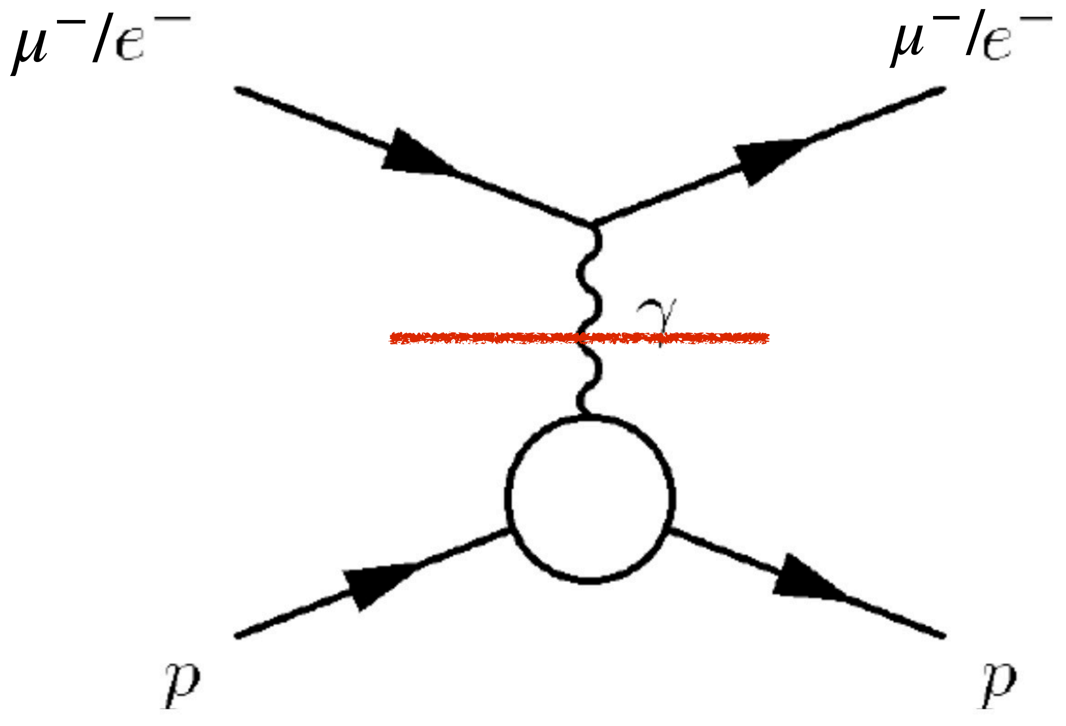

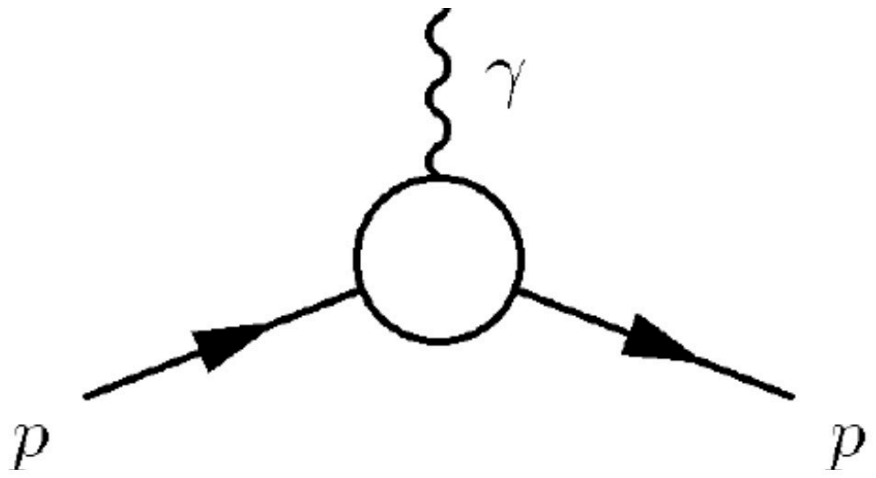

COVARIANT APPROACH WITH LEPTONIC-HADRONIC TENSORS

- The differential cross section of general lepton-lepton/hadron scattering can be obtained by:

$$d\sigma \sim L^{\mu\nu}L_{\mu\nu} \text{ or } d\sigma \sim L^{\mu\nu}W_{\mu\nu}$$

- where $W_{\mu\nu}$ is the hadronic tensor which in case of elastic $e^{-}p$ scattering:

$$W_{\mu\nu} = H_1 g_{\mu\nu} + H_2 p_{1\mu}p_{1\nu} + H_3 p_{2\mu}p_{2\nu} + H_4 p_{1\mu}p_{2\nu} + H_5 p_{2\mu}p_{1\nu} + H_6 \epsilon_{\mu,\nu,p_1,p_2}$$

where p_1 and p_2 are incoming and outgoing protons momenta. H_1, H_2, H_3, H_4, H_5 and H_6 are the hadronic structure functions which can be extracted from experimental data.

FULL **ELECTROWEAK** e^-p SCATTERING

- Elastic e^-p scattering is studied up to the NNLO level considering all SM particles in the loop.
- A longitudinally polarized e^- scatters off an unpolarized proton target

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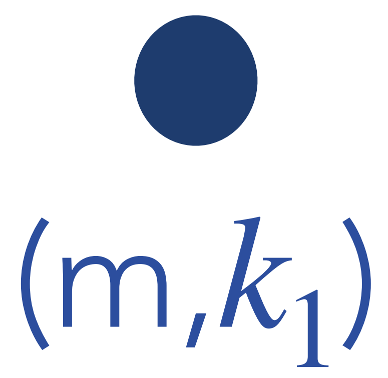

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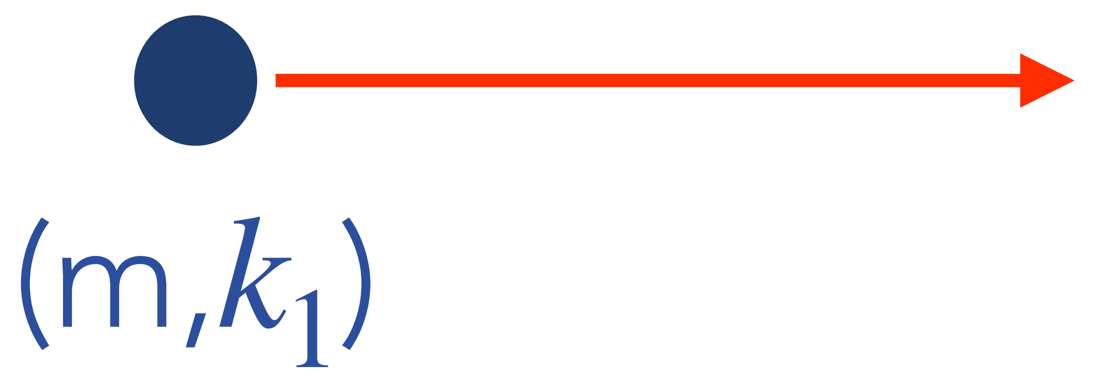

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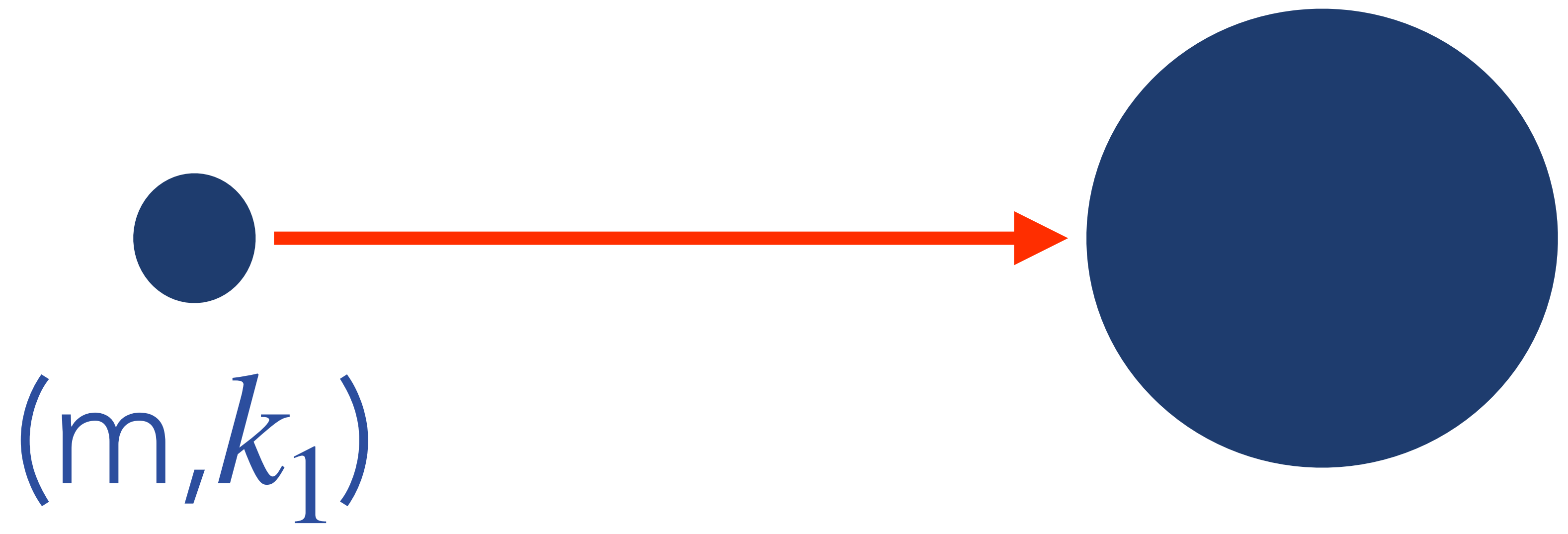

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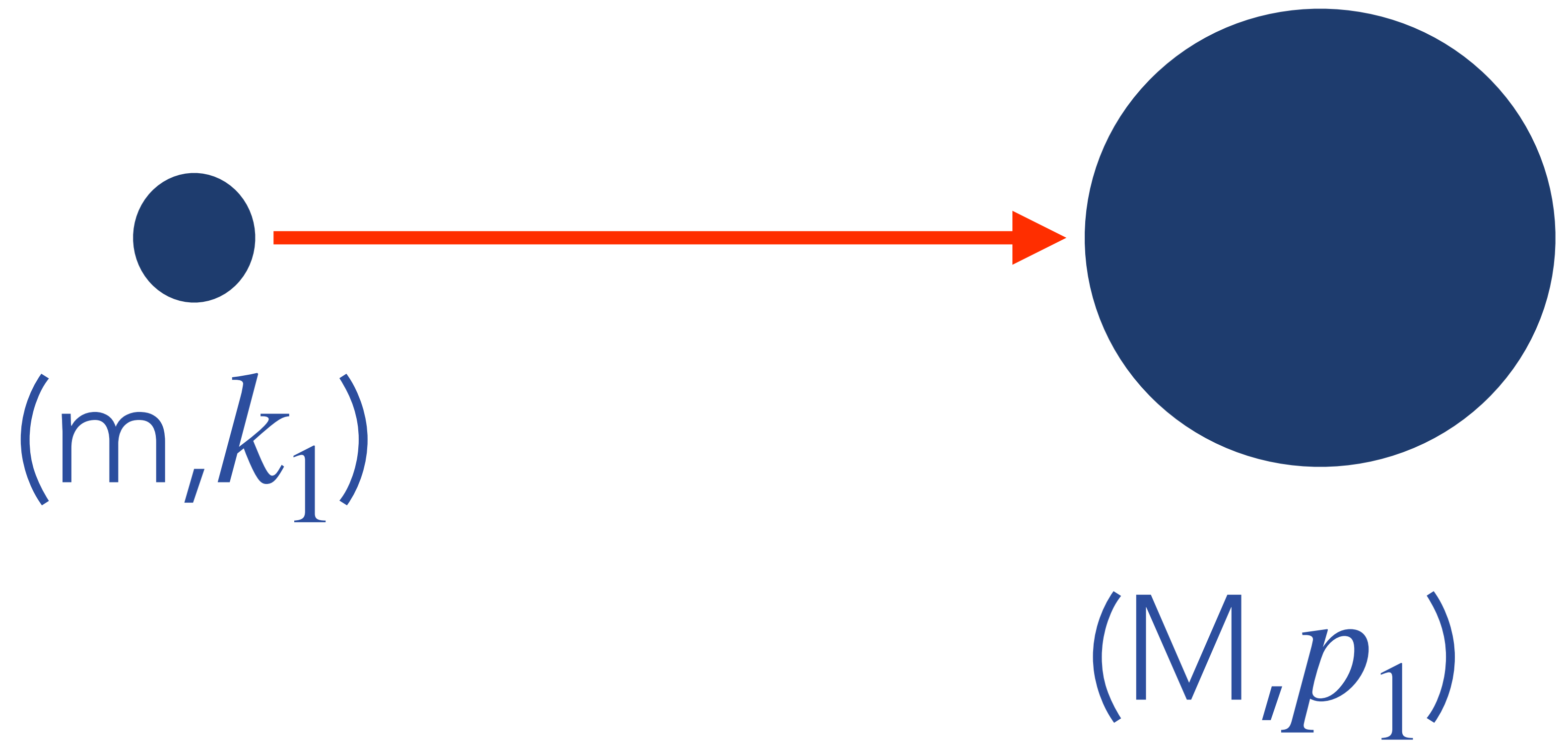

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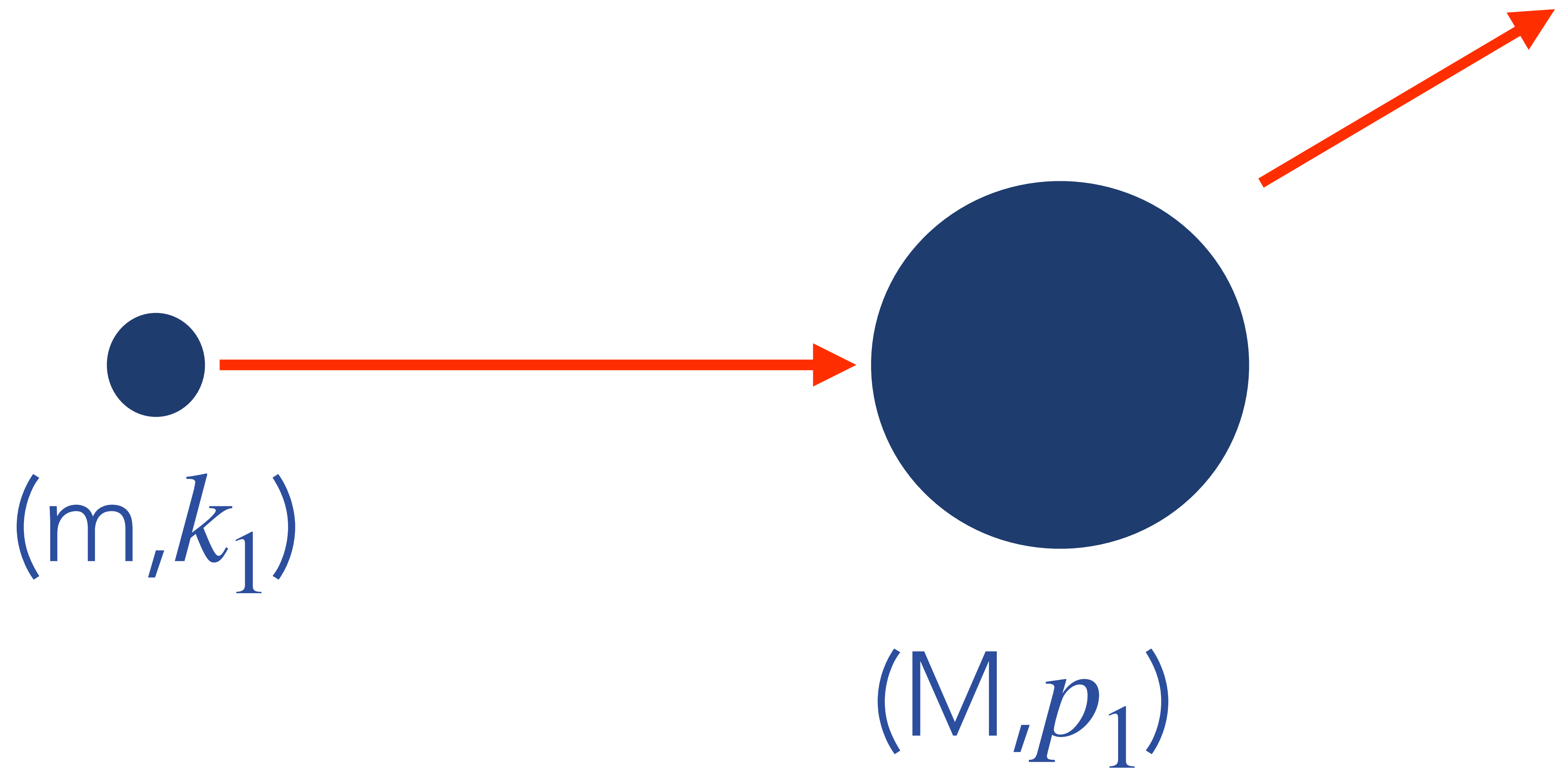

