Neutrino Physics

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1. Neutrinos and their masses

$$\mathcal{L} \supset \sum_{\alpha = e,\mu,\tau} \Big[\bar{\nu}_{\alpha L}\, i\not{\partial}\, \nu_{\alpha L}$$

$$+ \frac{g}{\sqrt{2}} \left(W^{\mu}\, \bar{\nu}_{\alpha L} \gamma_{\mu} e_{\alpha L} + h.c. \right)$$

$$+ \frac{g}{2\cos\theta_W} Z^{\mu}\, \bar{\nu}_{\alpha L} \gamma_{\mu} \nu_{\alpha L} \Big]$$

+ mass term

1.1 Dirac masses

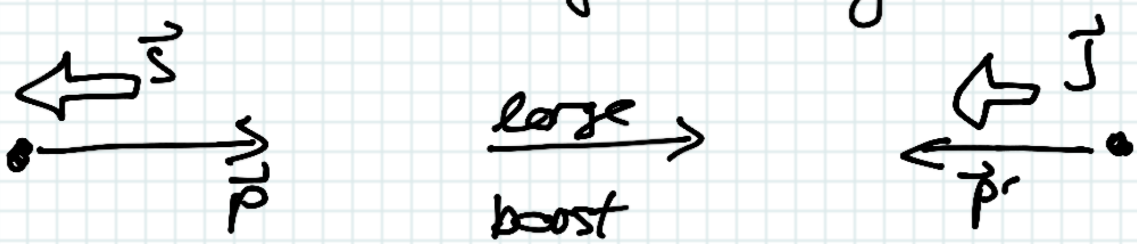
$\mathcal{L}$ involves only $\nu_L = \frac{1-\gamma^5}{2}\nu = \begin{pmatrix}\chi_1\\ \chi_2\\ 0\\ 0\end{pmatrix}$

Dirac mass: make also lower components physical

$$\nu_R = \frac{1+\gamma^5}{2}\nu = \begin{pmatrix}0\\ 0\\ \psi_1\\ \psi_2\end{pmatrix}$$

$$\longrightarrow \mathcal{L}_{mass} \equiv \sum_{\alpha,\beta=e,\mu,\tau} m_{\alpha\beta}\,\bar{\nu}_{\alpha L}\,\nu_{\beta R}$$

Physical reason for 4 dof:



Problem: Why are ν masses so much smaller than other fermion masses?

1.2 Majorana masses

Mass terms couple LH to RH fields

Antiparticle of LH field is RH

Could ν_R be identical to the antiparticle to ν_L?

More formally: charge conjugation

$$\hat{C} : \psi \longrightarrow \psi^c \equiv \underbrace{-i\gamma^2\gamma^0}_{\equiv C} \bar{\psi}^T$$

$$= -i\gamma^2\psi^*$$

Effect on chirality:

$$\gamma^5\psi^c = \gamma^5(-i\gamma^2\psi^*)$$

$$= +i\gamma^2\gamma^5\psi^* = -(\gamma^5\psi)^c$$

$\Rightarrow \hat{C}$ flips chirality, transforms LH particles to RH antiparticles.

$$\boxed{\text{Identify } \nu_R \equiv \nu_L{}^c}$$

In component notation: $\nu_L = \begin{pmatrix} \chi \\ 0 \end{pmatrix}$

$$\nu_L{}^c = -i \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix} \begin{pmatrix} \chi \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i\sigma^2\chi^* \end{pmatrix}$$

A new type of mass term:

$$\boxed{\mathcal{L}_m = \sum_{\alpha,\beta = e,\mu,\tau} \frac{1}{2} m_{\alpha\beta} \overline{(\nu_\alpha)^c} \nu_\beta + h.c.}$$

Questions:

- how to obtain this from an $SU(2)$-invariant theory?
- why are the masses so small

1.1 The seesaw mechanism

Dirac mass term + Majorana mass for RH ν:

$$\mathcal{L}_{seesaw} = -m_D \bar{\nu}_R \nu_L - \frac{1}{2} m_M \overline{\nu_R^c}\, \nu_R$$

$$+ h.c.$$

Define: $n = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$

$$M = \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix}$$

$$\Longrightarrow \mathcal{L}_{seesaw} = -\frac{1}{2}\, \overline{n^c} M n + h.c.$$

Here, we have used $(\nu^c)^c = \nu$ and

$$\overline{(\nu_R)^c}\, \nu_L^c = \bar{\nu}_L \nu_R$$

(to show this, use definition of $\hat{C}$, don't forget minus sign when anticommuting fermion fields.)

Next: diagonalize M:

$$\begin{pmatrix} \nu_L \\ \nu_R{}^c \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix}$$

Compute

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

and require off-diag. elements to vanish

$$\hookrightarrow \tan 2\theta = \frac{2m_D}{m_M}$$

Eigenvalues are

$$m_{1,2} = \frac{m_M}{2} \mp \sqrt{\frac{m_M^2}{4} + m_D^2}$$

Diagonalized mass term:

$$\mathcal{L}_{\text{seesaw}} = -\frac{1}{2} m_1 \overline{(\chi_{1L})^c}\, \chi_{1L} - \frac{1}{2} m_2 \overline{(\chi_{2L})^c}\, \chi_{2L} + h.c.$$

$\Rightarrow$ Two Majorana fields with

different masses and 2 dof each

Seesaw limit: $\boxed{m_M \gg m_D}$

$\hookrightarrow$ $m_2 \simeq m_M$

$$m_1 \simeq \frac{m_D^2}{m_M}$$

For instance: $m_D \sim 100$ GeV ($\sim v_H$)

