TRISEP '19: BSM LEC 3

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SLIGHTLY DIFFERENT:

1. **PIONS**: SYMMETRIES & SYMMETRY BREAKING
   motivation: HIGGS AS A GOLDSTONE
2. **AXIONS**: WHAT & WHY
   & relation to pions
   cf "POOLTABLE ANALOGY"
3. **WIMPS**: MOTIVATION
   DARK MATTER & HIGGS NATURALNESS
   ↑ slides

SYMMETRY

eg $i\bar{\psi}\not{\partial}\psi + V(\bar{\psi}\psi)$
invt under rephasing

GLOBAL: $e^{i\alpha T}$

GAUGE: $e^{i\alpha(x) T}$ (LOCAL)

↳ take a global symmetry &
"promote" it to a *mathematical redundancy*

$$\psi(x) \equiv \underbrace{e^{i\alpha(x) T}\psi(x)}$$

equivalent state
(not "transformed state w/ same physics")

introduces GAUGE BOSON (force)

eg $\bar{\psi}\, i\partial_\mu \gamma^\mu \psi \to \bar{\psi} e^{-i\alpha(x)}\, i\gamma^\mu \partial_\mu \left(e^{i\alpha(x)}\psi(x)\right)$

$$= i\bar{\psi}\not{\partial}\psi \underbrace{- [\partial_\mu \alpha(x)]\bar{\psi}\gamma^\mu\psi}_{\text{EXTRA TERM}}$$

SO PROMOTE $\partial_\mu \to \boxed{D_\mu = \partial_\mu - i A_\mu}$
s.t. $\boxed{A_\mu \mapsto A_\mu + \partial_\mu \alpha(x)}$

(for simplicity I set $g=1$

$$i\bar{\psi}\gamma^\mu D_\mu \psi \to i\bar{\psi}\gamma^\mu\partial_\mu\psi - [\partial_\mu\alpha(x)]\bar{\psi}\gamma^\mu\psi + \bar{\psi}\gamma^\mu A_\mu\psi + [\partial_\mu\alpha(x)]\bar{\psi}\gamma^\mu\psi$$

$$= i\bar{\psi}\gamma^\mu D_\mu\psi \quad \checkmark$$

<u>OBSERVE</u>: $A_\mu(x)$ IS A FIELD $\rightsquigarrow$ *force particle*

SYMMETRY BREAKING

GLOBAL SYM → EXACT
→ BROKEN → APPROX (EXPLICIT BREAKING) → CLASSICAL / QUANTUM (ANOMALY)
→ SPONTANEOUS (GOLDSTONE)

GAUGE SYM → EXACT
→ SPONTANEOUSLY BROKEN (GAUGE BOSON EATS GOLDSTONE)
⇢ *no explicit breaking*

SM PIONS: pGB → CHIRAL $SU(2)_L \times SU(2)_R$ (flavor) IS SPONTANEOUSLY BROKEN
→ ... <u>APPROX</u> SYM FROM QUARK MASSES
→ ... DECAY RELATED TO <u>ANOMALOUS</u> $U(1)_A$
→ EM IS UNBROKEN SUBGROUP THAT IS GAUGED

GLOBAL SYM of SM (FLAVOR)

<u>MATTER</u>: Q u d L e $\boxed{\times 3 \text{ generations}}$

eg $\underbrace{\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}}$

GLOBAL $U(3)$ sym $\cong SU(3) \otimes U(1)$

REPHASE

"if you know the Feynman rules of 1 gen, you know them for all 3" (up to specific numbers)

$$\mathcal{L} = \mathcal{L}_{KIN} + \mathcal{L}_{YUK} + \mathcal{L}_{HIGGS}$$

has full $U(3)^5$ global sym.

$$y^u \bar{Q} \cdot \tilde{H} u + y^d \bar{Q} \cdot H d + y^e \bar{L} \cdot H e + \text{h.c.}$$