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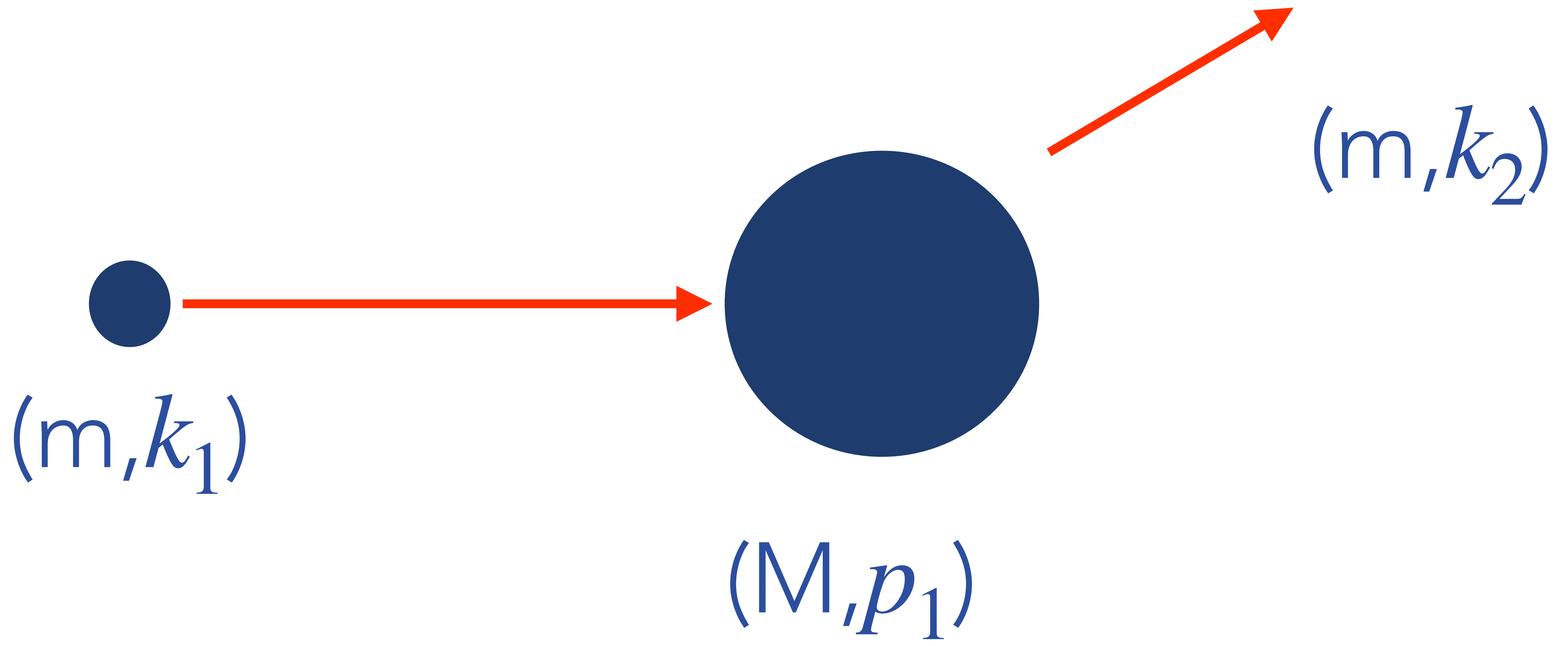

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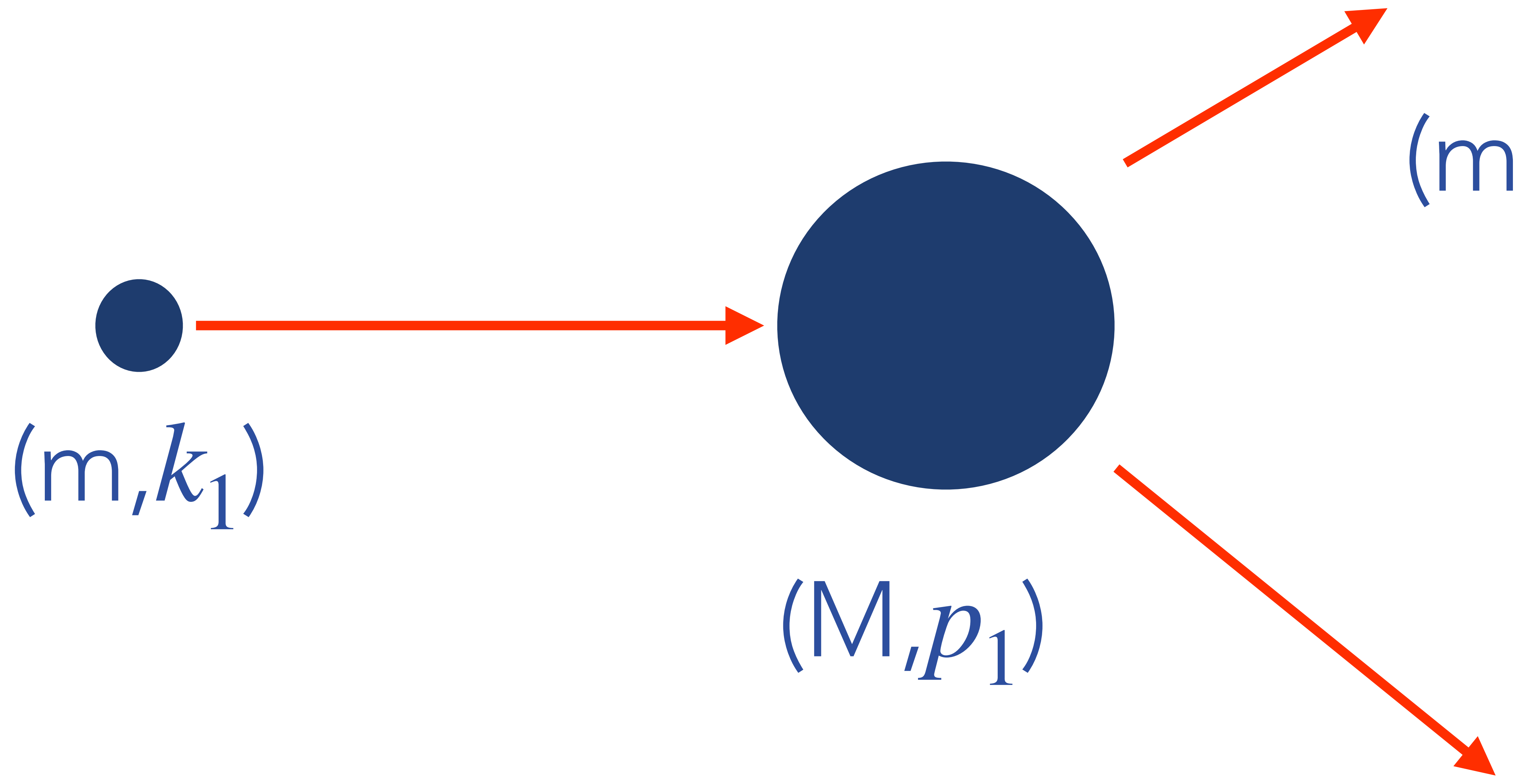

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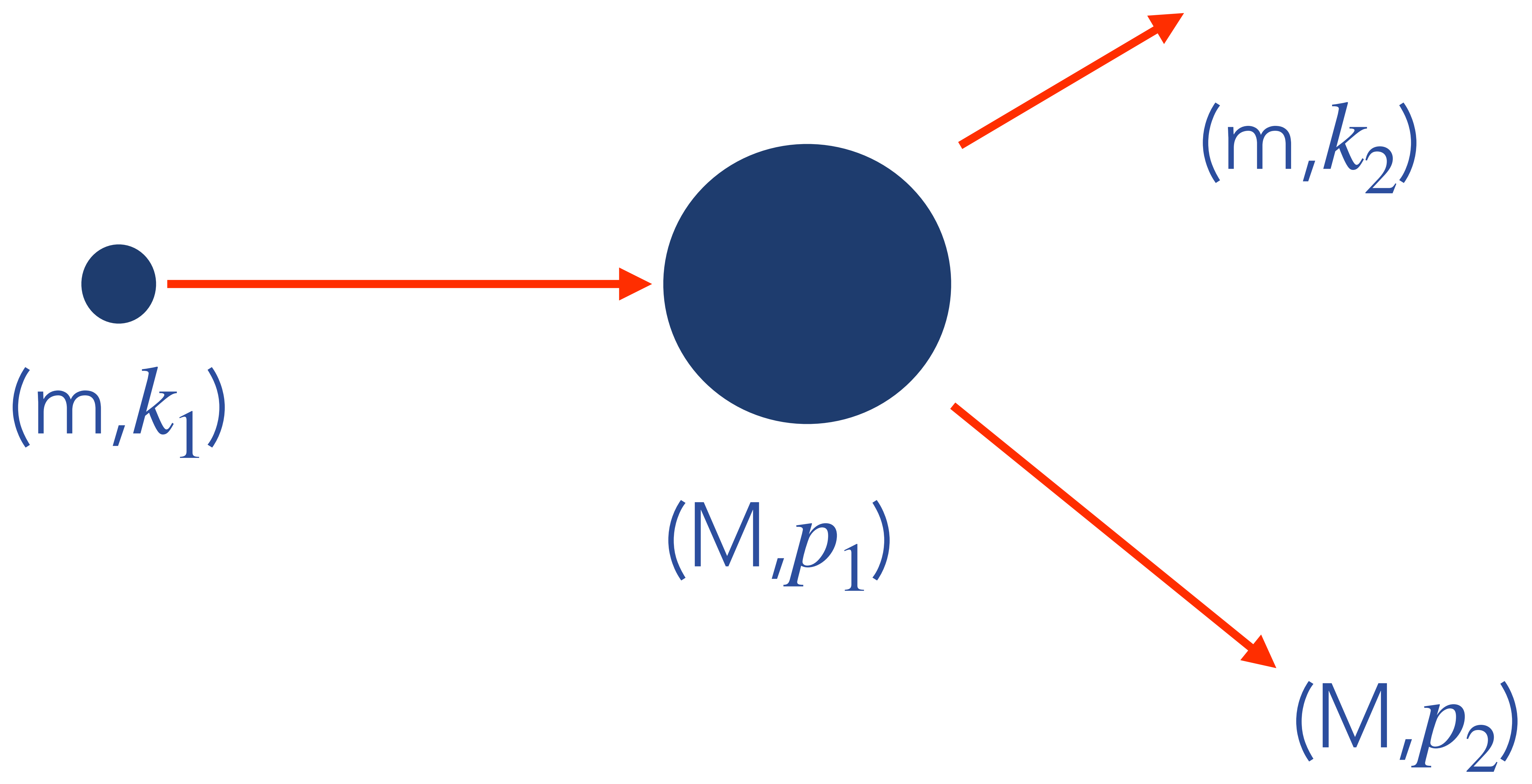

Full Electroweak Tree level Graphs →

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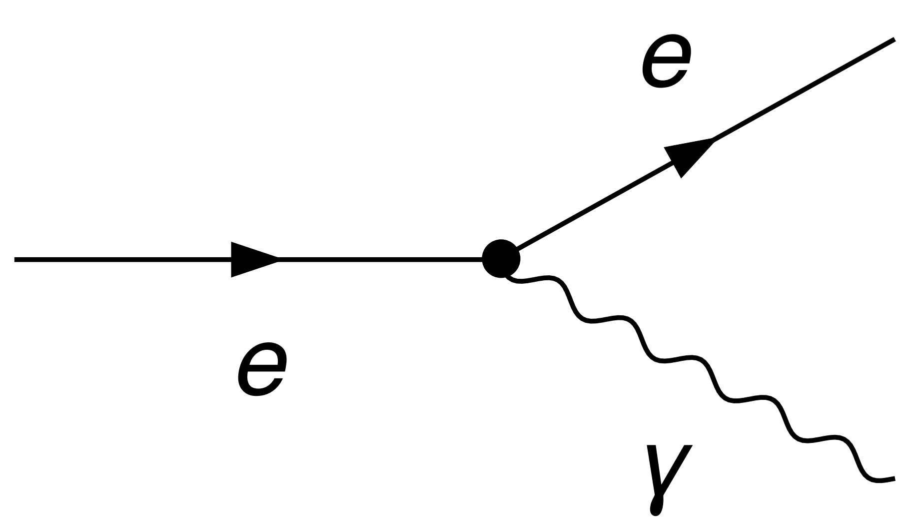

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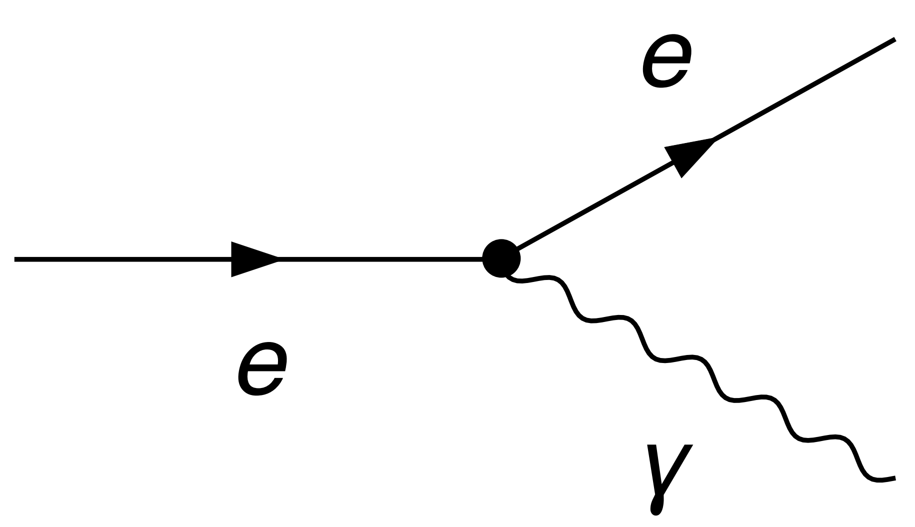

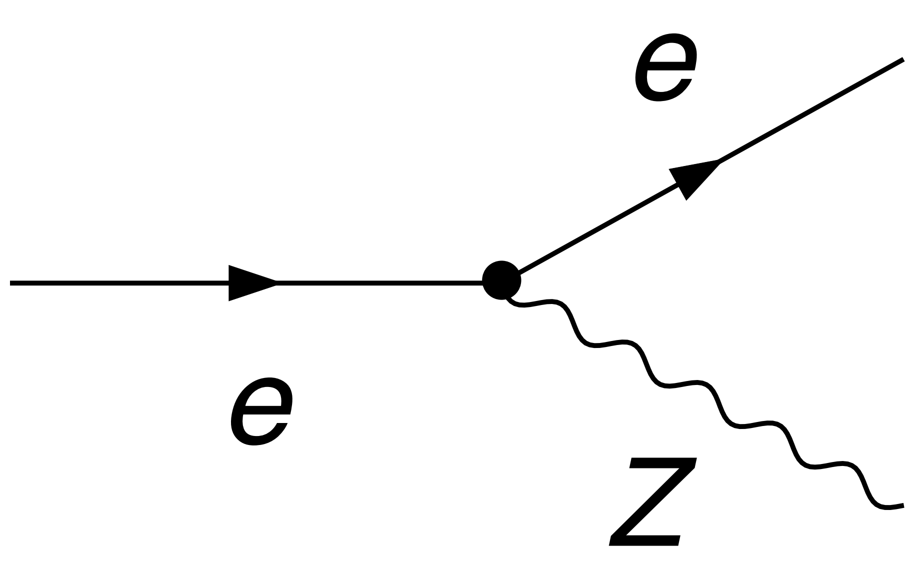

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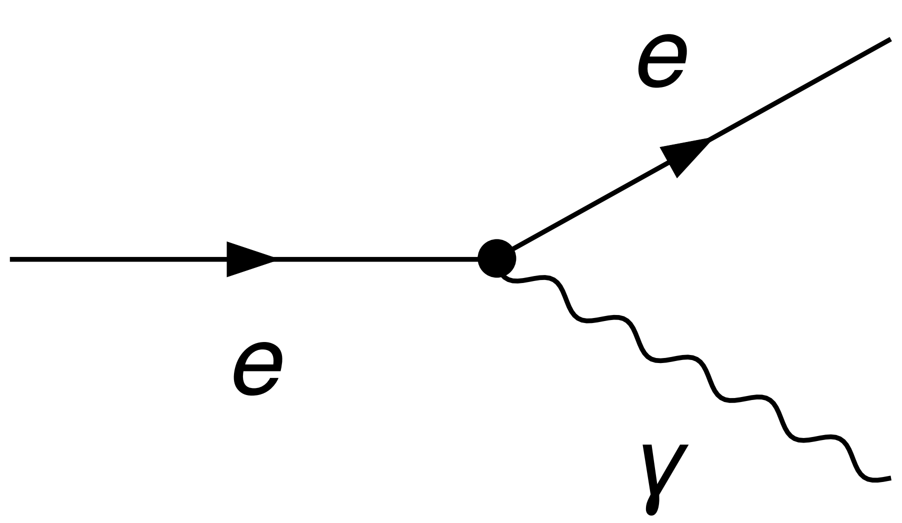

