The Standard Model

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“Periodic table” of elementary particles and forces

Fermions (spin ½)

	mass (approx.)	electric charge (units of proton charge e)
quarks		
up (u)	2 MeV	
charm (c)	1.3 GeV	+ ⅔
top (t)	173 GeV	
down (d)	5 MeV	
strange (s)	95 MeV	- ⅓
bottom (b)	4.2 GeV	
charged leptons		
electron (e)	0.511 MeV	
muon (μ)	106 MeV	-1
tau (τ)	1.8 GeV	
neutrinos		
ν_e		
ν_μ	0*	0
ν_τ		

* Standard Model defined with massless neutrinos, though neutrinos do have (small) masses. Two possibilities for including neutrino mass into SM and don’t know which is correct (Dirac or Majorana neutrinos).

“Periodic table” of elementary particles and forces

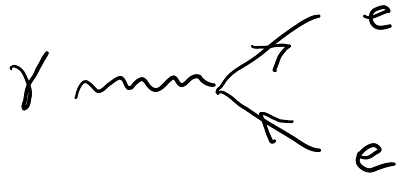
Bosons (spin 0 or 1)

	mass (approx.)	electric charge (units of proton charge e)
gauge bosons (s=1)		
photon/EM (γ)	0	0
gluon/strong (g)	0	0
weak force		
$W^{\pm}$	80.4 GeV	$\pm e$
Z	91.2 GeV	0
scalar (s=0)		
Higgs boson (h)	125 GeV	0

Feynman rules for gauge bosons

photon/EM (γ)

QED: U(1) gauge theory



$$= i e \gamma^{\mu}$$

Feynman rules for gauge bosons

photon/EM (γ)

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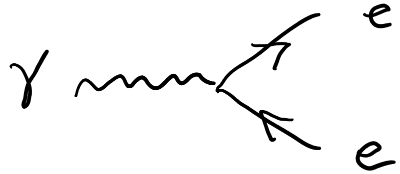
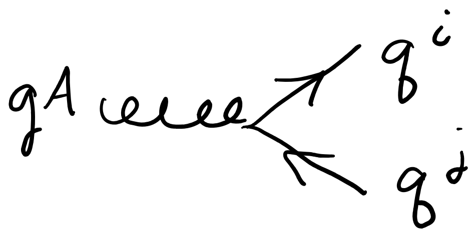


$= i e \gamma^\mu$

gluon/strong force (g)

QCD: SU(3) gauge theory

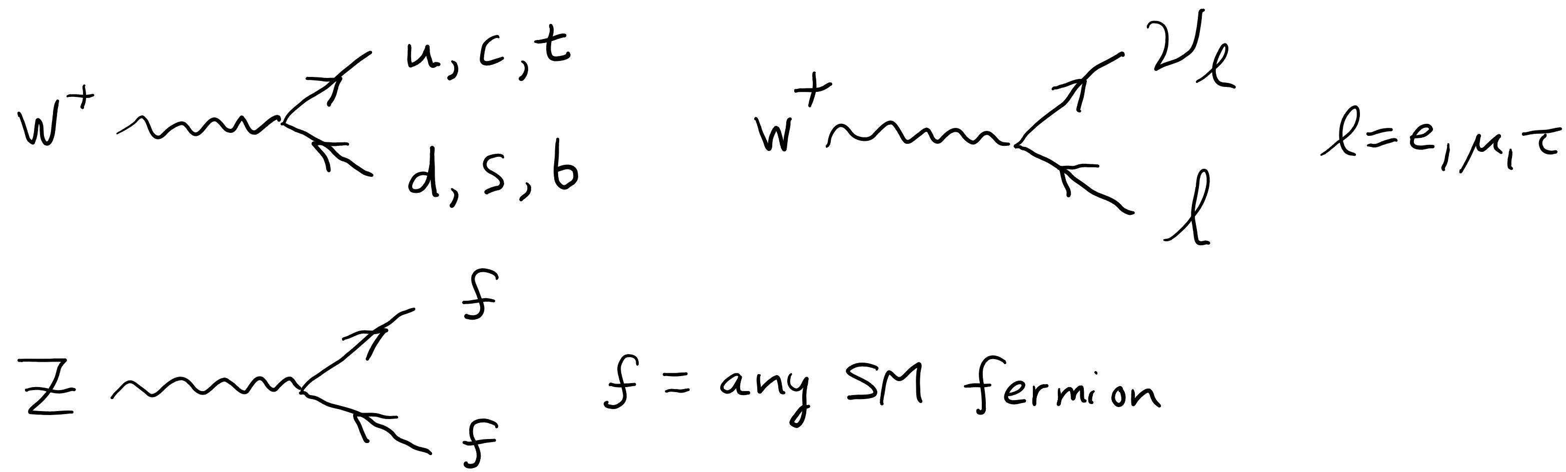


$= i g_s \gamma^\mu T^A_{ij}$

i,j = 1…3 (colors r,g,b)
A = 1…8 (8 types of gluons)

Feynman rules for gauge bosons

Weak force (W/Z)