3×3 $\mathbb{C}$ matrices in FLAVOR SPACE

this explicitly breaks $U(3)^5$

eg $Q \to U_q Q$
$u \to U_u u$

$\bar{Q} \cdot \tilde{H} u \to \tilde{H} \bar{Q} \boxed{U_q^\dagger U_u} u$

left over sym: $U_q^\dagger = U_u^\dagger$

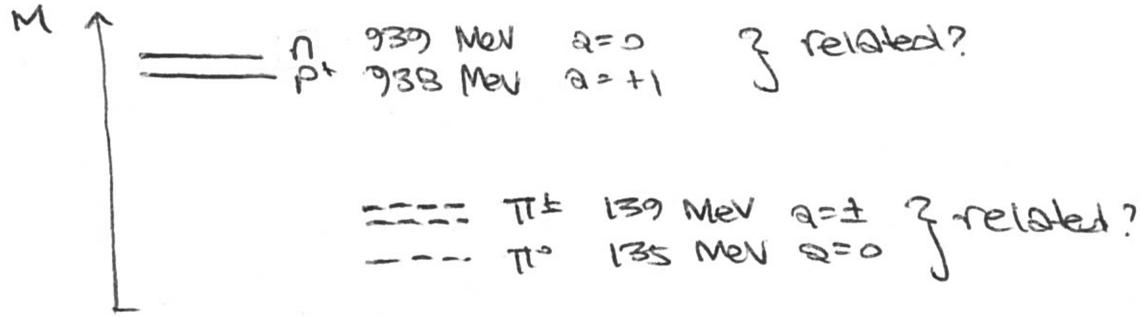
$U(3)^2 \to U(3)$

OR: $U(3)_q \times U(3)_u \to U(3)_V$

DIAGONAL

APPROXIMATE SYM : BOTTOM-UP

in ancient times:



Maybe : ∃ APPROX sym, isospin $SU(2)$ $(\neq SU(2)_L)$
w/

DOUBLET $\begin{pmatrix} p \\ n \end{pmatrix}$ ← cf HIGGS is $\underline{2}$ of $SU(2)_L$

TRIPLET $\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$ ← cf $W^\pm, W^3$ is $\underline{3}$ of $SU(2)_L$

what about CHARGE? appears that

$$Q = T^3 + \tfrac{1}{2} \underset{\uparrow\ \text{BARYON \#}}{B}$$

remark : when you add in the "strange hadrons" you also identify a "strangeness #" from selection rules

$$Q = T^3 + \tfrac{1}{2} \underbrace{(B+S)}_{\text{call this HYPERCHARGE, } Y}$$

eg $\begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$ $Q=1$ $T^3 = \frac{1}{2}$ $B=0$ $S=1$

$Q=0$ $T^3 = -\frac{1}{2}$ $B=0$ $S=1$

in fact

$SU(2) \times U(1)_Y \subset \underbrace{SU(3)_F}$

Gell-Mann 8-fold way
classification of Hadrons

relation to 'fundamental' physics?

sym of $\underline{QCD}$ when $\underbrace{\text{light quarks}}$ $\to$ massless

ignore EW $\qquad$ u, d, s

$U(3)_L \longrightarrow$ $u_L \quad c_L \quad t_L$
$\qquad\qquad\quad d_L \quad s_L \quad b_L$

$U(3)_R \longrightarrow$ u_R
$\qquad\qquad\quad d_R \quad s_R$

in fact - just take $U(2)_L \times U(2)_R$.

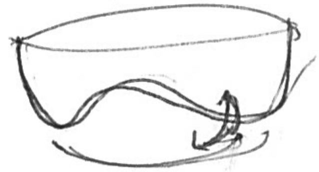
<u>SYMMETRY</u> : $U(2)_L \times U(2)_R = SU(2)_V \times SU(2)_A \times \underbrace{U(1)_B} \times U(1)_A$

BARYON #

GOOD SYM $\qquad$ AXIAL

$\boxed{\text{QCD}}$ BREAKS $SU(2)_A$ SPONTANEOUSLY

$\langle \bar{q} q \rangle$



Goldstone bosons

rule: · take vev [SPONTANEOUSLY]
· transform it w/ broken sym.

· promote sym param to field.

$$U(1): \langle\phi\rangle = v \to e^{i\alpha} v \to \boxed{e^{i\frac{\alpha(x)}{v}} v}$$

· PLACE $\langle\phi\rangle$ into your theory "before" sym breaking.

eg $|\partial\phi|^2 \to (\partial\alpha)^2$ kinetic term

FOR $SU(2)_L \times SU(2)_R \to SU(2)_V$:

WRITE Bifundamental $U \mapsto L U R^{\dagger}$ (L: $SU(2)_L$, R: $SU(2)_R$)

$$\langle U \rangle = f_\pi \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

invt under $L = R$ $SU(2)_V$
but breaks $L = R^{\dagger}$ $SU(2)_A$

$$\langle U \rangle \xrightarrow{SU(2)_A} e^{i\alpha^a T^a} \langle U \rangle e^{i\alpha^a T^a}$$

$$\boxed{U_\pi(x) \equiv e^{2i\pi^a(x)T^a/f_\pi}}$$

then: $\mathcal{L} \sim \mathrm{Tr}\left[(\partial U_\pi^{\dagger})(\partial U_\pi)\right] = \frac{4}{f_\pi^2}(\partial\pi^a)(\partial\pi^b)$ $\mathrm{Tr}(T^aT^b)$

so normalize by $f_\pi^2/4$ + (DERIV. INT.)