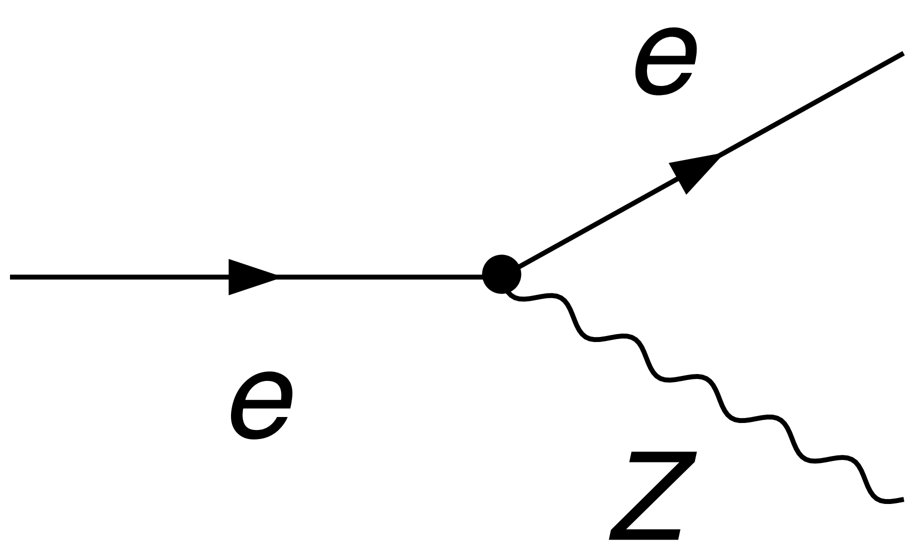

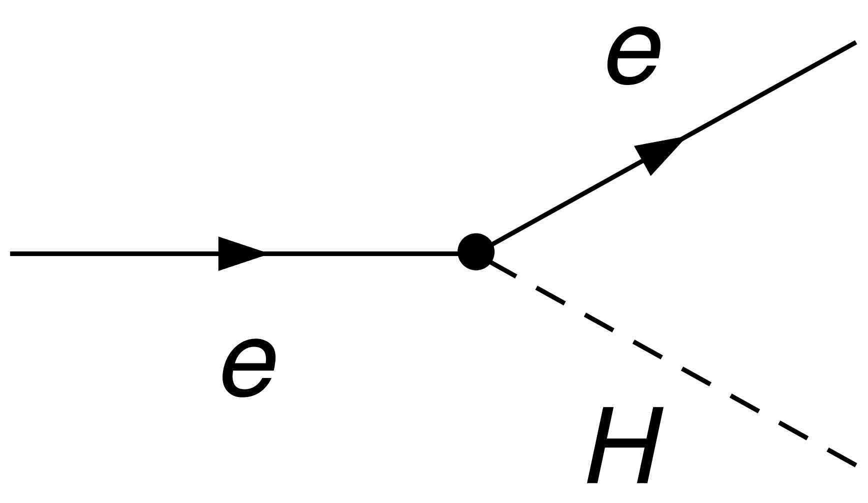

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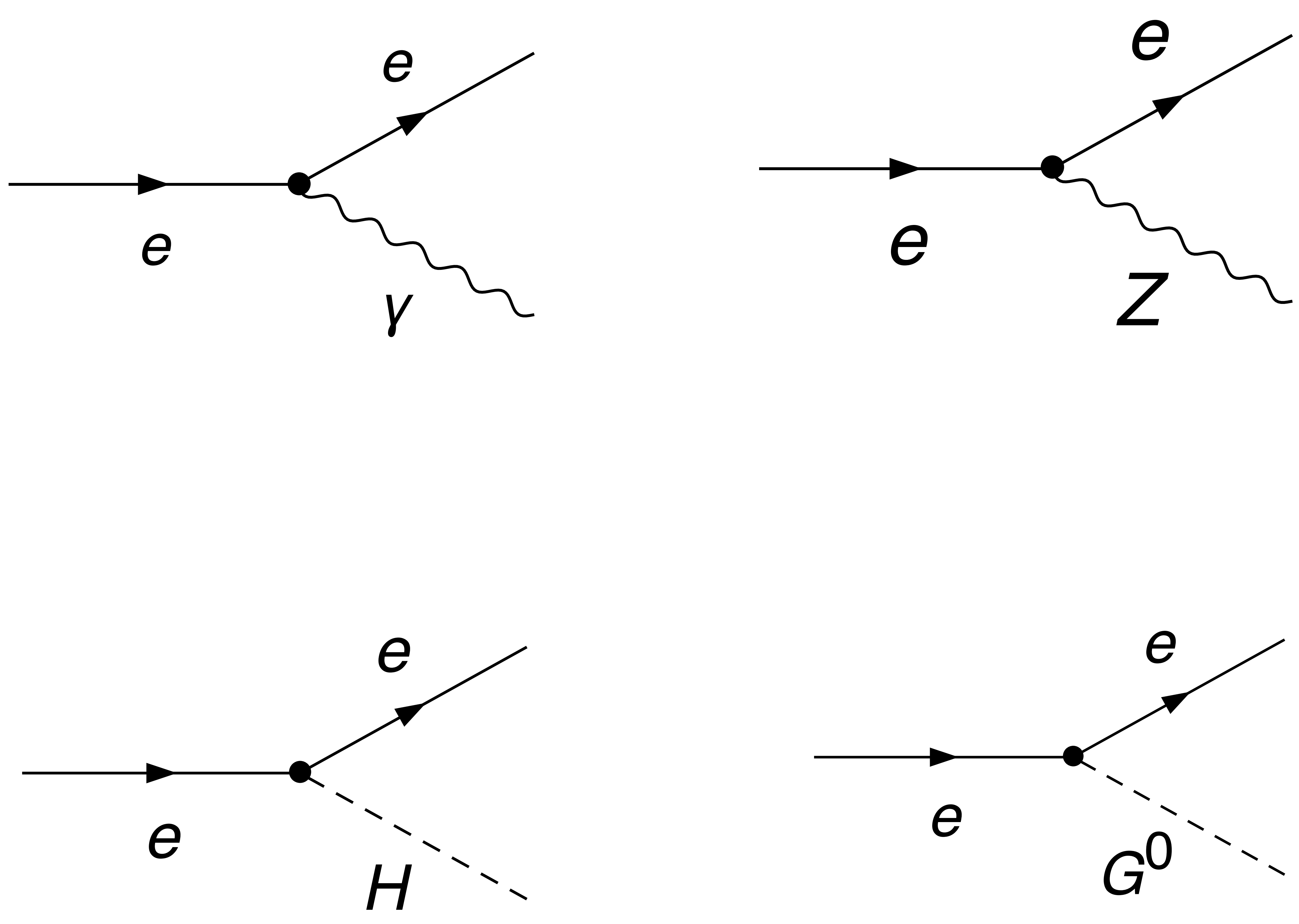

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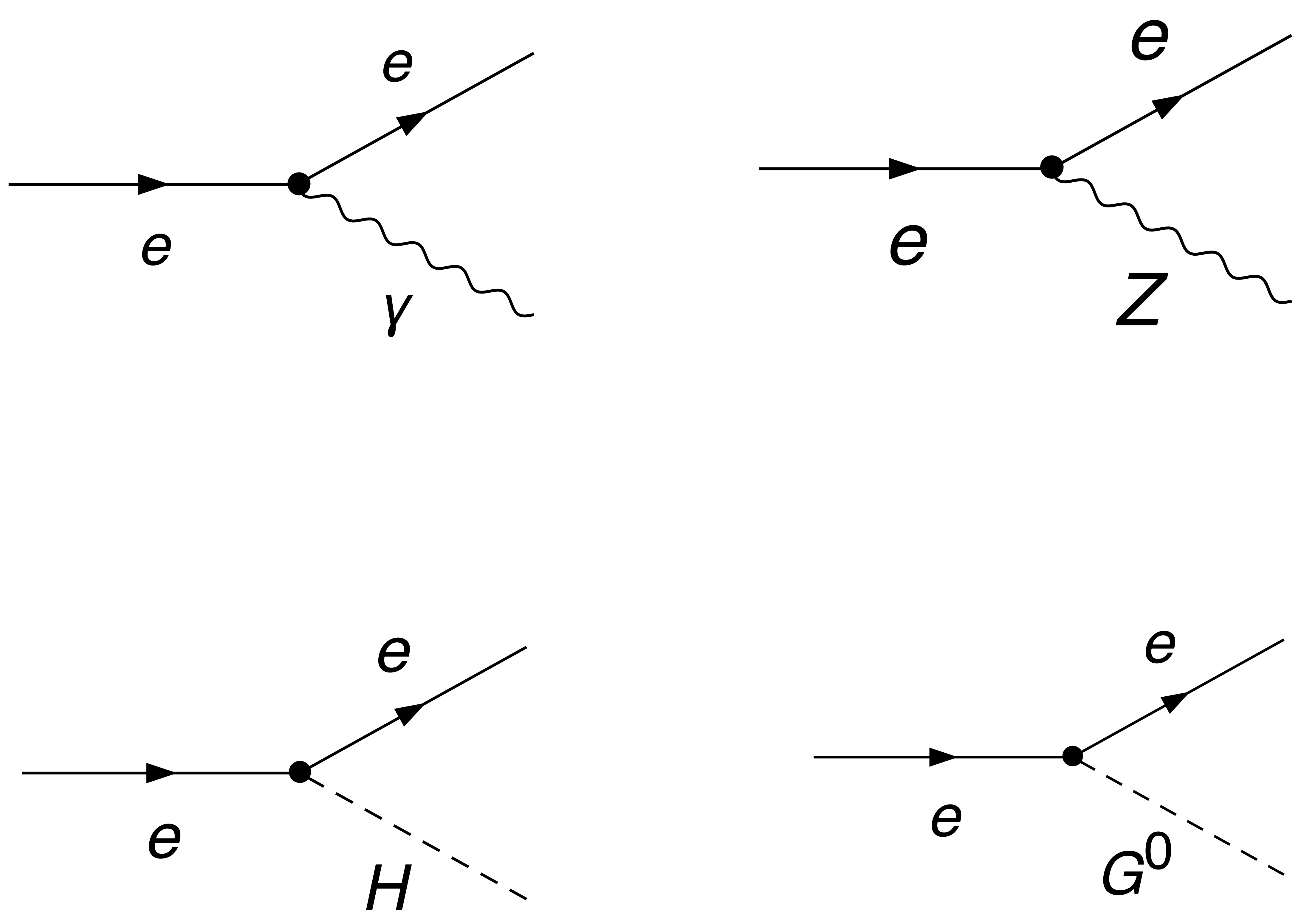
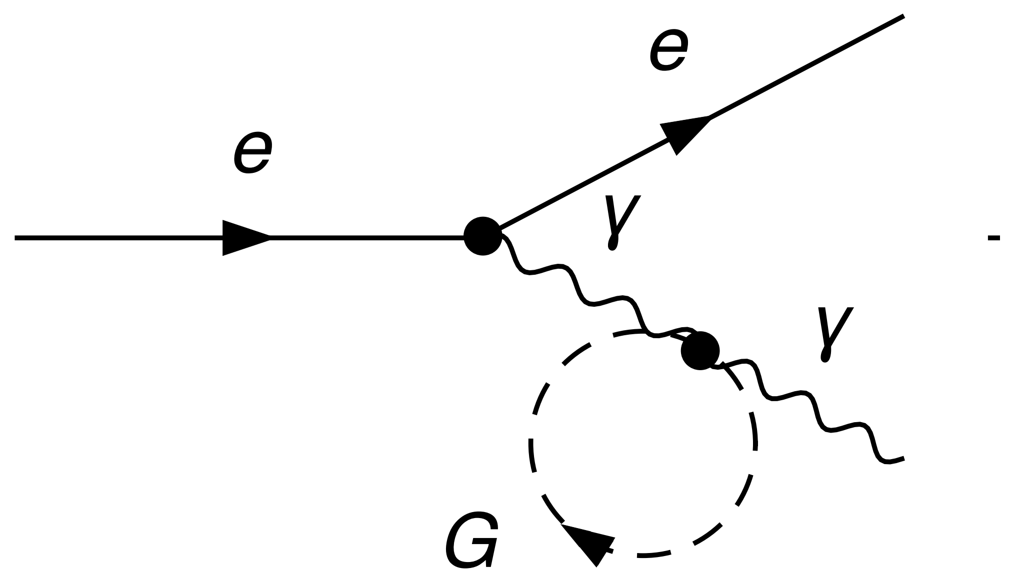


One loop level
Examples



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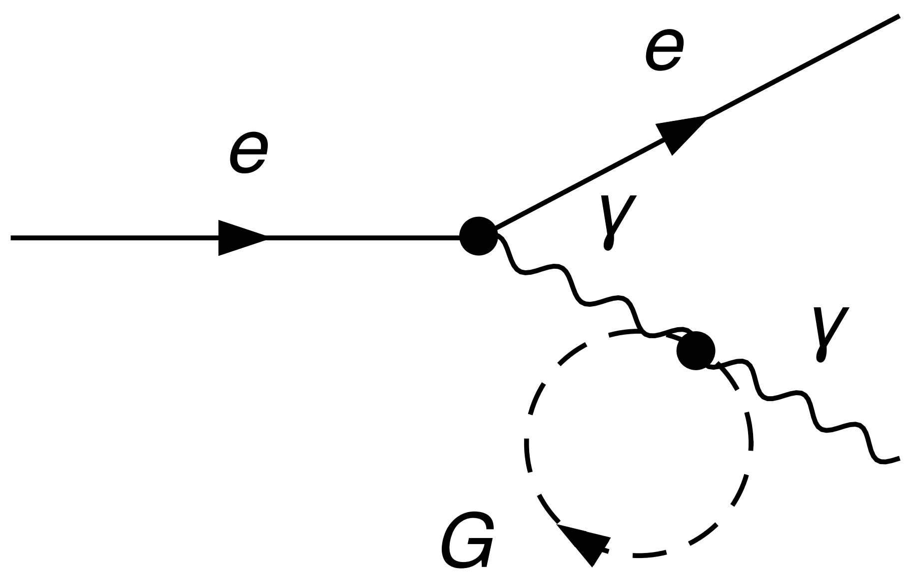

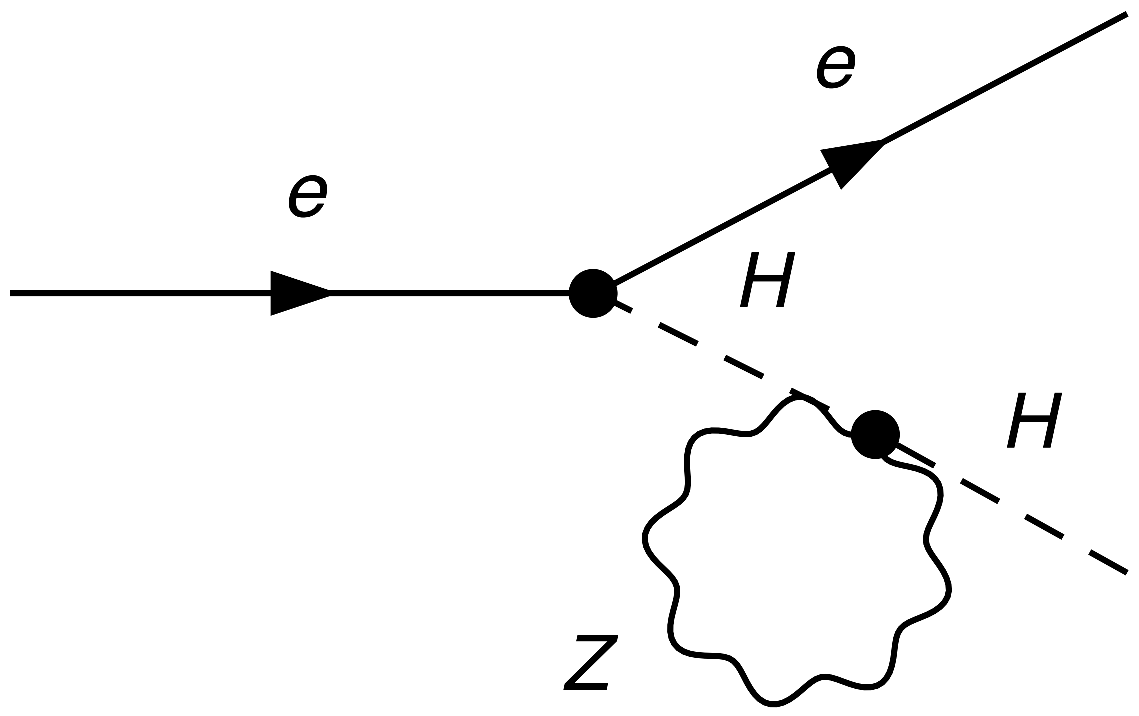

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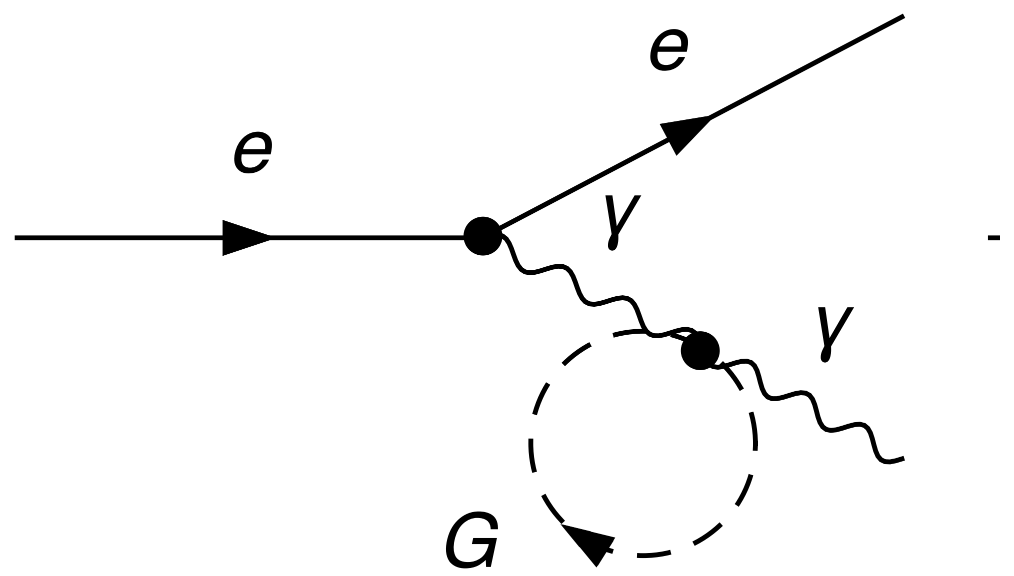