$m_M \sim 10^{14}$ GeV

$\Rightarrow \boxed{m_1 \sim 0{,}1 \text{ eV}}$

1.4 Measuring neutrino masses

1.4.1 Kinematics

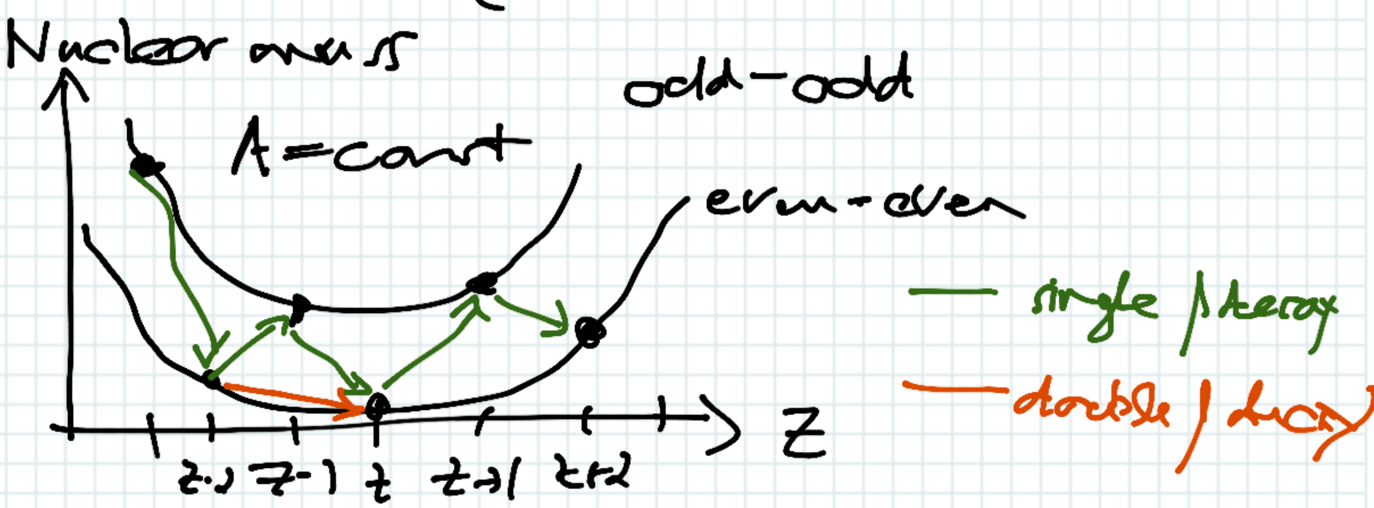
β decay $\;{}^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$

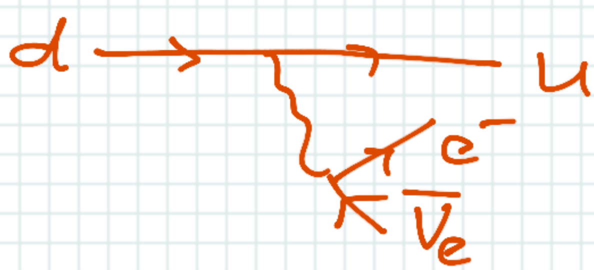
$$E_{e,max} = \underbrace{Q}_{= m_H - m_{He}} - m_\nu$$

measure E_e spectrum as precisely as possible

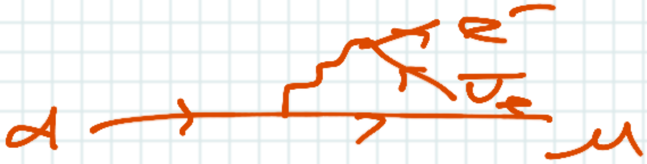
1.9.2 Neutrinoless double beta decay

std. β decay

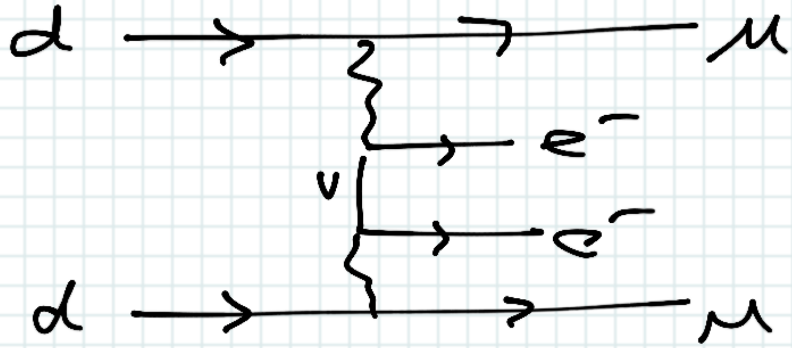




$$(A, Z) \rightarrow (A, Z+2) + 2e^- + 2\bar{\nu}_e$$



For Majorana neutrinos, also



"Lobster" diagram

Rate: $\Gamma_{0\nu 2\beta} = G_F^4 \, |M_{0\nu 2\beta}|^2 \cdot \left| \sum_j U_{ej}^2 m_j \right|^2 p_e^2$

sum over mass eigenstates

mixing matrix

↳ measuring $\Gamma_{0\nu 2\beta}$ would allow determination of $m_{e,2\beta} = \sum_j U_{ej}^2 m_j$