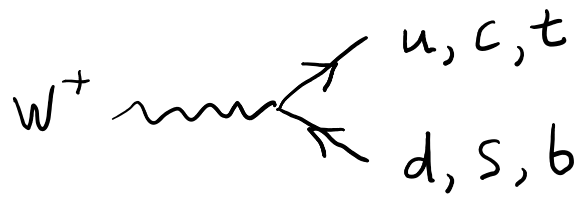


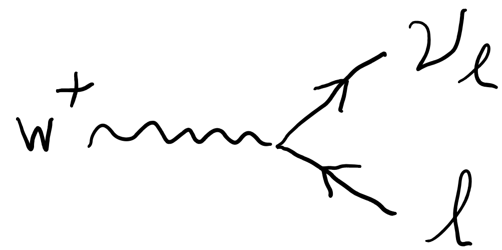
- Charged current interaction (W) is **flavor-changing** for quarks
  Converts any up-type quark to any down-type quark
  Trivially flavor-conserving for leptons if neutrinos are massless

- Neutral current interaction (Z) is **flavor-conserving** for quarks and leptons

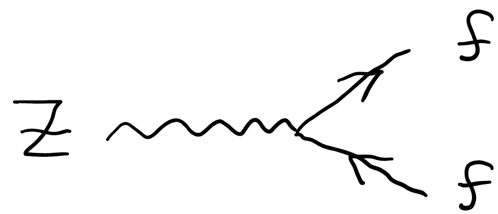
Feynman rules for gauge bosons

Weak force (W/Z)





$\ell = e, \mu, \tau$



f = any SM fermion

- W,Z interactions are **chiral**
  Left-handed and right-handed fermions have different gauge couplings

$$i e \gamma^\mu \longrightarrow i \left(g_L \gamma^\mu P_L + g_R \gamma^\mu P_R \right) \qquad g_L \neq g_R$$

$$P_{L,R} = \frac{1 \pm \gamma_5}{2}$$

Goal: Write down Lagrangian for all known particles and interactions (except neutrino masses, dark matter, gravity, etc.)

Key ingredients we want:

1. Renormalizability (want predictive theory at all energy scales)
2. Gauge symmetry (abelian and nonabelian)

Goal: Write down Lagrangian for all known particles and interactions (except neutrino masses, dark matter, gravity, etc.)

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2. Gauge symmetry (abelian and nonabelian)

Problem:

- Weak interactions have: (1) massive gauge bosons and (2) chiral interactions with fermions
- Inconsistent with gauge symmetry and renormalizability

Fix: Higgs mechanism and spontaneous symmetry breaking

Let's illustrate this with a simplified version of the SM

QED with (1) massive photon and (2) chiral fermion couplings

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\not{D} - m_\psi)\psi + \underbrace{\frac{1}{2} m_A^2 A_\mu A^\mu}_{\text{(1) gauge boson mass}}$$

$$D_\mu = \partial_\mu + i\underbrace{(g_L P_L + g_R P_R)}_{\text{(2) chiral gauge couplings } g_L \neq g_R} A_\mu$$

In usual QED: $m_A = 0$ and $g_L = g_R = e$

Is the theory gauge invariant?

Gauge transformations:

$$A_\mu \to A_\mu + \partial_\mu \alpha(x), \qquad \psi_{L,R} \to e^{-i g_{L,R} \alpha(x)} \psi_{L,R}$$

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$$A_\mu \to A_\mu + \partial_\mu \alpha(x), \qquad \Psi_{L,R} \longrightarrow e^{-ig_{L,R}\alpha(x)}\Psi_{L,R}$$

Kinetic terms are gauge invariant since $F_{\mu\nu}$, $\bar{\Psi}\not{D}\Psi$ are invariant

Gauge boson mass term is not invariant $A_\mu A^\mu \longrightarrow A_\mu A^\mu + 2A^\mu \partial_\mu \alpha + (\partial_\mu \alpha)^2$

Fermion mass term is not invariant unless $g_L = g_R$

$$\bar{\Psi}\Psi = \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L \longrightarrow e^{i(g_L - g_R)\alpha}\bar{\Psi}_L \Psi_R + e^{i(g_R - g_L)\alpha}\bar{\Psi}_R \Psi_L$$

Weak interactions: Maybe not a gauge theory?

- W,Z are massive
- SM fermions have both masses and chiral couplings to W,Z

Reasons to want a gauge theory for W,Z:

1. Automatically explains experimental observations for weak interactions:
    - No flavor-changing neutral currents (FCNCs)
      Z boson (and γ) doesn't change one fermion flavor into another
    - Same coupling of the W,Z to e, μ, τ (universality)

   More on this later

2. Renormalizability
    - Theory with massive gauge bosons also nonrenormalizable

Let’s argue that a massive gauge theory is **not** renormalizable

We need to know the propagator for a massive gauge boson

Massive gauge boson propagator

Recall: Feynman propagator for massless photon

$$D_{\mu\nu}(k) = \frac{-i\eta_{\mu\nu}}{k^2 + i\varepsilon} = \frac{i\sum_{\substack{\text{polar.} \\ i=1,2}} \varepsilon_\mu^{(i)}(k)\,\varepsilon_\nu^{(i)}(k)^*}{k^2 + i\varepsilon}$$

Propagator for massive photon: (1) Shift the pole and (2) include new polarization

$$D_{\mu\nu}(k) = \frac{i\sum_{\substack{\text{pol.} \\ i=1,2,3}} \varepsilon_\mu^{(i)}(k)\,\varepsilon_\nu^{(j)}(k)^*}{k^2 - m_A^2 + i\varepsilon}$$

Massive gauge boson propagator

Consider gauge boson rest frame

Polarization vectors

$$\varepsilon_\mu^{(1)}(0) = (0,1,0,0)$$

$$\varepsilon_\mu^{(2)}(0) = (0,0,1,0)$$

$$\varepsilon_\mu^{(3)}(0) = (0,0,0,1)$$

Now sum over polarization vectors

$$\sum_i \varepsilon_\mu^{(i)}(0)\,\varepsilon_\nu^{(i)}(0) = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}_{\mu\nu}$$