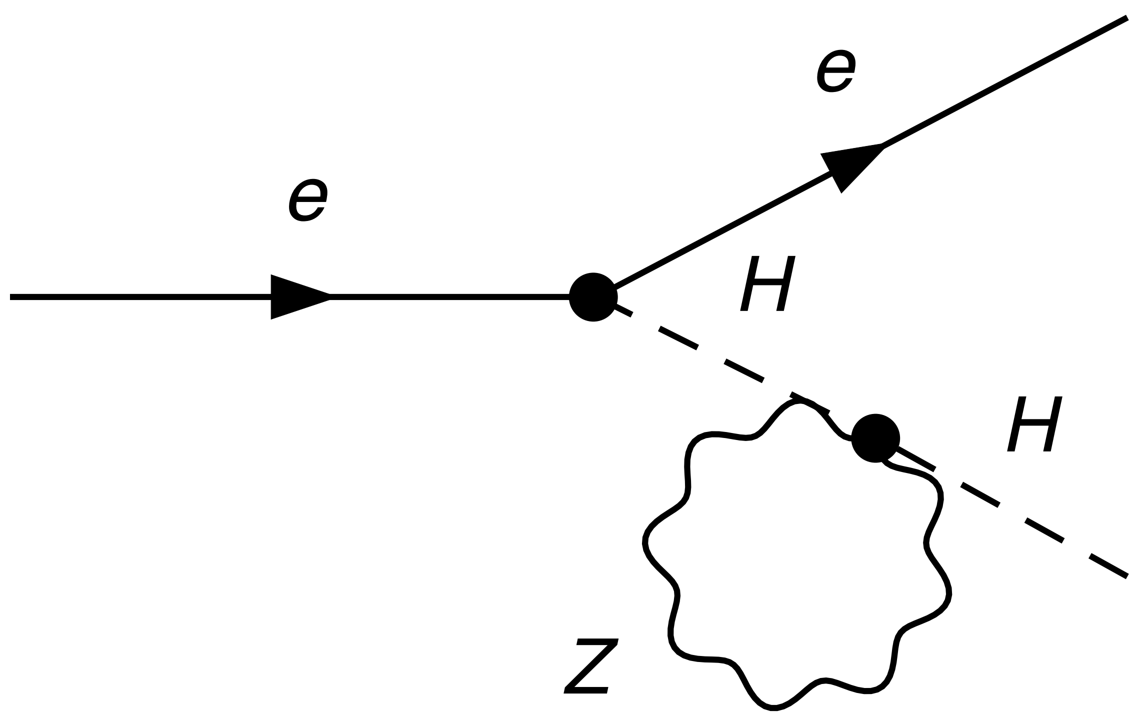

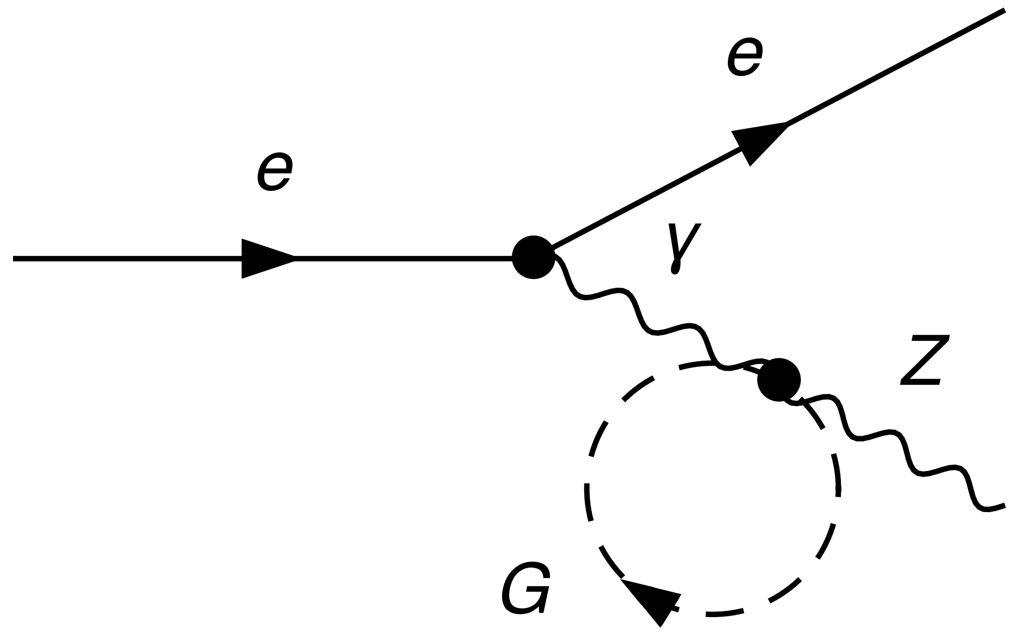

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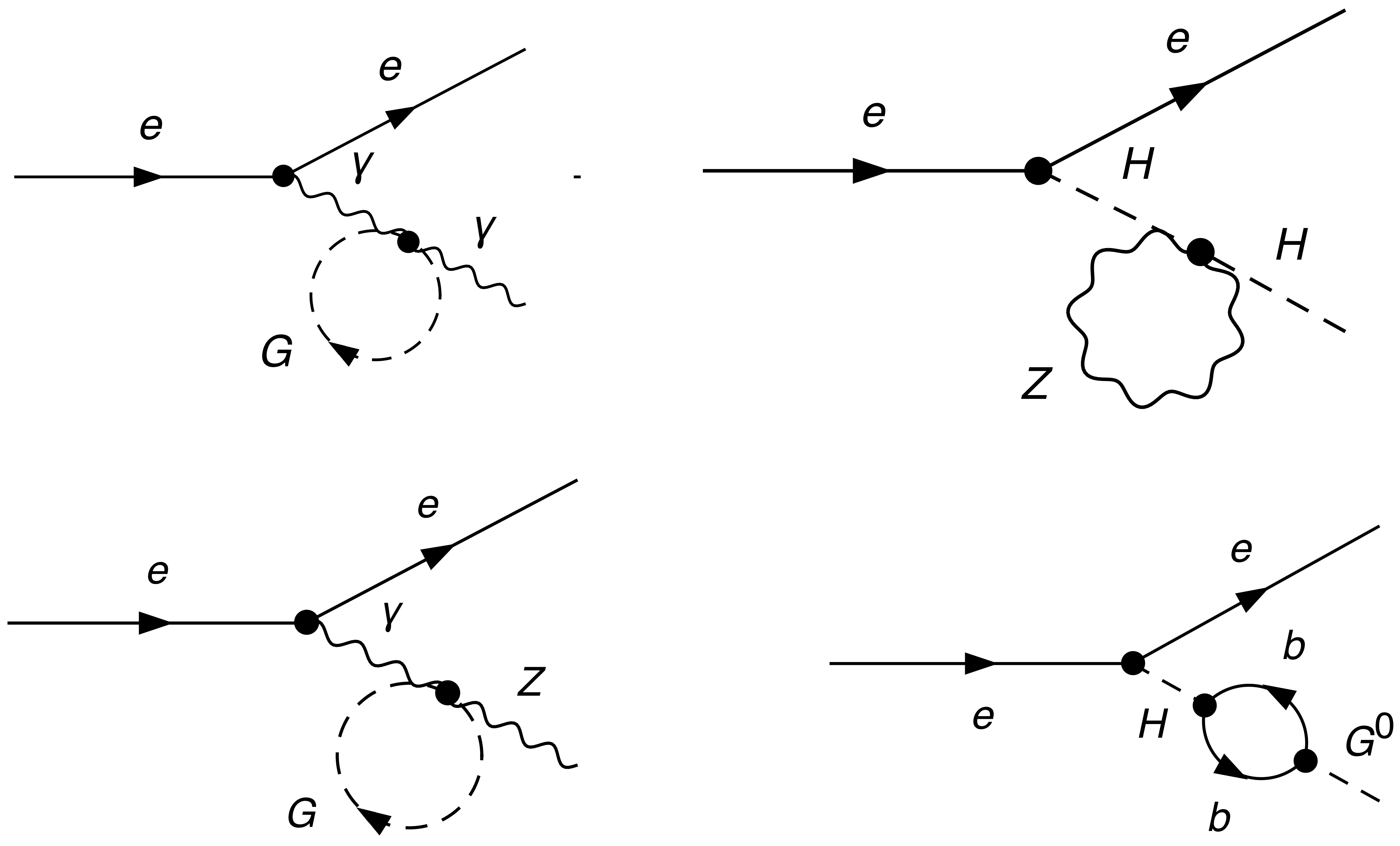

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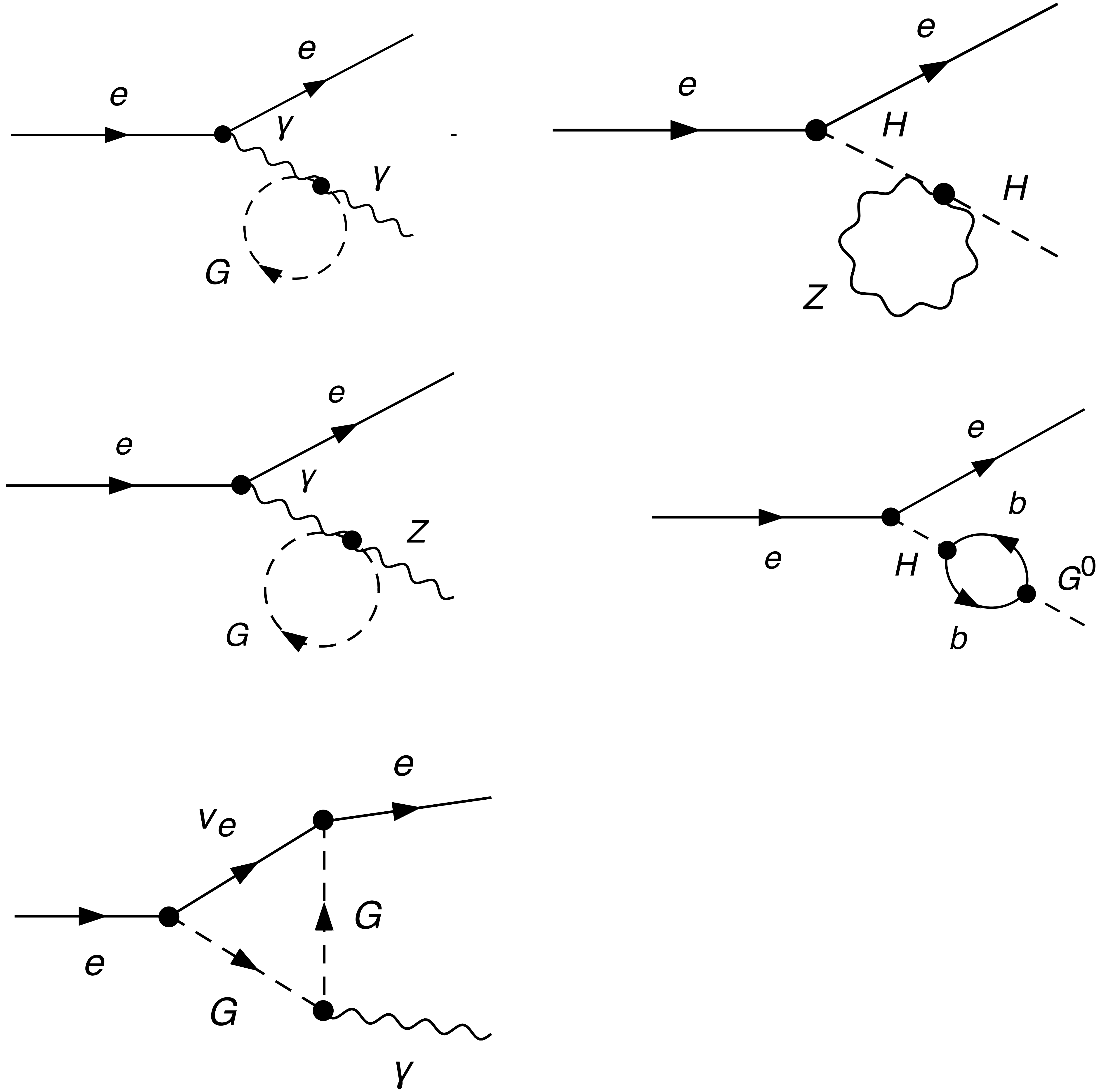

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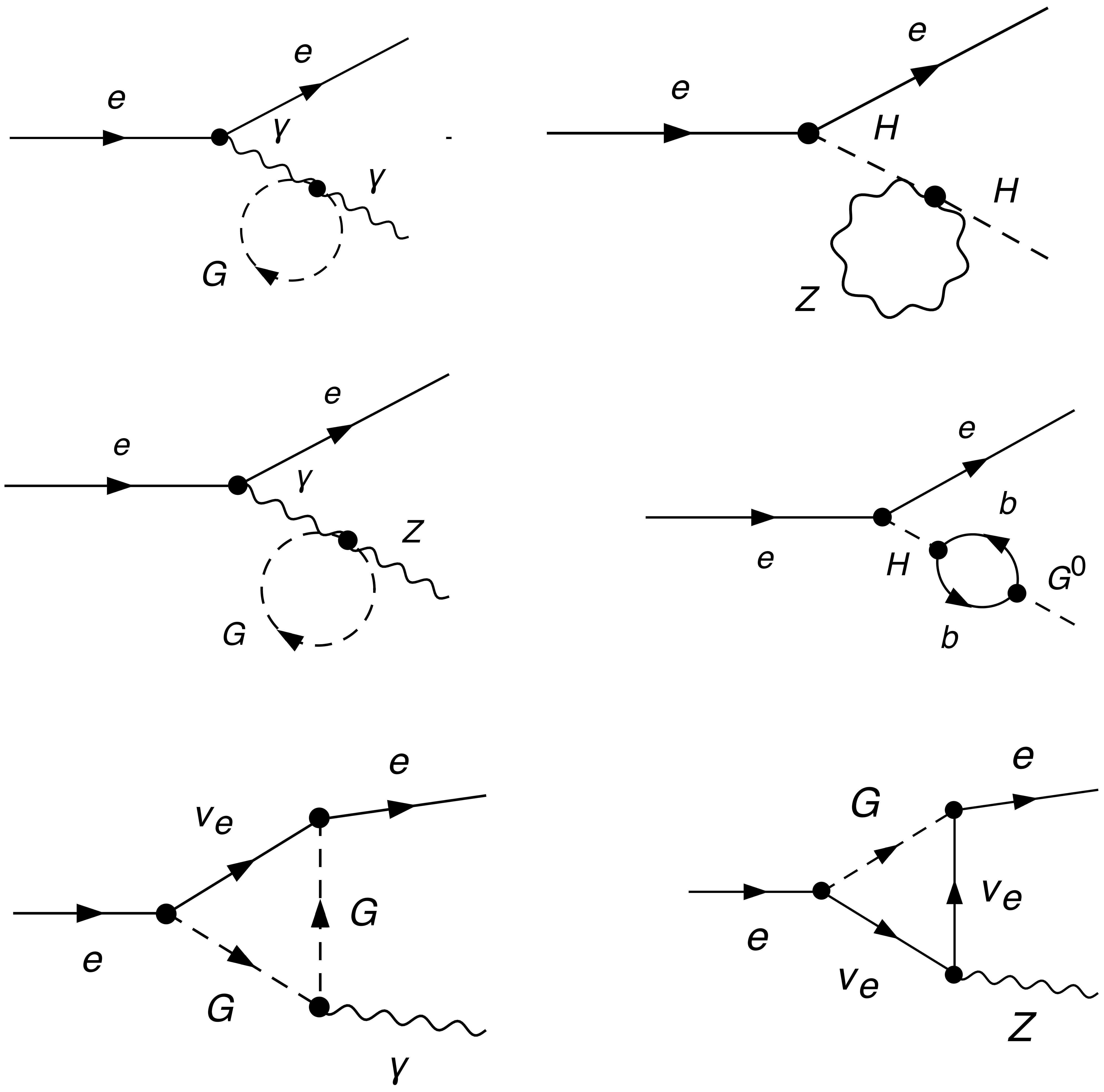