$$\Gamma \quad U = \mathbb{1} + \frac{2i}{f_\pi}\pi^a T^a - \frac{2}{f_\pi^2}\pi^a\pi^b T^a T^b + \dots \qquad \dashv$$

SYMMETRY:

		$\underline{SU(3)_L}$	$\underline{SU(3)_R}$	$\underline{U(1)_B}$
$SU(2)_L$ GAUGE	$\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix}$	$\underline{3}$	$\mathbb{1}$	$1/3$
	$\begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$	$\mathbb{1}$	$\underline{3}$	$1/3$

$\leftarrow$ cf GLOBAL SYM of SM

$\subset SU(3)_L$ $\qquad \boxed{T^3_R + \frac{1}{2}(B-L)}$

GAUGE $SU(2)_L \times U(1)_Y \subset SU(3)_L \times SU(3)_R \times U(1)_B$

$$\partial_\mu \mapsto D_\mu = \partial_\mu - igW^a_\mu \frac{\tau^a}{2} - ig'\frac{\tau^3}{2}B_\mu$$

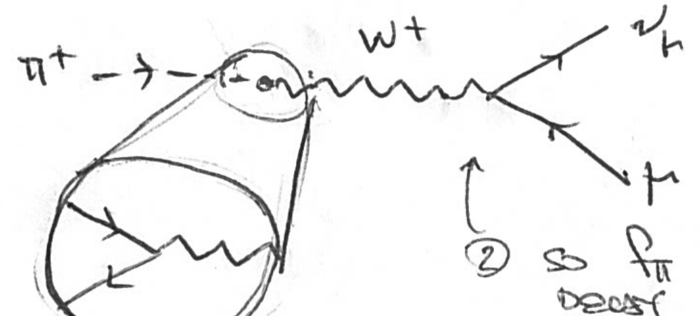
acts from left on U

ACTS from RIGHT on U

this explicitly breaks global sym

$\hookrightarrow$ introduces terms that are not symmetric.

including: $g\frac{f_\pi}{2}W^+_\mu\partial^\mu\pi^- + h.c.$



② so f_π called PION DECAY CONSTANT

① LOOKS LIKE $W^\pm$ EATS $\pi^\pm$

indeed: $\frac{f_\pi^2}{4} \mathrm{Tr}\,(DU_\pi^\dagger)(DU_\pi) \supset$ GAUGE BOSON MASSES

$\langle U_\pi \rangle = \mathbb{1}$

$f_\pi \simeq 93$ MeV

$\frac{g^2 f_\pi^2}{4} W^+W^- + \frac{(g^2+g'^2)}{4} f_\pi^2 Z^2$

nb: not so far fetched that this framework may be adapted to $\mathcal{O}(100$ GeV$)$ EWSB!

↳ ORIGINAL IDEA: just scale this up : TECHNICOLOR
but need radial mode → "HIGGS AS pNGB"

WHAT ABOUT PION MASS?

from explicit breaking → quark YUKAWAS (MASSES) ; gauging (LOOPS)

$$\boxed{q_L^\dagger M q_R + \text{h.c.}}$$

$q_L^\dagger$: $\bar{3}$ of $SU(3)_L$ $\qquad$ q_R: 3 of $SU(3)_R$

trick: PRETEND M is a field s.t. this is invariant

⇒ M is $(3,\bar{3})$ under $SU(3)_L \times SU(3)_R$
"bifundamental"

⇝ then set to a "vev" $M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$

PUT IN SIMPLEST (lowest order) INTERACTION

$$\mathcal{L} \supset f_\pi^3 \, \mathrm{Tr}\left(M^\dagger U_\pi + h.c.\right) \leftarrow \text{eg. } -m f_\pi^3 \cos(\pi^0/f_\pi)$$

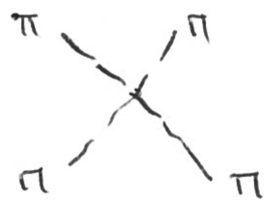
& work out spectrum

↳ in ANCIENT TIMES: did not know m_u, m_d, m_s
so derive mass relations
(Gell-Mann — Okubo) ← assume $m_u = m_d$

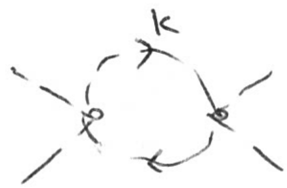
$$3m_\eta^2 + m_\pi^2 = 4m_K^2$$

last remarks

Perturbativity:



$\sim \dfrac{p^2}{f_\pi^2}$



shift sym.