Massive gauge boson propagator

Now boost to a new frame with gauge boson momentum $k^\mu = (E_k, 0, 0, k)$

The Lorentz transformation matrix is $\Lambda = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix}$ where $\beta = k/E_k$, $\gamma = \dfrac{E_k}{m_A}$

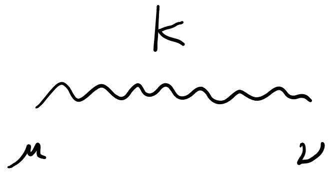
Then we have

$$\sum_i \varepsilon_\mu^{(i)}(k)\, \varepsilon_\nu^{(i)}(k)^* = \Lambda \sum_i \varepsilon_\mu^{(i)}(0)\, \varepsilon_\nu^{(i)}(0)^* \Lambda^T = \begin{pmatrix} k^2/m_A^2 & 0 & 0 & kE_k/m_A^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ kE_k/m_A^2 & 0 & 0 & E_k^2/m_A^2 \end{pmatrix}$$

$$= -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2}$$ expressed in Lorentz-covariant form

Massive gauge boson propagator

$$D_{\mu\nu}(k) =$$



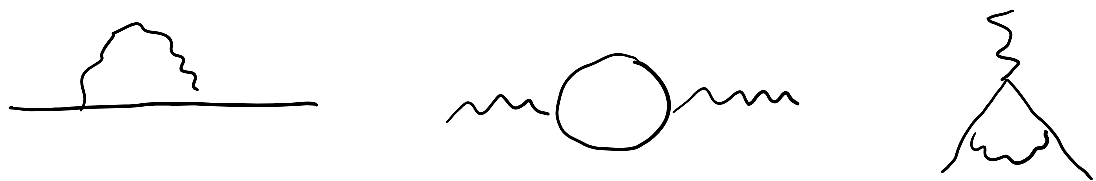
$$= \frac{-i}{k^2 - m_A^2 + i\epsilon}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}\right)$$

Extra $k_\mu k_\nu$ term introduces extra ultraviolet divergences that are absent in massless theory (QED)

Renormalizability

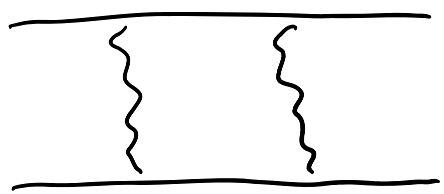
Recall in QED:

- Ultraviolet divergences appear at one-loop order in the two- and three-point functions



- Can be absorbed by renormalizing mass, electric charge, wavefunctions

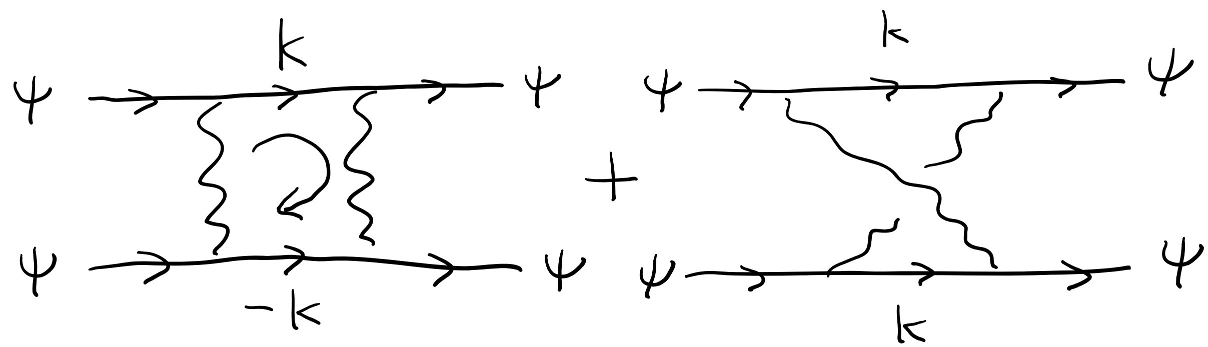
- After this is done, the theory gives finite predictions. No new divergences appear for higher-point functions, e.g., four-point function (scattering)



Renormalizability

Consider scattering at one-loop in our massive gauge theory

Keep only the leading divergent terms. Schematically, we have:



$$\sim g^4 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{\not{k} + m_\psi}{k^2} \right)^2 \left(\frac{k_\mu k_\nu}{k^2 m_A^2} \right)^2$$

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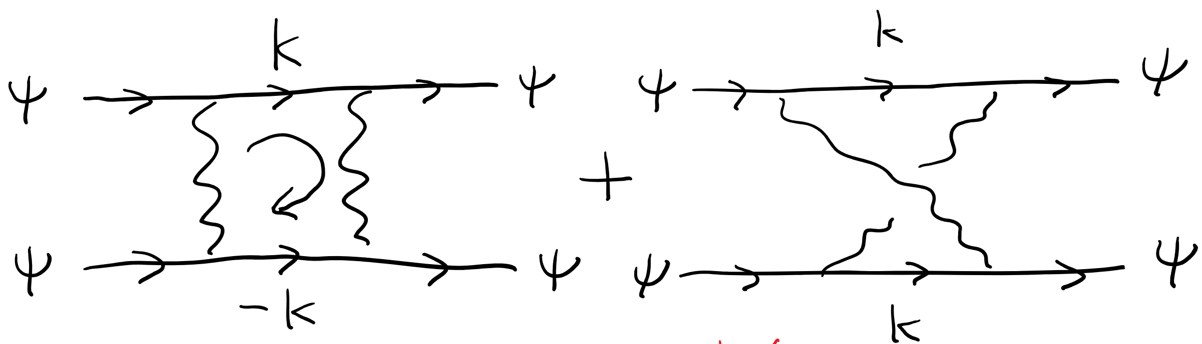