One loop level
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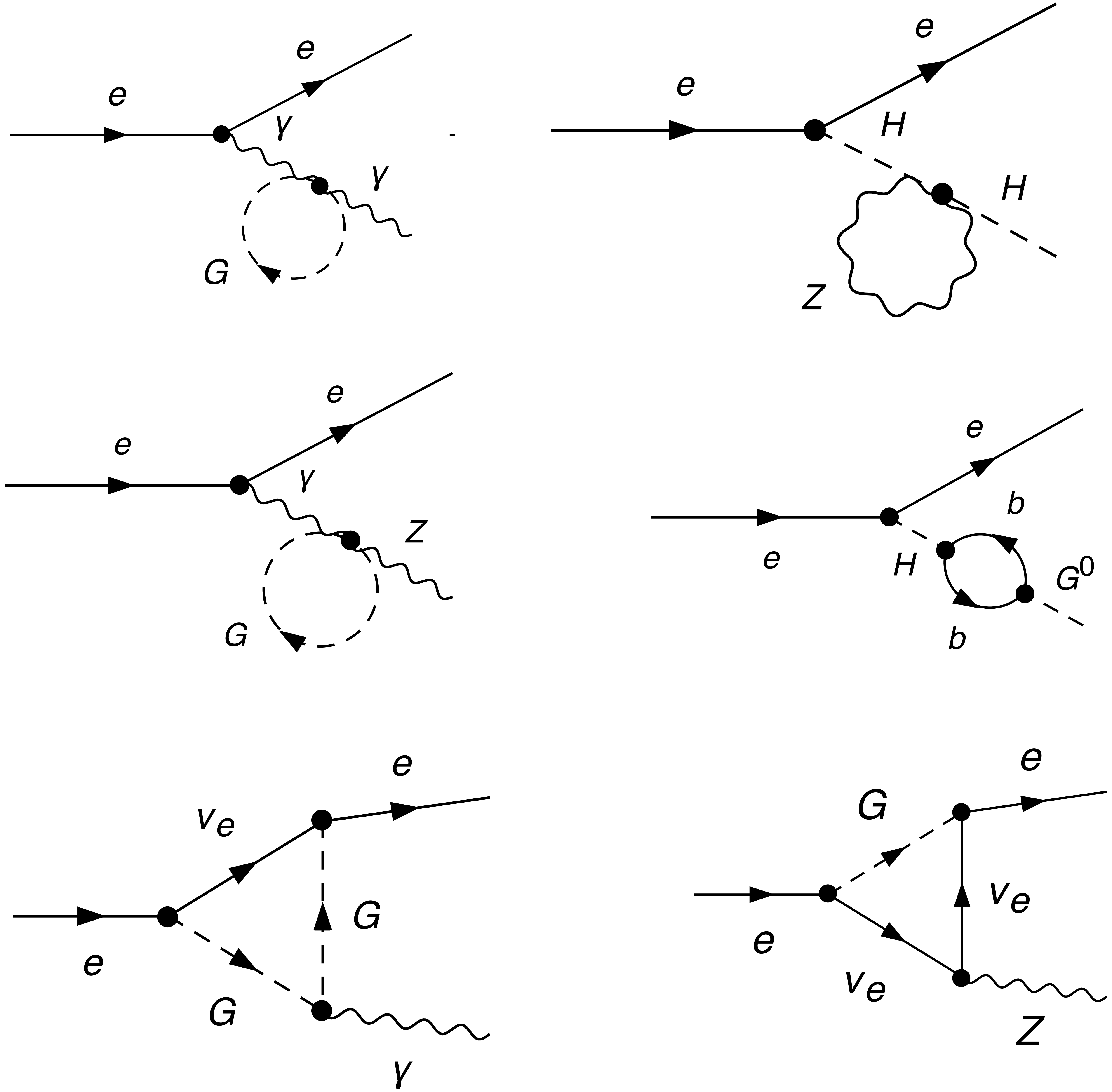
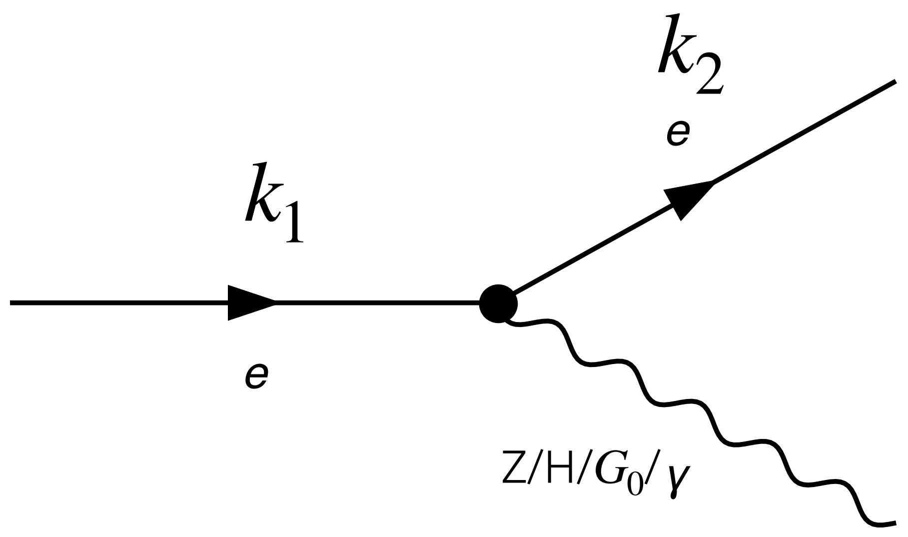
307 graphs



TREE-LEVEL LEPTONIC TENSOR (α-ORDER)

$$|M|^2 \propto \left| \quad \right|^2_{\mu\nu}$$



- For **tree-level** upper part of the diagram (say e^-p scattering), one can calculate leptonic tensor which is:

$$L^0_{\mu\nu} \propto 4\pi\alpha((l_1)g_{\mu\nu} + (l_2)k_{2\mu}k_{1\nu} + (l_3)k_{1\mu}k_{2\nu} + \ldots)$$

where k_1, k_2 are incoming and outgoing e^- momenta and $l_{1,2..}$ are tree level leptonic tensor structure functions.

ELECTROWEAK LEPTONIC TENSOR STRUCTURE FUNCTIONS

- In case of **tree level** polarized $e^- p$ scattering:

  - With photon (γ) as a mediator → **Five** leptonic tensor structure functions

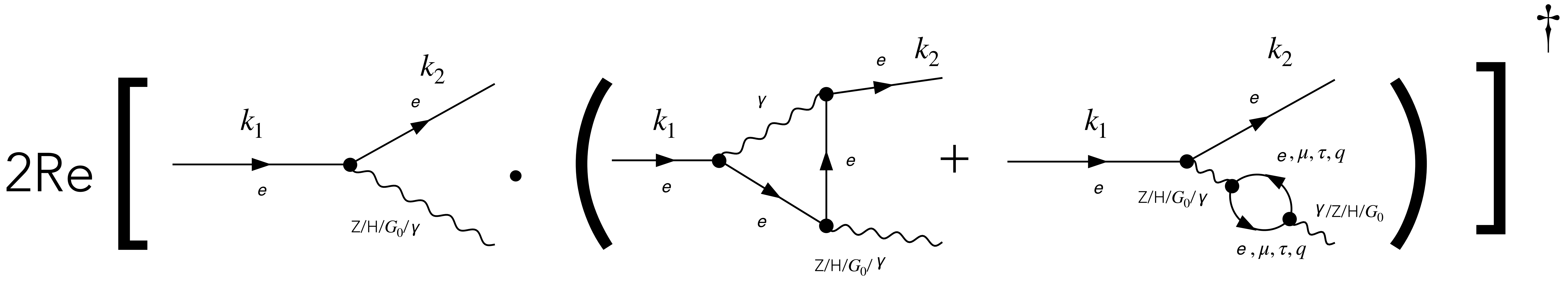
$$g^{\mu\nu},\ k_2^{\mu}k_1^{\nu},\ k_1^{\mu}k_2^{\nu},\ \epsilon^{s_1\mu\nu k_1},\ \epsilon^{s_1\mu\nu k_2}$$

where s_1 → helicity reference vector of the incoming electron.

  - With Z boson or γZ mixing → **Eight** leptonic tensor structure functions

$$g^{\mu\nu},\ k_2^{\mu}s_1^{\nu},\ k_2^{\nu}s_1^{\mu},\ {k_1}^{\mu}{k_2}^{\nu},\ k_2^{\mu}k_1^{\nu},\ \epsilon^{s_1\mu\nu k_1},\ \epsilon^{s_1\mu\nu k_2},\ \epsilon^{\mu\nu k_1 k_2}$$

NEXT TO THE LEADING ORDER (NLO) LEPTONIC TENSOR (α^2-ORDER)



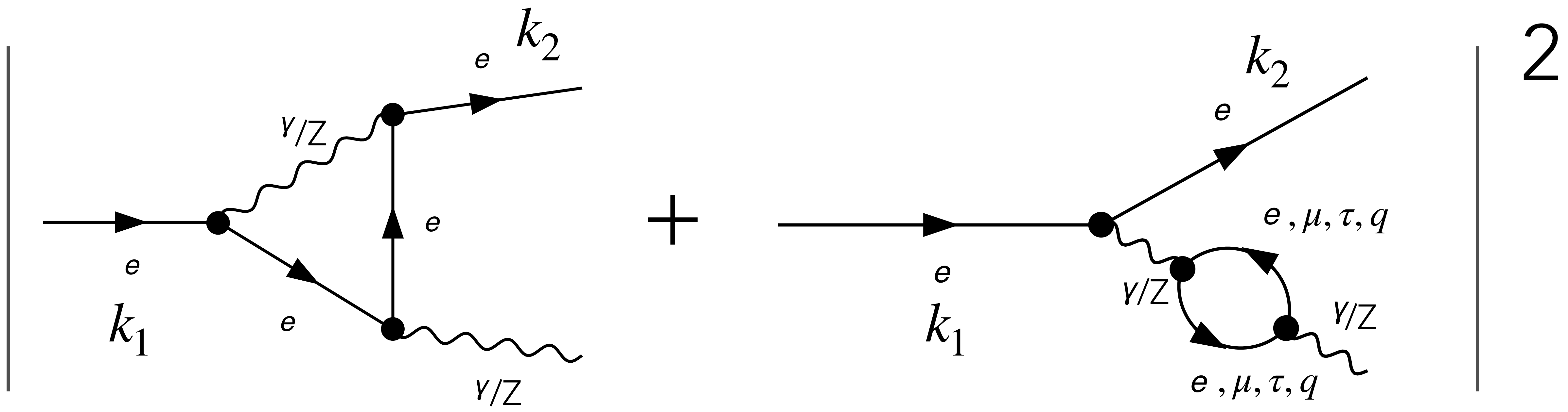
- The **NLO** leptonic tensor can be obtained by multiplying tree-level upper diagram with the sum of one-loop level SE and triangular diagrams.

$$L_{\mu\nu}^{NLO} = (m_1)g_{\mu\nu} + (m_2)k_{1\nu}k_{2\mu} + (m_3)k_{1\mu}k_{2\nu} + (m_4)k_{1\mu}k_{1\nu} + (m_5)k_{2\mu}k_{2\nu} + \ldots..$$

Where $m_{1,2,3...}$ are leptonic structure functions which depend on the momentum transfer "t" and written in terms of Passarino-Veltman integral functions. We used LoopTools Mathematica package to calculate them.

- With photon (γ), Z boson or γZ mixing → **19** leptonic tensor structure functions

NEW RESULTS: **QED AND ELECTROWEAK** QUADRATIC LEPTONIC TENSOR (α^3-ORDER)



- The **quadratic** leptonic tensor can be obtained by squaring the sum of one-loop level SE and triangular diagrams. Tensor form is the same as that of NLO and is given by:

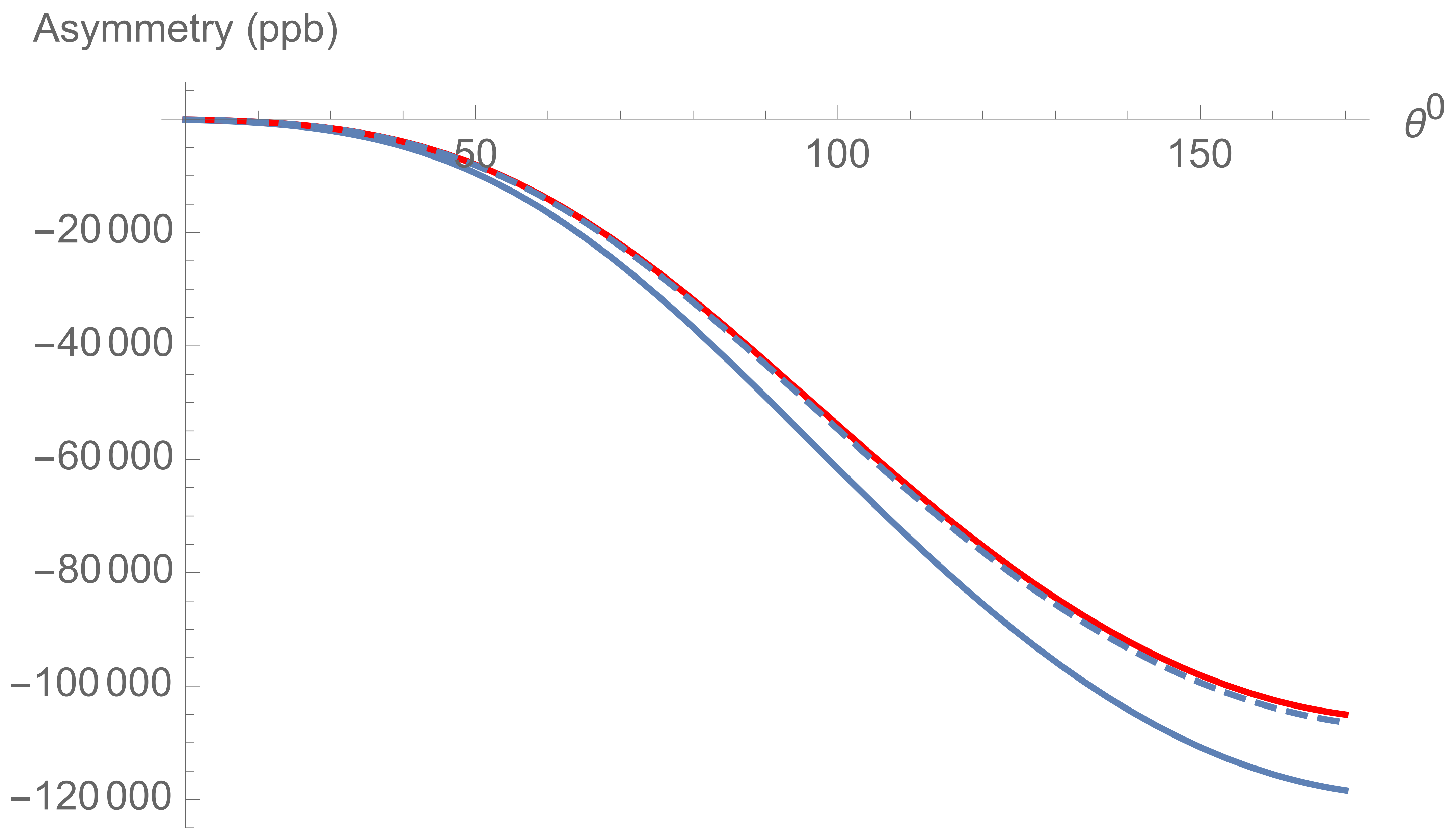
$$L_{\mu\nu}^{Quadratic} = (n_1)g_{\mu\nu} + (n_2)k_{1_\nu}k_{2_\mu} + (n_3)k_{1_\mu}k_{2_\nu} + (n_4)k_{1_\mu}k_{1_\nu} + (n_5)k_{2_\mu}k_{2_\nu} + \dots$$

where $n_{1,2,3..}$ are quadratic leptonic structure functions of the order of α^3.