$$\sim \frac{p^2}{f_\pi^4} \underbrace{\int d^4k \, \frac{k^2}{k^4}}_{\Lambda^2/(4\pi)^2}$$

$$\Rightarrow \quad \underset{\sim \text{GeV}}{\Lambda} \lesssim 4\pi \underset{\sim 100 \text{ MeV}}{f}$$

scale @ which χRAL PERT. THY breaks as an effective thy

all of this carries over to building theory of HIGGS as a "PION"

AXIONS

recall symm. of low-E QCD

$$U(2)_L \times U(2)_R = SU(2)_V \times \boxed{SU(2)_A} \times \underline{U(1)_B} \times U(1)_A$$

approx. BROKEN BY EW

GOOD SYM (well, B-L)

SPONT. BREAK + expl. or by MASS, GAUGING

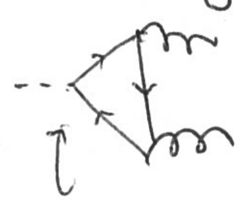
small mass

spont. broken by $\langle \bar{q}q \rangle$
... where's the GOLDSTONE?

η' ← "eta-prime problem"
(η' is the SU(3) version)

<u>ANS</u>: <u>EXPLICITLY</u> BROKEN
$m_{\eta'} \sim$ GeV !! ← <u>BIG</u> MASS ($m_{\eta'} \sim \Lambda$)

broken by $\boxed{\text{ANOMALY}}$
↑ mathematical consistency of QFT



QCD cannot be anomalous...

$\Rightarrow \partial_\mu j^\mu_A \neq 0$ @ loop level in fact: explicit breaking → mass terms

$$V \sim -m_u f_\pi^3 \cos\left(\frac{\pi}{f_\pi} + \frac{\eta'}{f_\pi}\right)$$
$$- m_d f_\pi^3 \cos\left(\frac{\pi}{f_\pi} - \frac{\eta'}{f_\pi}\right)$$
$$- \Lambda^4_{QCD} \cos\left(\frac{2\eta'}{f_\pi}\right)$$

?

comes from

$$\delta \mathcal{L}_{AXIAL} \supset \underbrace{\frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}}_{} \times 4\alpha$$

AXIAL TRANSF.

why isn't this in $\mathcal{L}_{QCD}$?

... it is.

... & it's a PUZZLE

$$\mathcal{L}_{QCD} \supset \frac{\alpha_s}{8\pi} \Theta_{YM} G\tilde{G}$$ ← CP VIOLATING

theta angle
just some param
(has to do w/ QCD vacuum)

* further: MASS TERMS IN QCD could have ℂ PHASE → these can be absorbed into Θ_{YM} ...
now contains contributions from very diff. types of phys! //

<u>OBSERVABLE</u> : Electric dipole moment of the neutron

observed? $\boxed{\Theta_{YM} \lesssim 10^{-11}}$

CP VIOLATION IS SMALL IN QCD

eh?!
expect: $\mathcal{O}(1)$

shows up in Goldstone pot:

$$
\begin{aligned}
V \sim & -m_u f^3 \cos\left(\frac{\pi}{f} + \frac{\eta'}{f}\right) \\
& - m_d f^3 \cos\left(\frac{\pi}{f} - \frac{\eta'}{f}\right) \\
& - \Lambda^4 \cos\left(\frac{2\eta'}{f} - \Theta_{YM}\right)
\end{aligned}
$$

← nonpert. QCD

<u>strong CP problem</u> : if $\Theta_{YM} = 0$ → nice
min of pot @ $\langle\pi\rangle = \langle\eta'\rangle = 0$
& no CP VIOLATION in strong sector
(explicit C phases)

but: no reason for this!

<u>Solution</u> : introduce a new $U(1)_{PQ}$ symmetry that is also anomalous w/rt QCD & spontaneously broken

↳ Goldstone is <u>axion</u>

then $V \to -m_u f^3 \cos\left(\frac{\pi}{f_\pi} + \frac{\eta'}{f_\pi}\right)$
$\quad - m_d f^3 \cos\left(\frac{\pi}{f_\pi} - \frac{\eta'}{f_\pi}\right)$
$\quad - \Lambda^4 \cos\left(\frac{2\eta'}{f_\pi} - \theta_{YM} + \frac{a}{f_a}\right)$

can have
$\langle a/f_a \rangle = \theta_{YM}$

$\Rightarrow \boxed{\langle \pi \rangle = \langle \eta' \rangle = 0}$
no CPV in QCD.

must have some

$a G\tilde{G}$

interaction.

usually you get other interactions, too
eg $aF\tilde{F}$

(inherited from mixing w/ π^0)

but details are model dependent)
(HOW you SPONT. BREAK PQ & ARRANGE for IT TO BE ANOMALOUS W/RT QCD)

see: The POOL-TABLE analogy to Axion Physics
Phys Today 49, 12 (1996)
Pierre Sikivie