$$\sim g^4 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{\overbrace{\cancel{k}}^{\text{vanishes}} + m_\psi}{k^2}\right)^2 \left(\frac{k_\mu k_\nu}{k^2 m_A^2}\right)^2$$

$$\sim \frac{g^4 m_\psi^2}{m_A^4} \log \Lambda^2$$

Renormalizability

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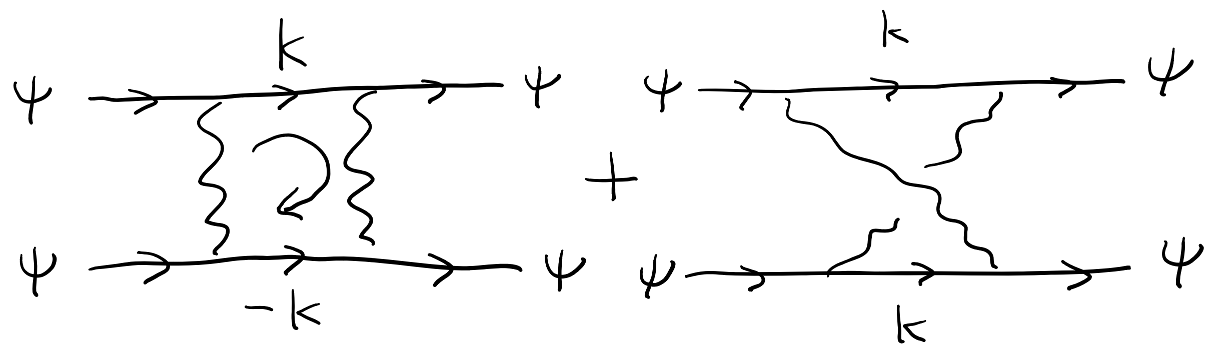
Keep only the leading divergent terms. Schematically, we have:



$$\sim \frac{g^4 m_\psi^2}{m_A^4} \log \Lambda^2$$

Requires new counter terms from higher dimensional operators to cancel divergence

$$\mathcal{L}_{counterterm} \sim \bar{\psi}\psi\bar{\psi}\psi$$

Spontaneous symmetry breaking and Higgs mechanism

Basic idea:

- Start with a Lagrangian that is gauge invariant
- Gauge symmetry is broken **spontaneously** by the vacuum, not by explicit terms in the Lagrangian
- Symmetry is no longer manifest in the spectrum of states
  Particles can have masses that seem to violate the gauge symmetry
- Parameters of the theory are not all independent, but are correlated due to the original gauge symmetry
  Ultraviolet divergences cancel out and theory is renormalizable

Abelian Higgs model

Complex scalar field ϕ with a U(1) abelian gauge symmetry (scalar QED)

$$\mathcal{L} = (D_\mu \phi^\dagger)(D^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Scalar potential $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$

Covariant derivative $D_\mu = \partial_\mu + i g A_\mu$

What is the vacuum of the theory? Minimize the Hamiltonian

$$\mathcal{H} = \underbrace{|\dot{\phi}|^2 + |\vec{\nabla}\phi|^2}_{\text{minimized for } \phi = \text{const.}} + V(\phi) + \underbrace{\text{gauge field terms}}_{\text{minimized for } A_\mu = 0 \text{ (or pure gauge)}}$$

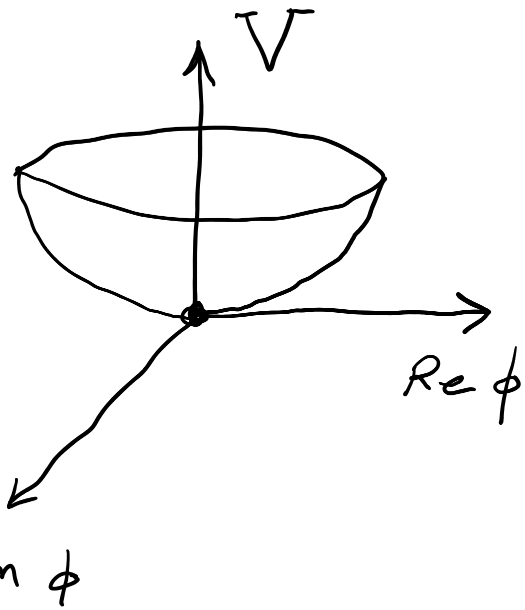
Abelian Higgs model

Vacuum state for value of ϕ that minimizes the potential V(ϕ)