- With photon (γ), Z boson or γZ mixing → **21** leptonic tensor structure functions.

- We kept **mass of the electron** throughout these calculations to account precision.

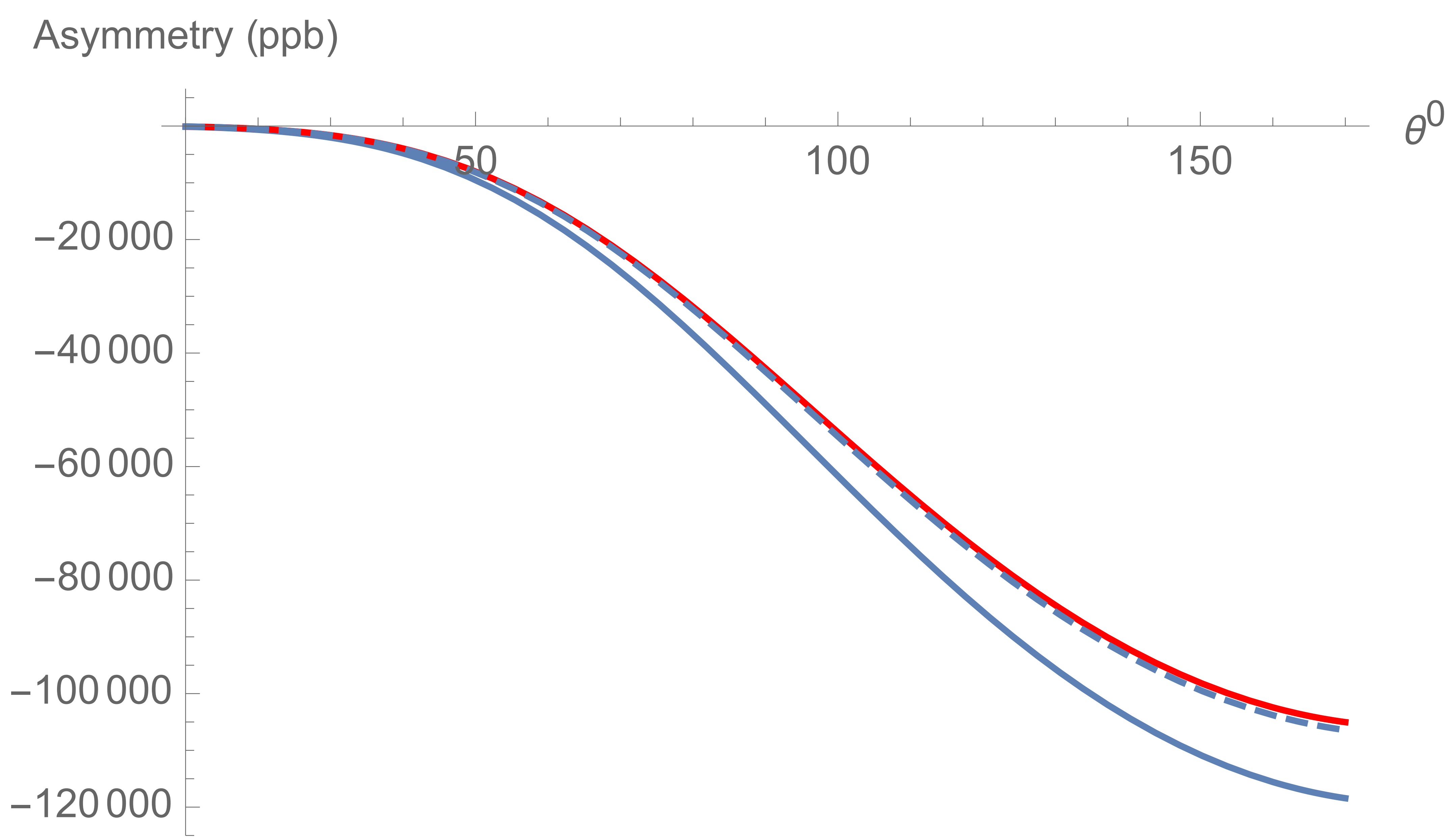
Graphs for Tree level, NLO and Quadratic level Parity Violating Asymmetry (A_{PV}) for $(e^- p)$ Scattering versus Scattering angle θ_{CM}



$(e^- p)$ Tree level, NLO and NNLO level A_{PV} versus θ_{CM}

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

Graphs for Tree level, NLO and Quadratic level Parity Violating Asymmetry (A_{PV}) for $(e^{-}p)$ Scattering versus Scattering angle θ_{CM}



Tree $A_{PV} \sim -294ppb$

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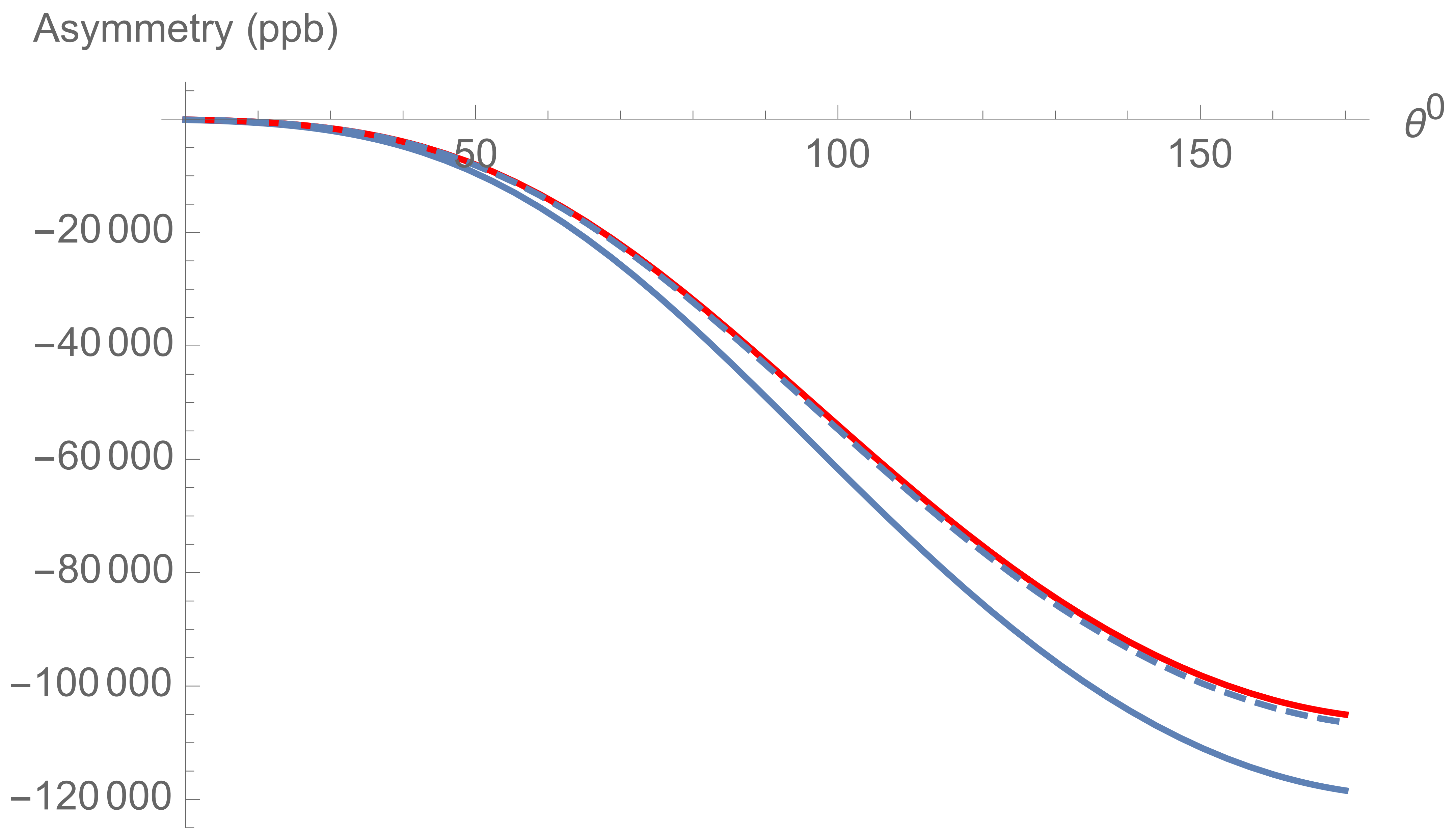

Tree $A_{PV} \sim -294ppb$

NLO $A_{PV} \sim -214ppb$

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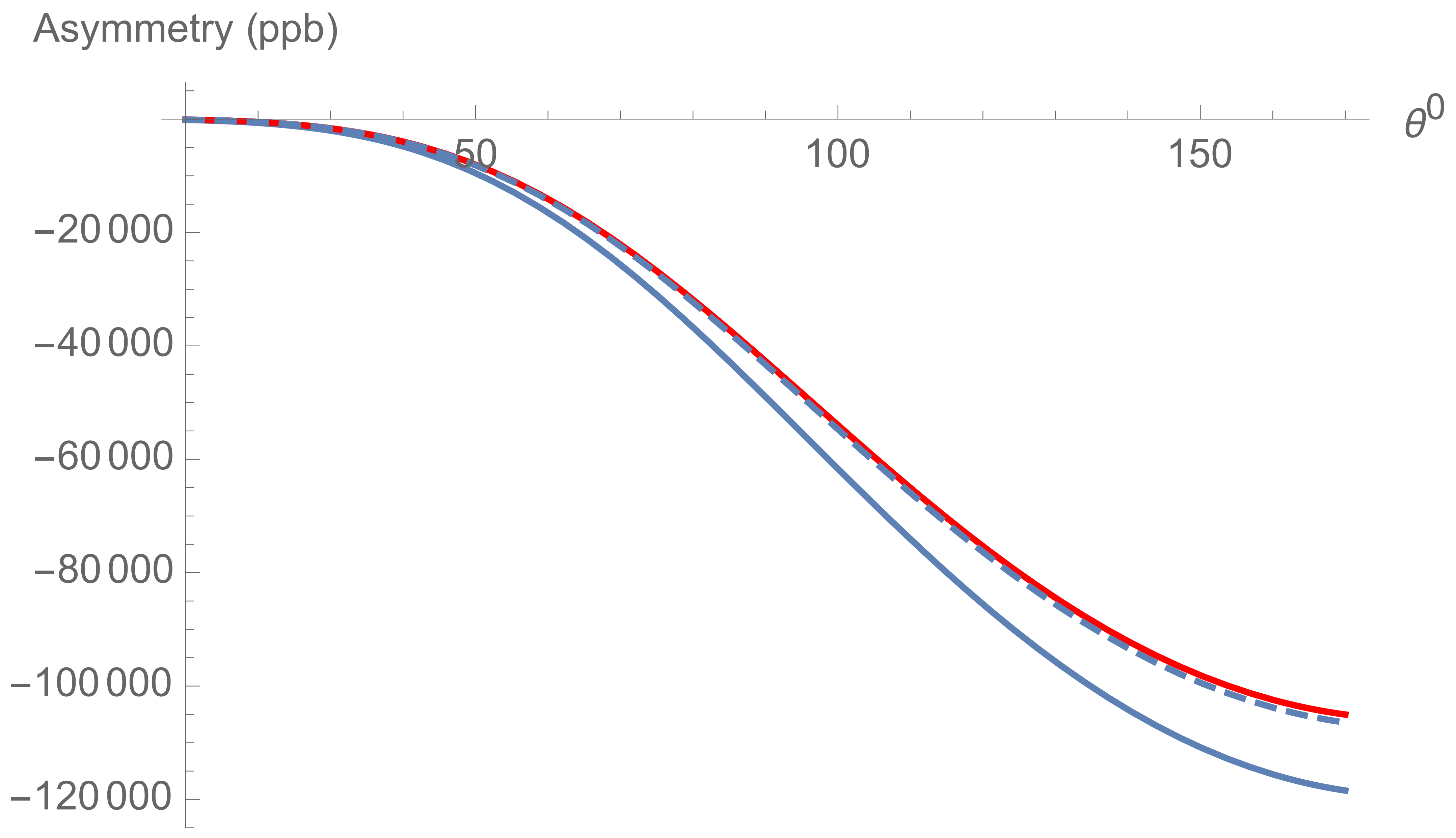

$(e^{-}p)$ Tree level, NLO and NNLO level A_{PV} versus θ_{CM}

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Quadratic $A_{PV} \sim -218ppb$

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

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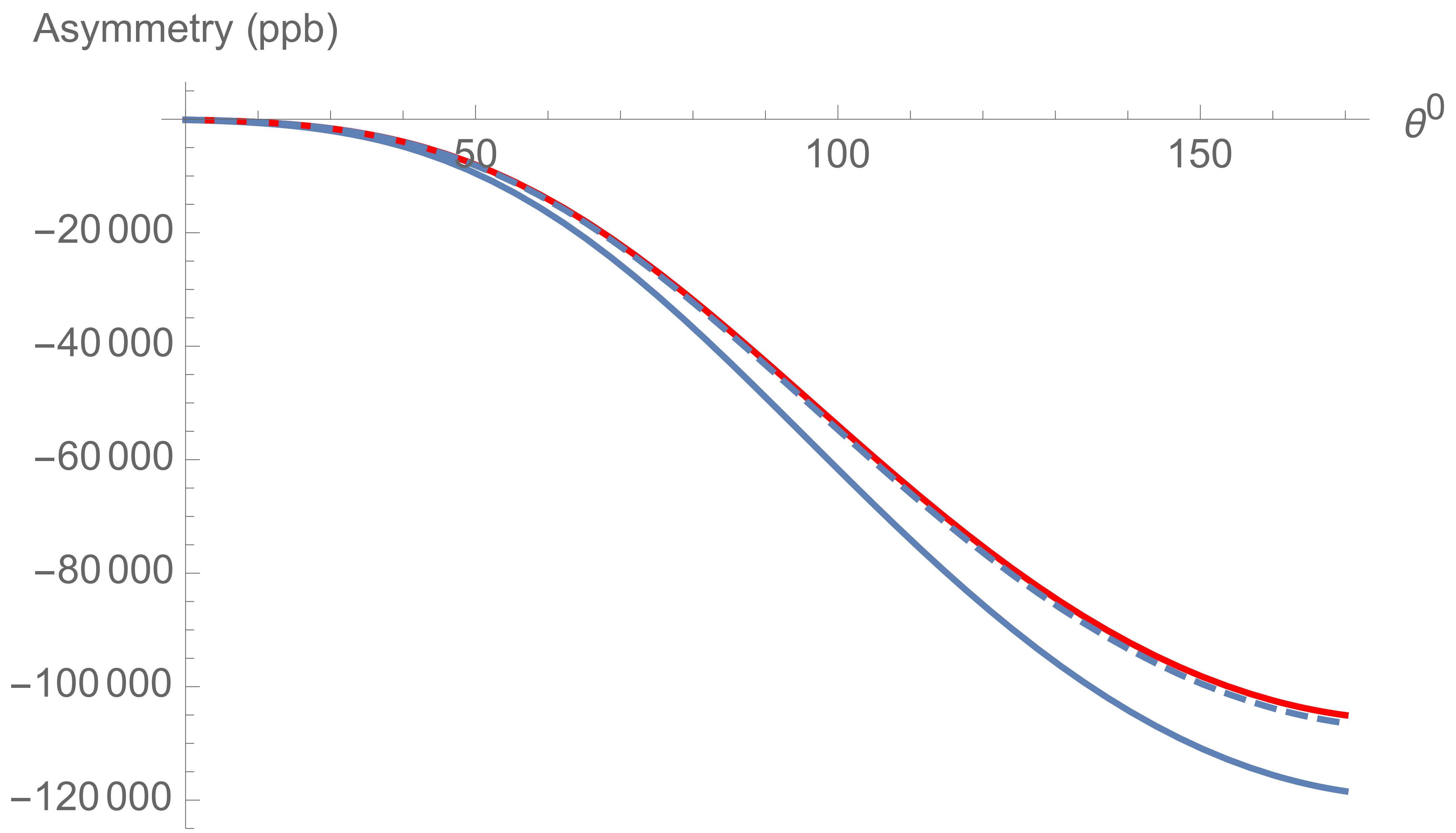

$(e^- p)$ Tree level, NLO and NNLO level A_{PV} versus θ_{CM}

$(\theta = 14.6^0)$

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$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