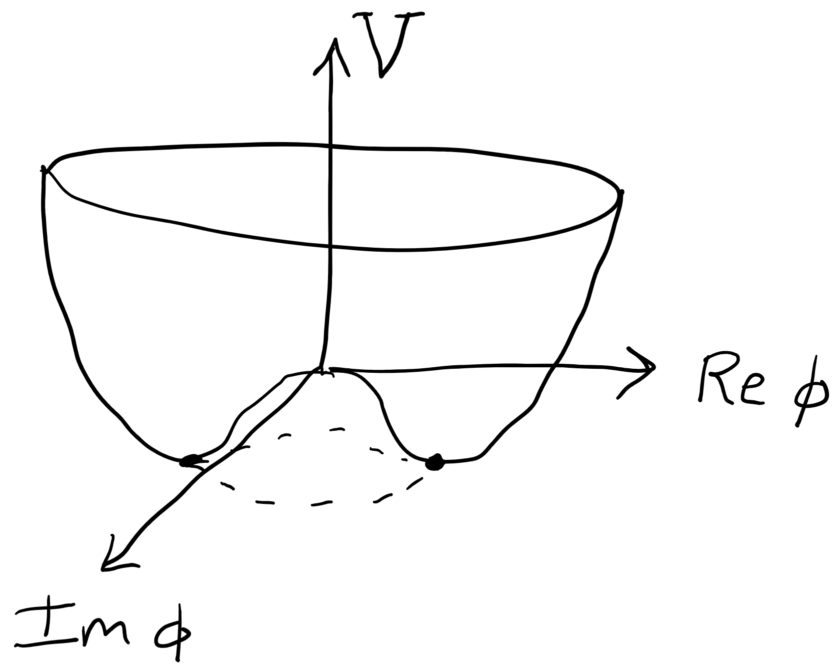
$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 ,$$

$\lambda > 0$ required but μ^2 can have either sign

$\mu^2 > 0$



$\mu^2 < 0$



Abelian Higgs model

Minimize the potential $$\frac{\partial V}{\partial \phi} = \left(\mu^2 + 2\lambda\phi^\dagger\phi\right)\phi^\dagger = 0$$

$\mu^2 > 0$ case: $|\phi| = 0$ Gauge symmetry remains intact

$\mu^2 < 0$ case: $|\phi| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$ Gauge symmetry is broken

Infinite number of degenerate minima, all related by gauge transformation
Free to pick one such that ϕ is real and positive

Scalar field ϕ has acquired a **vacuum expectation value (vev)** $$\langle 0|\phi|0\rangle = \frac{v}{\sqrt{2}}$$

Abelian Higgs model

Physical particles are quantum fluctuations above the vacuum
To find the spectrum, expand the scalar field around its vev (polar form)

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + h(x)\right) e^{i\xi(x)/v}$$

where $h(x)$, $\xi(x)$ are real scalar fields

Again, free to remove the phase of ϕ using a gauge transformation, writing

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + h(x)\right) \qquad \text{unitary gauge}$$

Abelian Higgs model

Covariant derivative term:

$$|D_\mu \phi|^2 = \frac{1}{2} \left| (\partial_\mu + i g A_\mu)(v+h) \right|^2$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \underbrace{\frac{1}{2} g^2 A_\mu A^\mu (v+h)^2}$$

Gauge boson mass term $\frac{1}{2} m_A^2 A_\mu A^\mu$ with $m_A = gv$

Abelian Higgs model

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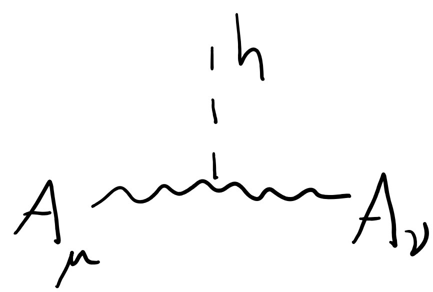
Gauge boson mass term $\frac{1}{2}m_A^2 A_\mu A^\mu$ with $m_A = gv$

Residual real scalar degree of freedom **h**: the Higgs boson

Interactions of the Higgs boson are **not** free parameters. Fixed by mass and vev.



$$= i\underbrace{g^2 v}_{m_A^2/v}\,\eta_{\mu\nu}$$

Abelian Higgs model

Next, consider fermions. How do we get fermion masses for a chiral gauge theory?

Add a fermion term to the Lagrangian: $\bar{\Psi} i \not{D} \Psi$

$$D_\mu = \partial_\mu + i(g_L P_L + g_R P_R) A_\mu$$

no mass allowed if $g_L \neq g_R$

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no mass allowed if $g_L \neq g_R$

But we can construct another term for a gauge invariant Lagrangian

$$\mathcal{L}_{Yukawa} = -y \bar{\Psi}_L \Psi_R \phi + h.c. \qquad y = \text{Yukawa coupling}$$

$$\phi \to e^{-ig\alpha} \phi, \quad \Psi_{L,R} = e^{-ig_{L,R}\alpha} \Psi_{L,R}$$

This term is allowed if $g + g_R = g_L$

Abelian Higgs model

Expanding around the vev:

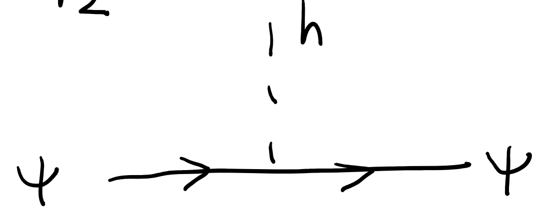
$$\mathcal{L}_{Yukawa} = -y\bar{\Psi}_L \Psi_R \phi + h.c.$$

$$= -y\bar{\Psi}_L \Psi_R \left(\frac{v+h}{\sqrt{2}}\right) - y\bar{\Psi}_R \Psi_L \left(\frac{v+h}{\sqrt{2}}\right)$$

$$= -\frac{yv}{\sqrt{2}}\bar{\Psi}\Psi - \frac{y}{\sqrt{2}} h\bar{\Psi}\Psi$$

We now have a fermion mass $m_\Psi = \frac{yv}{\sqrt{2}}$

Higgs boson interaction with fermion is fixed in terms of mass and vev



$$= -i\underbrace{\frac{y}{\sqrt{2}}}_{m_\Psi/v}$$

Abelian Higgs model

Higgs mechanism:

- Generates mass for gauge bosons and chiral fermions in a gauge theory
- Prediction: extra residual degree of freedom (Higgs boson)
- Higgs boson couplings to a particle are fixed by particle’s mass and the vev

Counting degrees of freedom (dof):

Original gauge invariant theory	Spontaneously broken theory
1 massless gauge boson (2 pol.)	1 massive gauge boson (3 pol.)
+ 1 complex scalar (2 dof)	+ 1 real scalar (1 dof)
4 total degrees of freedom	4 total degrees of freedom

Higgs boson mass:

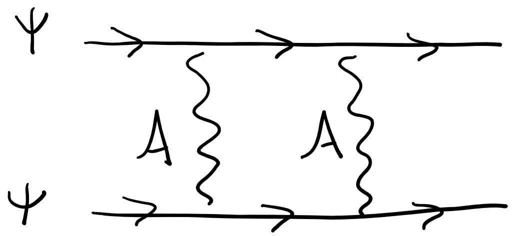
$$\mathcal{L} = |D_\mu \phi|^2 - V(\phi) + \ldots = \frac{1}{2}(\partial_\mu h)^2 + \mu^2 h^2 + \ldots$$

$$m_h = \sqrt{-2\mu^2}$$

Free parameters $(\mu^2, \lambda) \longleftrightarrow (m_h, v)$

Ultraviolet divergences revisited

Theory with massive gauge bosons led to a divergence for scattering



$$\sim \frac{g^4 m_\psi^2}{m_A^4} \log \Lambda^2$$

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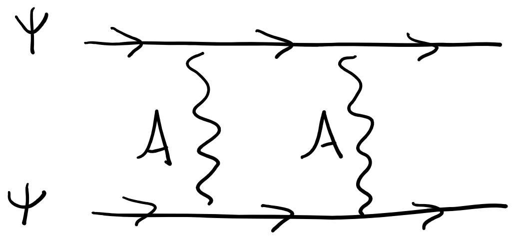
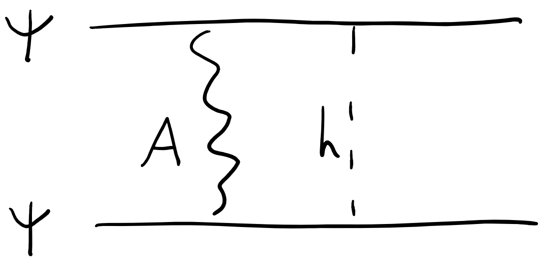


$$\sim \frac{g^4 m_\psi^2}{m_A^4} \log \Lambda^2$$

Now we have additional diagrams involving the Higgs boson to exactly cancel the divergences



$$\sim g^2 y^2 \int \frac{d^4 k}{(2\pi)^4} \left(\frac{1}{\not{k}}\right)^2 \left(\frac{k_\mu k_\nu}{m_A^2}\right)\left(\frac{1}{k^2}\right)$$

Ultraviolet divergences revisited

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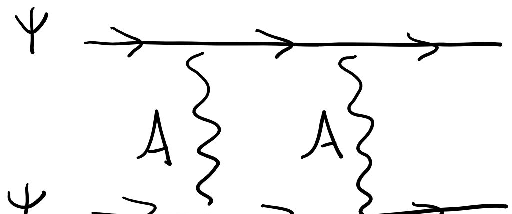

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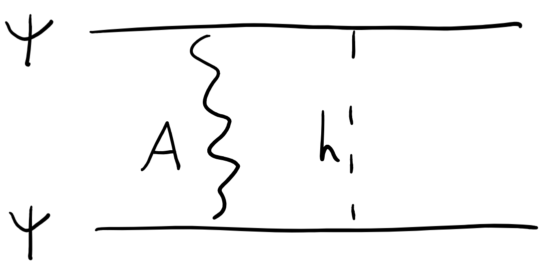

$$\sim g^2 y^2 \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{\not{k}}\right)^2 \left(\frac{k_\mu k_\nu}{m_A^2}\right)\left(\frac{1}{k^2}\right)$$

$$\sim \frac{g^2 y^2}{m_A^2} \log \Lambda^2 \sim \frac{g^4 m_\psi^2}{m_A^4} \log \Lambda^2$$

Higgs couplings are related to masses and vev

$$\text{using} \quad y^2 = \frac{2 m_\psi^2}{v^2} = \frac{2 g^2 m_\psi^2}{m_A^2}$$

Nonabelian Higgs model

SU(2) gauge theory with a complex scalar doublet field

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) - \frac{1}{2} \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$$

Complex doublet $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ Both ϕ_1, ϕ_2 are complex scalar fields

$$D_\mu \Phi = \left(\partial_\mu + ig \sum_{a=1}^{3} \frac{\sigma^a}{2} A_\mu^a \right) \Phi$$

Three gauge fields $A_\mu^1, A_\mu^2, A_\mu^3$

$\sigma^a =$ Pauli matrices

Nonabelian Higgs model

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Complex doublet $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ Both ϕ_1, ϕ_2 are complex scalar fields

$$D_\mu \Phi = \Big(\partial_\mu + i g \underbrace{\sum_{a=1}^{3} \frac{\sigma^a}{2} A_\mu^a}\Big) \Phi$$

$$\sum_{a=1}^{3} \sigma^a A_\mu^a = \begin{pmatrix} A_\mu^3 & A_\mu^1 - i A_\mu^2 \\ A_\mu^1 + i A_\mu^2 & -A_\mu^3 \end{pmatrix}$$