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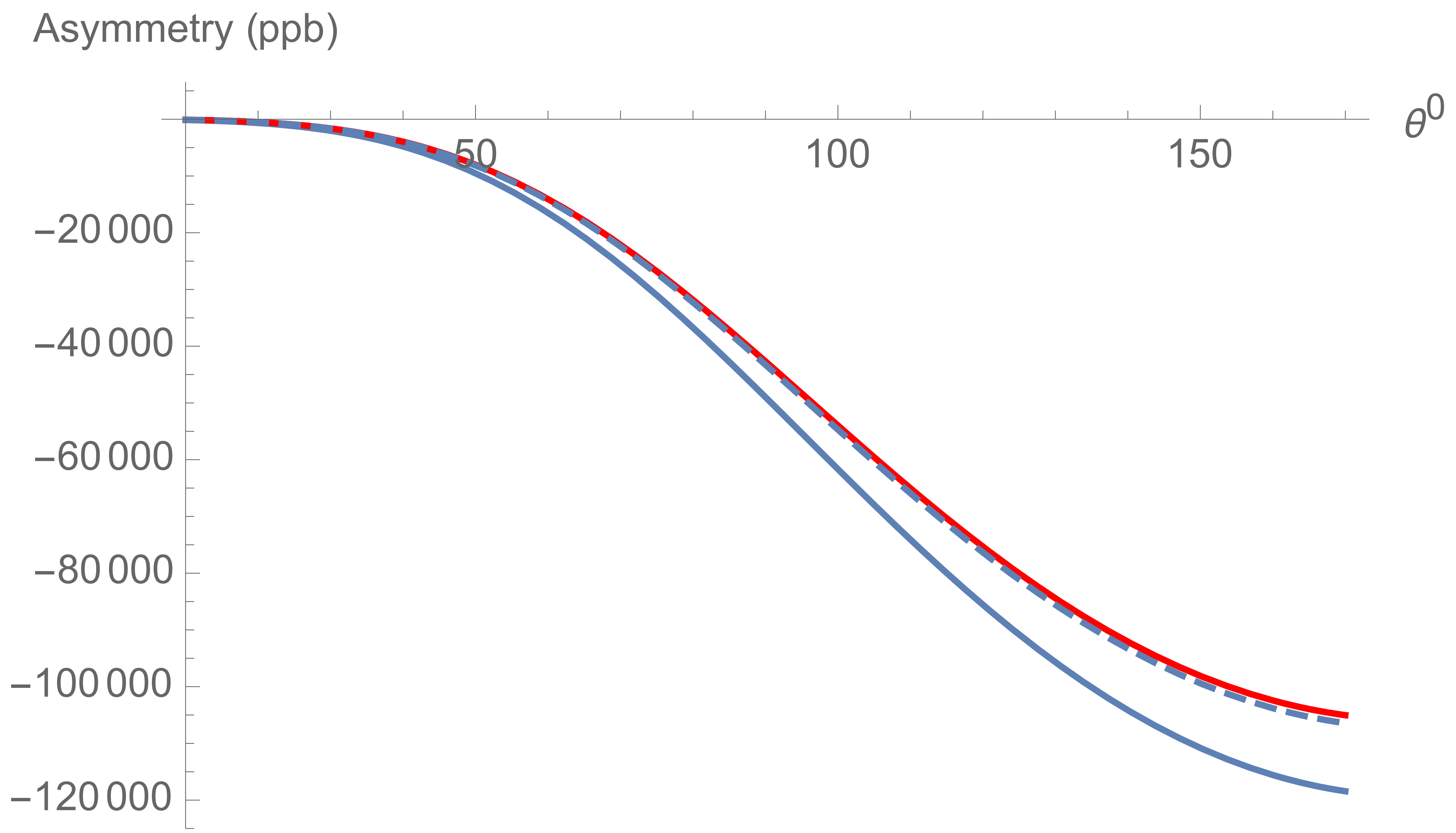

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QWEAK Measured

$\sim -226.5 \pm 7.3(stat) \pm 5.8(sys)ppb$

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

ADVANTAGES OF USING COVARIANT APPROACH

- This is a general approach and can be used to calculate any scattering process with a distinguishable target ($e^-\mu$, μ^-p).

- A good approach to calculate higher order effects by squaring one loop level diagrams e.g. our Quadratic leptonic tensor (α^3).

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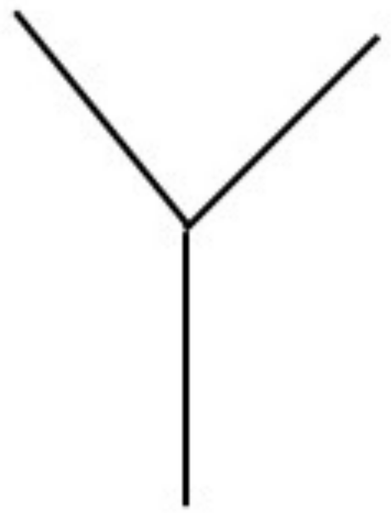

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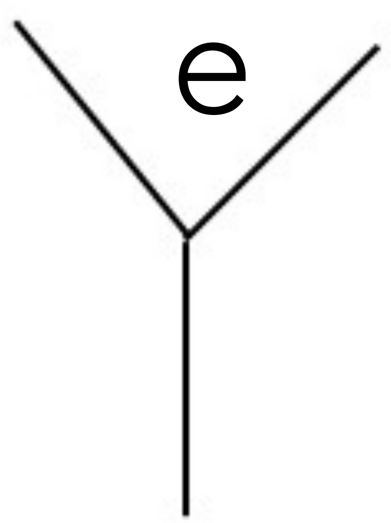

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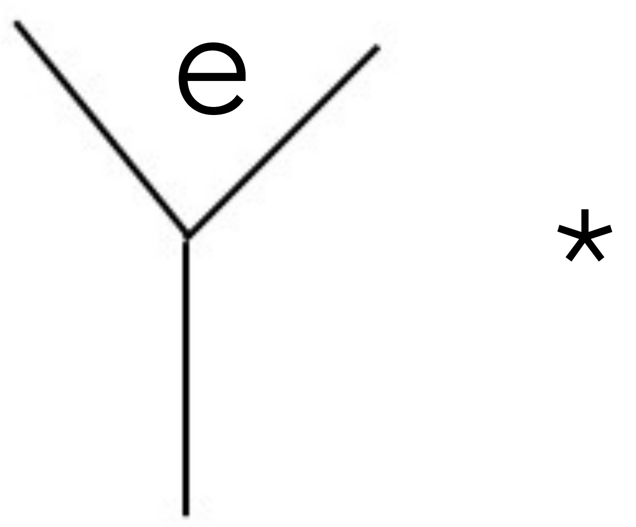

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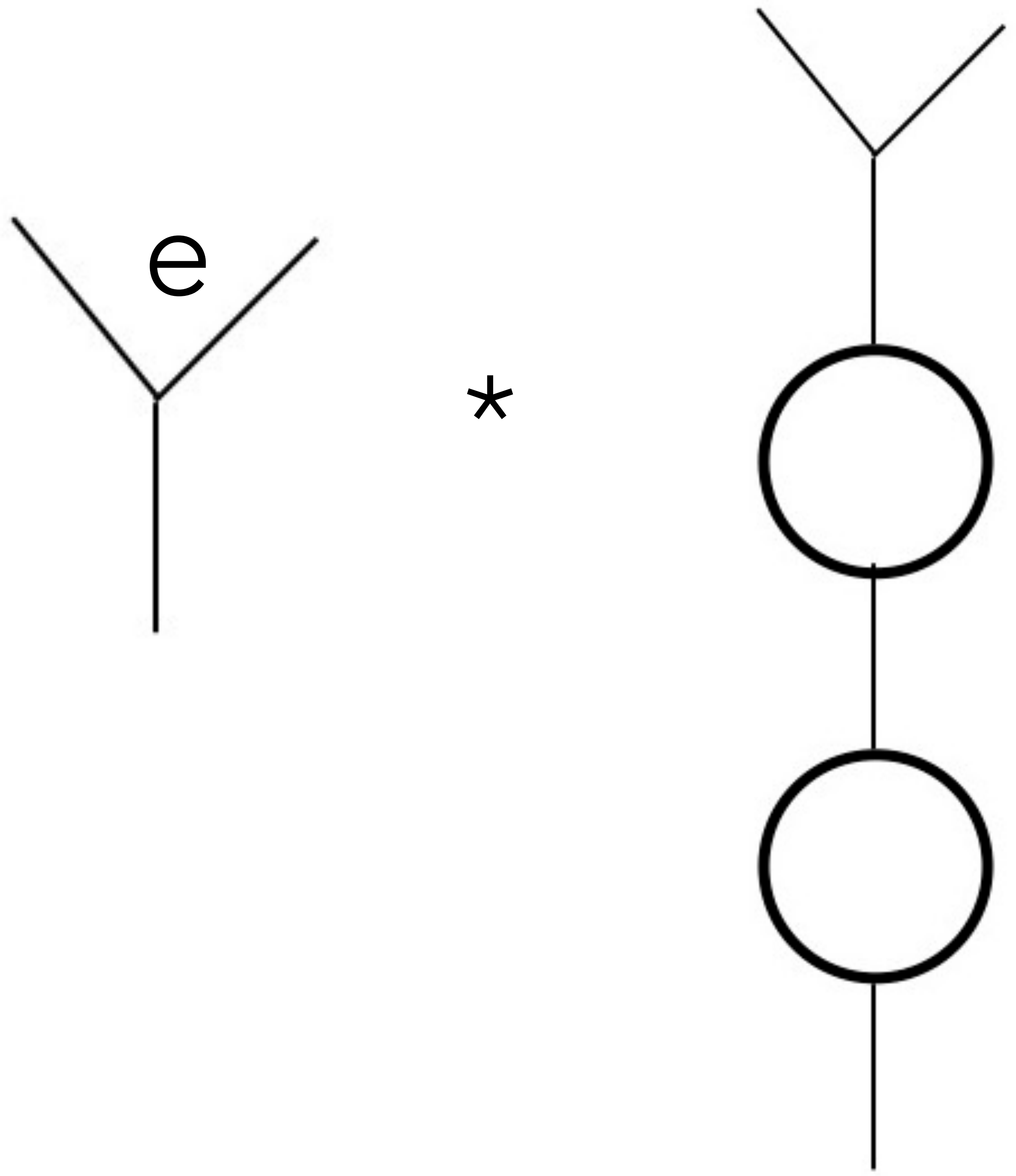

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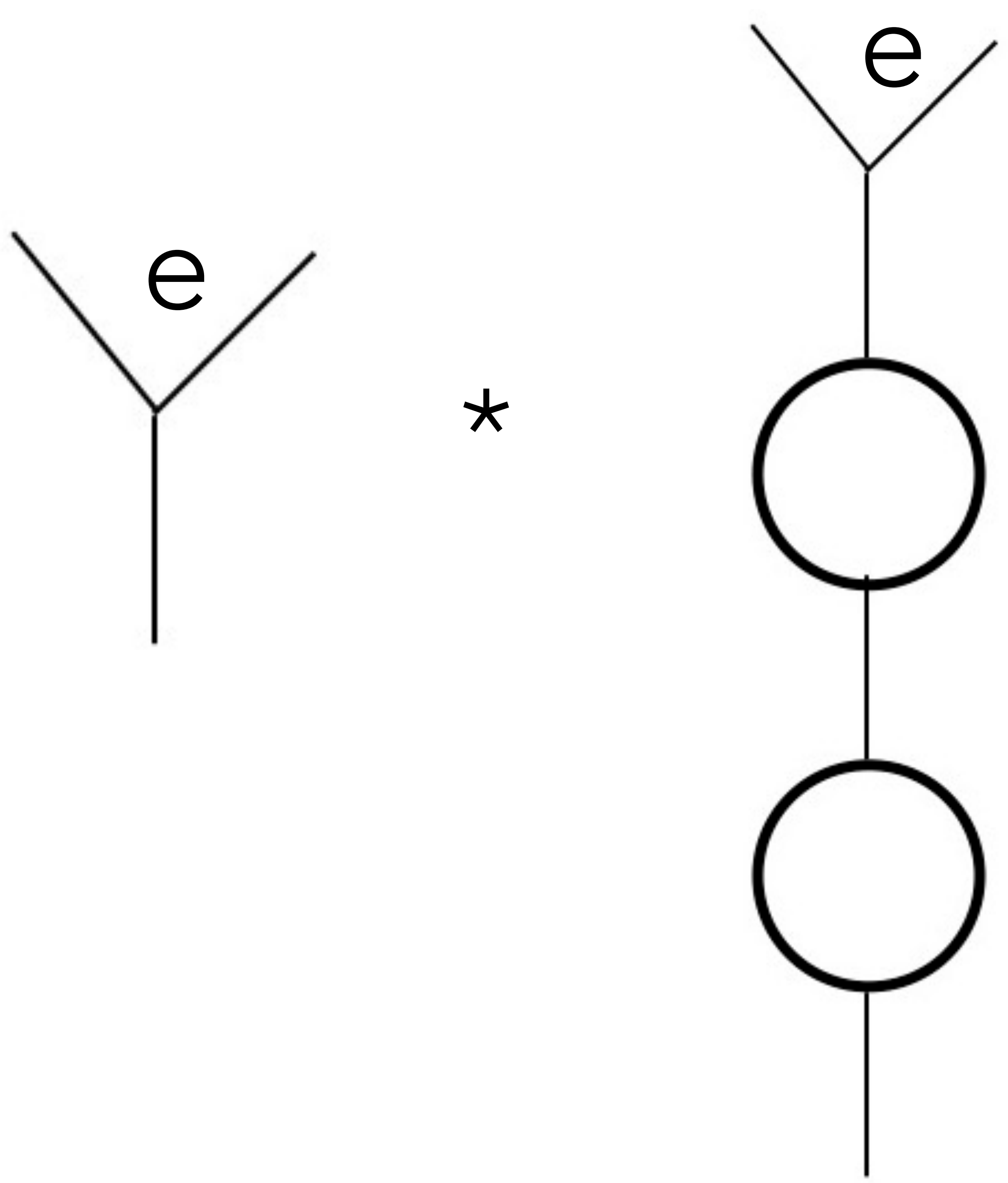

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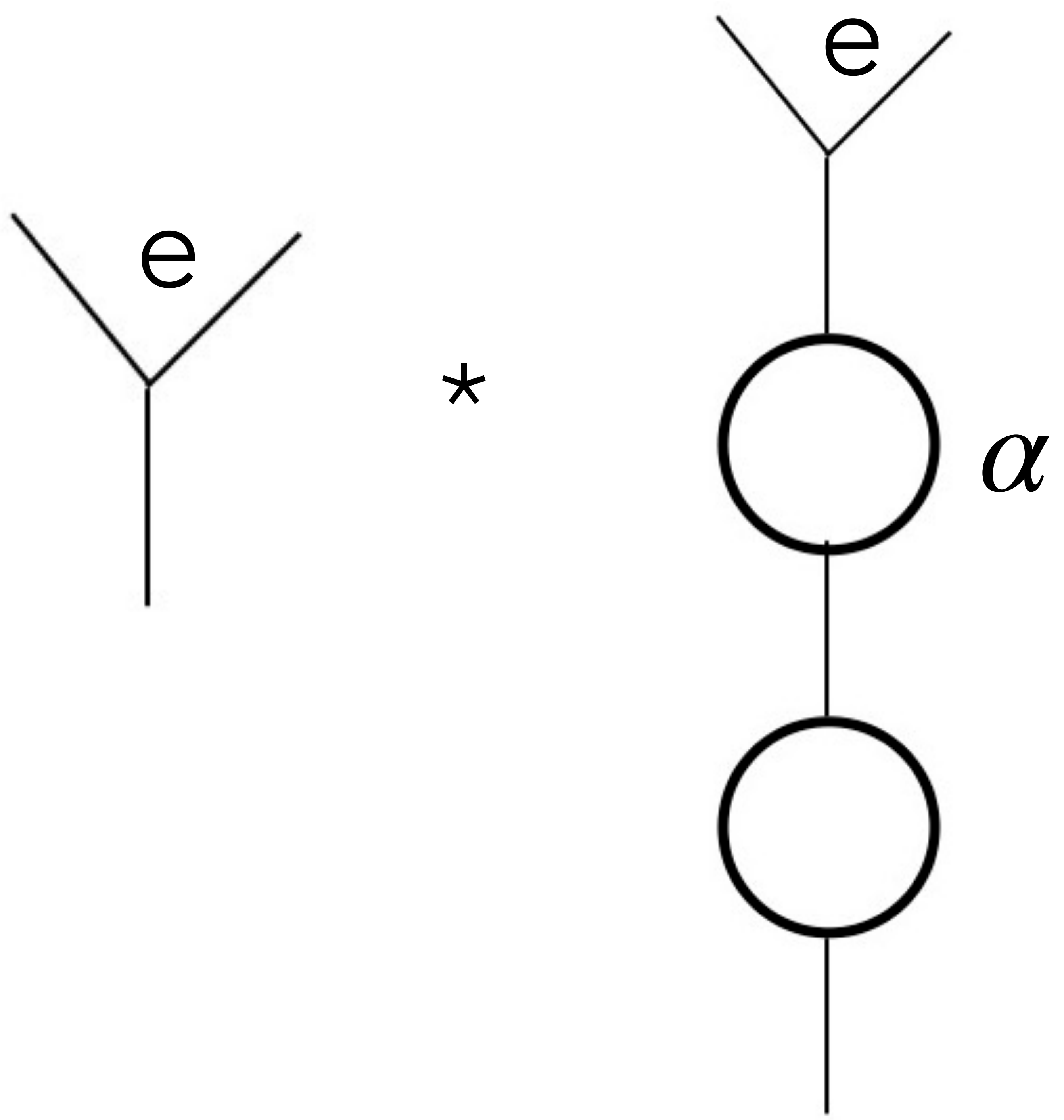