Nonabelian Higgs model

Spontaneous symmetry breaking

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$

if $\mu^2 < 0$, minimum at $\Phi^\dagger \Phi = \frac{v^2}{2}$, $v = \sqrt{\frac{-\mu^2}{\lambda}}$

Can expand the scalar doublet as

$$\Phi(x) = \exp\left(i \frac{\sigma^a}{2} \xi^a(x)/v\right) \begin{pmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{pmatrix}$$

by making a gauge transformation to unitary gauge

Nonabelian Higgs model

Expanding around the vev:

$$(D_\mu \Phi^\dagger)(D^\mu \Phi) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(0, v+h)\begin{pmatrix} A_\mu^3 & A_\mu^1 - iA_\mu^2 \\ A_\mu^1 + iA_\mu^2 & -A_\mu^3 \end{pmatrix}^2 \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}\left(\frac{gv}{2}\right)^2 \left((A_\mu^1)^2 + (A_\mu^2)^2 + (A_\mu^3)^2\right)\left(1 + \frac{h}{v}\right)^2$$

Nonabelian Higgs model

Expanding around the vev:

$$(D_\mu \Phi^\dagger)(D^\mu \Phi) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(0, v+h)\begin{pmatrix} A_\mu^3 & A_\mu^1 - iA_\mu^2 \\ A_\mu^1 + iA_\mu^2 & -A_\mu^3 \end{pmatrix}^2 \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}\left(\frac{gv}{2}\right)^2 \left((A_\mu^1)^2 + (A_\mu^2)^2 + (A_\mu^3)^2\right)\left(1+\frac{h}{v}\right)^2$$

We get three degenerate massive gauge bosons, with $m_1 = m_2 = m_3 = \frac{gv}{2}$

Almost like the Standard Model

Standard Model has three massive gauge bosons, but $m_W \neq m_Z$

Plus one massless photon $m_\gamma = 0$

Standard Model Lagrangian

Need to specify the gauge group, the degrees of freedom (fields), and their quantum numbers

Gauge group: $\underbrace{SU(3)_C}_{\substack{\text{QCD} \\ (C = \text{color})}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\substack{\text{Electroweak} \\ (L = \text{left},\ Y = \text{hypercharge})}}$

Gauge bosons: gluon g_μ^A $(A = 1 \ldots 8)$ $\quad W_\mu^a$ $(a = 1 \ldots 3)$ $\quad B_\mu$

Gauge couplings: g_s $\quad g$ $\quad g'$

Standard Model Lagrangian

Fermions

Quantum numbers ($SU(3)_C$, $SU(2)_L$, $U(1)_Y$)

$$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix} \qquad \left(3, 2, \tfrac{1}{6}\right)$$

$$u_R^i \qquad \left(3, 1, \tfrac{2}{3}\right)$$

$$d_R^i \qquad \left(3, 1, -\tfrac{1}{3}\right)$$

quarks

$$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} \qquad \left(1, 2, -\tfrac{1}{2}\right)$$

$$e_R^i \qquad \left(1, 1, -1\right)$$

leptons

i = 1,2,3 labels **generation**. All fermions with same quantum numbers come in three copies.

Scalar

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \qquad \left(1, 2, \tfrac{1}{2}\right)$$

Higgs scalar doublet

Some group theory notation

Abelian gauge symmetry $U(1)_Y$:

Quantum numbers are in units of gauge coupling g'

e.g. Q_L^i has $U(1)_Y$ charge $\frac{g'}{6}$

Quantum numbers indicate representation of $SU(N)$

$N \to$ fundamental rep.

$1 \to$ trivial rep. i.e. doesn't transform

e.g. Q_L^i is in fundamental rep. of $SU(3)_c$ and $SU(2)_L$, but L_L^i is in trivial rep of $SU(3)_c$ since leptons don't carry color

Standard Model Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{scalar} + \mathcal{L}_{Yukawa}$$

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$$\mathcal{L}_{gauge} = -\frac{1}{2}\mathrm{Tr}\left(g_{\mu\nu}\, g^{\mu\nu}\right) - \frac{1}{2}\mathrm{Tr}\left(W_{\mu\nu} W^{\mu\nu}\right) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Standard Model Lagrangian

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$$\mathcal{L}_{scalar} = (D_\mu H^\dagger)(D^\mu H) - V(H)$$

$$V(H) = \mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2 \qquad (\mu^2 < 0)$$

Same as nonabelian Higgs model with $\Phi \to H$

Electroweak symmetry breaking

Let's see how masses arise for the W,Z bosons via the Higgs mechanism

Covariant derivative $$D_\mu H = \left(\partial_\mu + i\frac{g}{2}\sum_{a=1}^{3} \sigma^a W_\mu^a + \frac{ig'}{2} B_\mu \right) H$$

Expand Higgs field around the vev in unitary gauge $$H(x) = \begin{pmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{pmatrix}$$

Plug in and evaluate the covariant derivative term $$|D_\mu H|^2$$

(exercise for tutorial)