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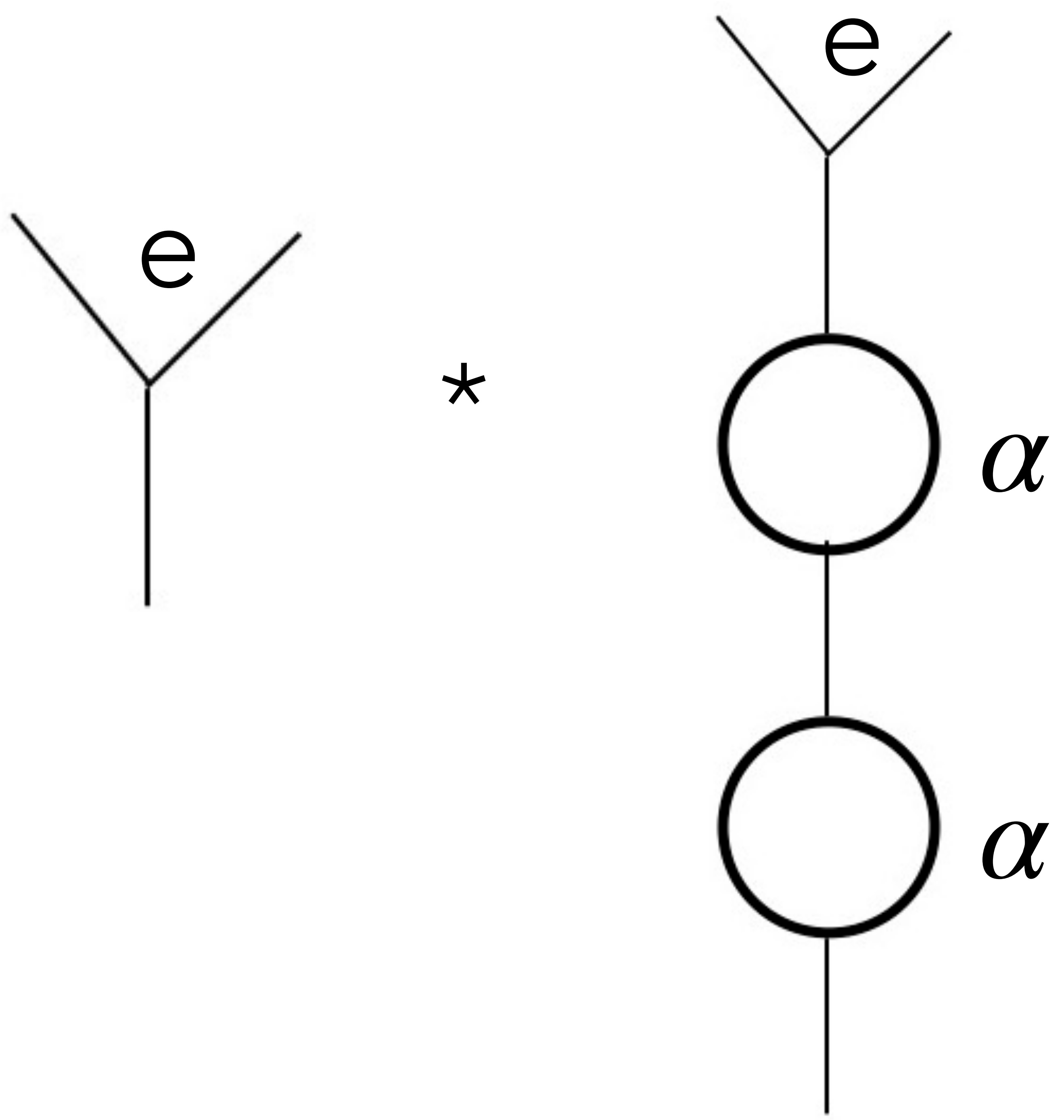

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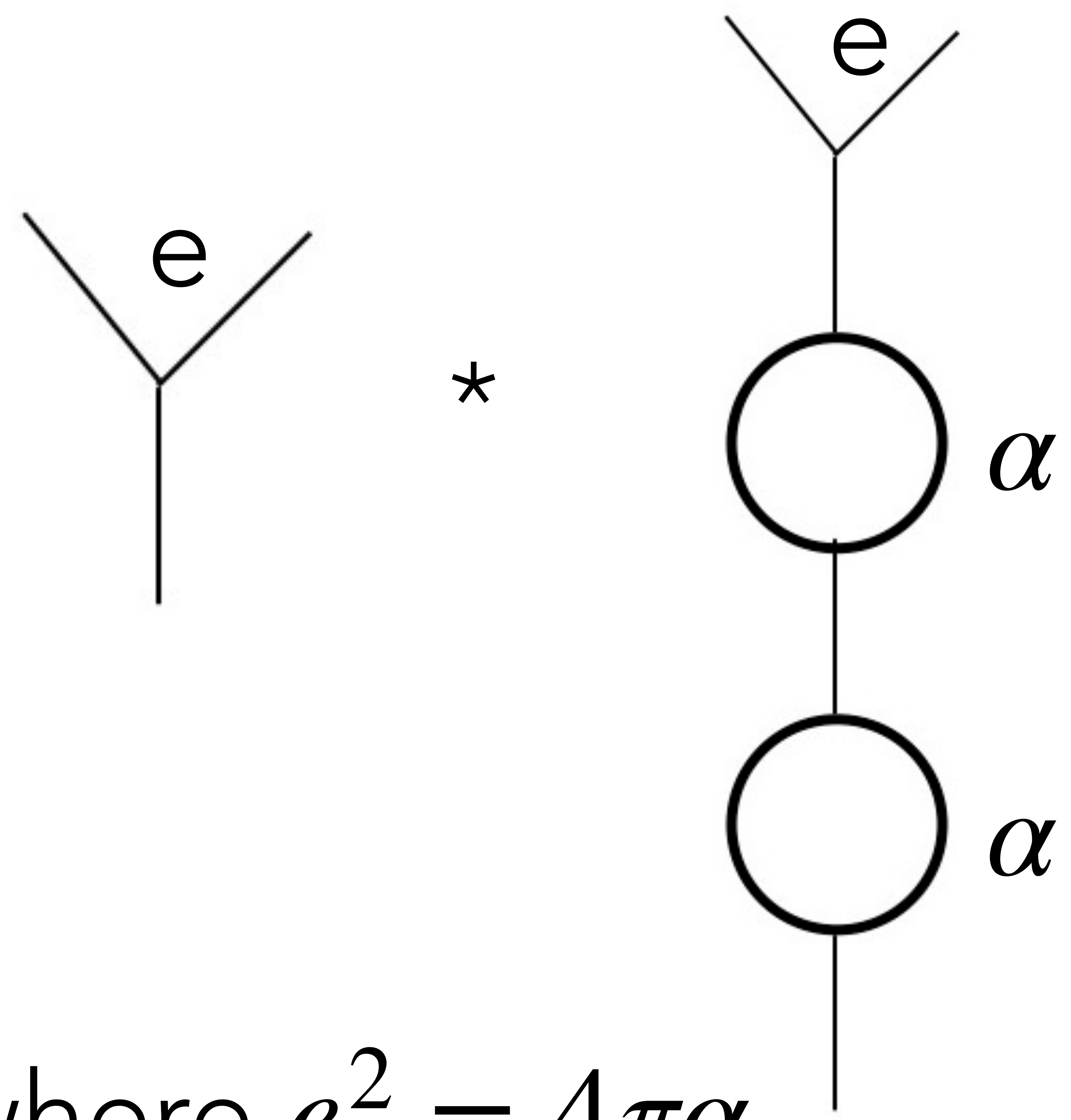

where $e^2 = 4\pi\alpha$

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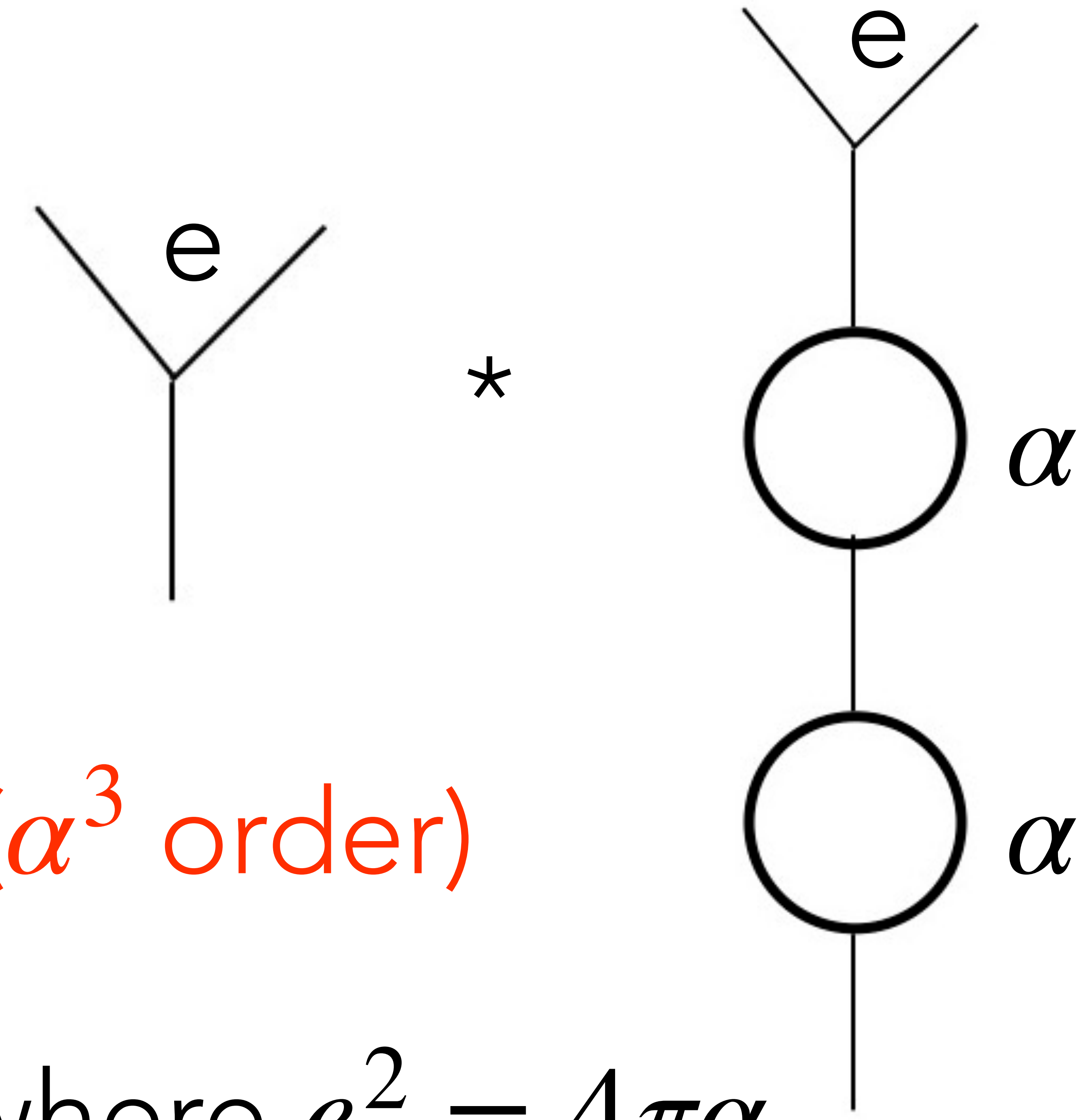


(α^3 order)

where $e^2 = 4\pi\alpha$

RESULTS:

- We have produced results for the QED and full electroweak quadratic leptonic tensors which were not calculated previously. We cross checked results using non covariant approach.

- We make predictions for the $e^{-}p$ NNLO (quadratic) level radiative corrections. Our $e^{-}p$ results are particularly useful in background analysis for the proposed Electron-Ion Collider experiment.

- Radiative corrections in $A_{PV} \rightarrow$ calculate the most precise value of the proton's weak charge: $Q_W^P = 1 - 4\sin^2\theta_W \rightarrow$ Any discrepancy may enable us to search for the physics beyond the Standard Model.

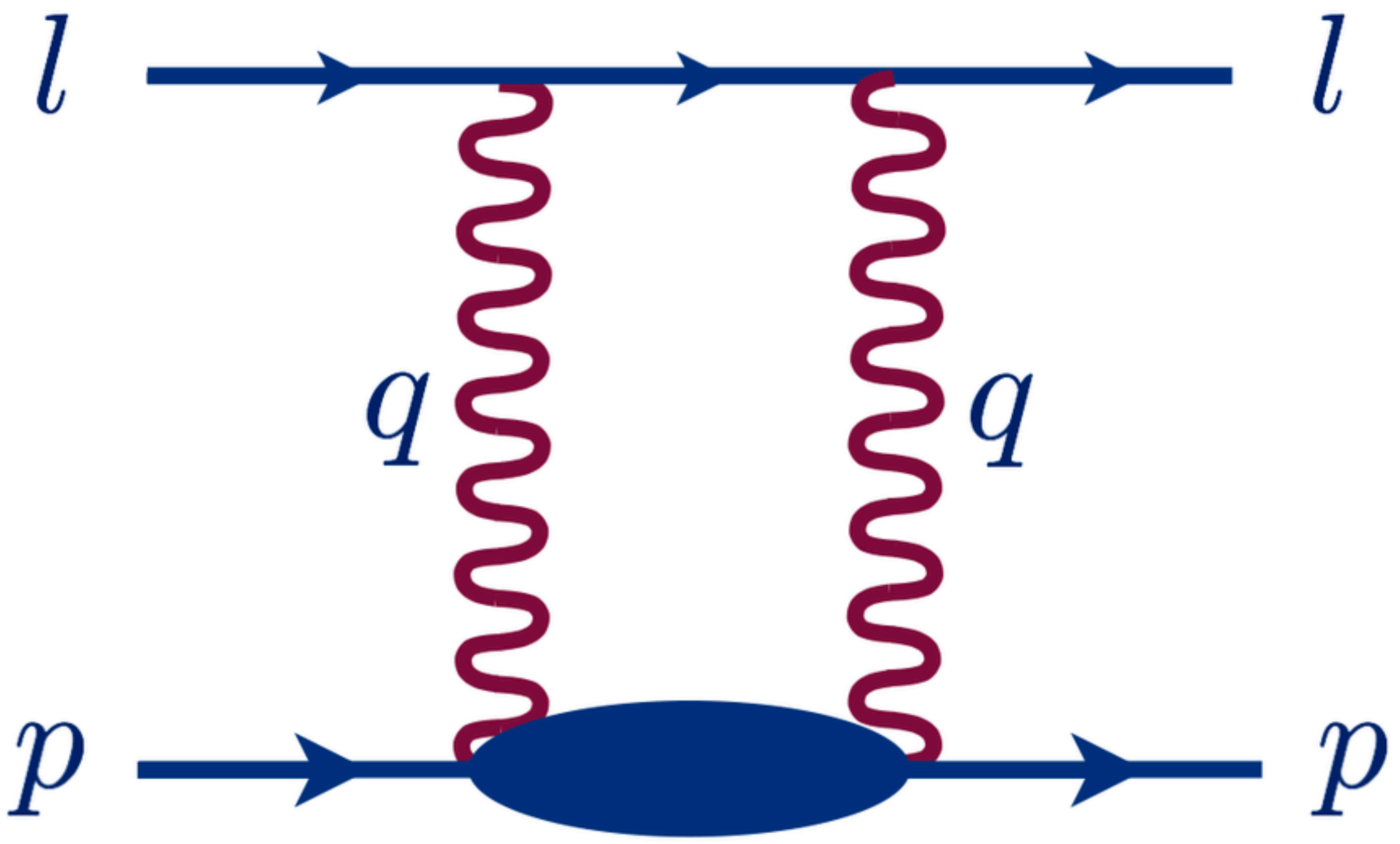
FUTURE GOALS

- We are calculating reducible up to two loop level electroweak leptonic tensors $\rightarrow$ another way to calculate quadratic level radiative corrections.
- For completeness, our next goal is to also include soft and hard photon bremsstrahlung cross sections in the results.
- These theoretical predictions will be important for many experimental programs such as QWEAK, MOLLER (background studies), EIC etc. searching for physics beyond the Standard Model at the precision frontier.

REFERENCES FOR BOX DIAGRAMS

[1] M. Gorchtein, Phys. Rev. C **73**, 055201 (2006)

[2] Peter G. Blunden et al., Physical Review Letters 91(14)



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Thanks for listening!