Fermion Lagrangian

Let's write out all the fermion terms explicity

$$\mathcal{L}_{fermion} = \sum_{\psi} \bar{\psi} i \not{D} \psi$$

$$= \sum_{i=1}^{3} \bar{Q}_L^i i \left(\not{\partial} + i \frac{g}{2} \sigma^a \not{W}^a + i \frac{g'}{6} \not{B} + i g_s \frac{\lambda^A}{2} \not{g}^A \right) Q_L^i$$

$$+ \bar{u}_R^i i \left(\not{\partial} + i g' \frac{2}{3} \not{B} + i g_s \frac{\lambda^A}{2} \not{g}^A \right) u_R^i$$

$$+ \bar{d}_R^i i \left(\not{\partial} + i g' \left(-\frac{1}{3}\right) \not{B} + i g_s \frac{\lambda^A}{2} \not{g}^A \right) d_R^i$$

$$+ \bar{L}_L^i i \left(\not{\partial} + i \frac{g}{2} \sigma^a \not{W}^a - i \frac{g'}{2} \not{B} \right) L_L^i$$

$$+ \bar{e}_R^i i \left(\not{\partial} - i g' \not{B} \right) e_R^i$$

Charged current interactions

Interactions with the W bosons are only for left-handed fields Q^i_L and L^i_L

$$W^{\pm}_{\mu} = \frac{W^1_{\mu} \mp i W^2_{\mu}}{\sqrt{2}}$$

$$\sigma^1 W^1 + \sigma^2 W^2 = \begin{pmatrix} 0 & W^1 - iW^2 \\ W^1 + iW^2 & 0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix}$$

$$\mathcal{L}_{cc} = i\,\underbrace{(\bar{u}_L^i, \bar{d}_L^i)}_{\bar{Q}_L^i}\left(i\frac{g}{\sqrt{2}}\right)\begin{pmatrix} 0 & \cancel{W}^+ \\ \cancel{W}^- & 0 \end{pmatrix}\underbrace{\begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}}_{Q_L^i}$$

$$+\ i\,\underbrace{(\bar{\nu}_L^i, \bar{e}_L^i)}_{\bar{L}_i}\left(i\frac{g}{\sqrt{2}}\right)\begin{pmatrix} 0 & \cancel{W}^+ \\ \cancel{W}^- & 0 \end{pmatrix}\underbrace{\begin{pmatrix} \nu_L^i \\ e_L \end{pmatrix}}_{L_L^i}$$

$$= -\frac{g}{\sqrt{2}}\,\bar{u}_L^i \cancel{W}^+ d_L^i - \quad \bar{\nu}_L^i \cancel{W}^+ e_L^i + h.c.$$

Neutral current interactions

Interactions with Z and γ are for all SM fermions

$$\mathcal{L}_{NC} = -\bar{Q}_L^i \left(\frac{g}{2} \begin{pmatrix} \not{W}^3 & 0 \\ 0 & -\not{W}^3 \end{pmatrix} + \frac{g'}{6} \begin{pmatrix} \not{B} & 0 \\ 0 & \not{B} \end{pmatrix} \right) Q_L^i$$

$$- \bar{u}_R^i \left(\frac{2}{3} g' \not{B} \right) u_R^i - \bar{d}_R^i \left(-\frac{1}{3} g' \not{B} \right) d_R^i$$

$$- \bar{L}_L^i \left(\frac{g}{2} \begin{pmatrix} \not{W}^3 & 0 \\ 0 & -\not{W}^3 \end{pmatrix} - \frac{g'}{2} \begin{pmatrix} \not{B} & 0 \\ 0 & \not{B} \end{pmatrix} \right) L_L^i$$

$$- \bar{e}_R^i \left(-g' \not{B} \right) e_R^i$$

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$$- \bar{e}_R^i \left(-g' \cancel{B} \right) e_R^i$$

Plug in redefined gauge fields and expand

$$W_\mu^3 = \cos\theta_W Z_\mu + \sin\theta_W A_\mu$$

$$B_\mu = -\sin\theta_W Z_\mu + \cos\theta_W A_\mu$$

Neutral current interactions

End result (tutorial exercise):

$$\mathcal{L}_{NC} = -g s_W \left(\frac{2}{3} \bar{u}^i \not{A} u^i - \frac{1}{3} \bar{d}^i \not{A} d^i - \bar{e}^i \not{A} e^i \right)$$

$$- \frac{g}{c_W} \Big[\bar{u}^i \not{Z} \left(\frac{1}{2} P_L - \frac{2}{3} s_W^2 \right) u^i$$

$$+ \bar{d}^i \not{Z} \left(-\frac{1}{2} P_L + \frac{1}{3} s_W^2 \right) d^i$$

$$+ \bar{e}^i \not{Z} \left(-\frac{1}{2} P_L + s_W^2 \right) e^i$$

$$+ \bar{\nu}^i \not{Z} \left(\frac{1}{2} P_L \right) \nu^i \Big]$$

Neutral current interactions

End result (tutorial exercise):

$$\mathcal{L}_{NC} = -g s_W \left(\frac{2}{3} \bar{u}^i \not{A} u^i - \frac{1}{3} \bar{d}^i \not{A} d^i - \bar{e}^i \not{A} e^i \right) + Z \text{ terms}$$

Even though we started with chiral theory, able to assign charges such that massless gauge field A is **not** chiral

$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}$$ remains unbroken gauge symmetry

Neutral current interactions

End result (tutorial exercise):

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Even though we started with chiral theory, able to assign charges such that massless gauge field A is **not** chiral

Electric charge $e = g \sin\theta_W$

fields	EM charge
u^i	$+\frac{2}{3}e$
d^i	$-\frac{1}{3}e$
e^i	$-e$
ν^i	0