TRISEP Tutorial 1

(Dated: July 21, 2019)

I. HIGGS MECHANISM IN THE STANDARD MODEL

The electroweak gauge symmetry of the Standard Model is $SU(2)_L \times U(1)_Y$. The Higgs field $H(x)$ has electroweak quantum numbers $(2, \frac{1}{2})$, meaning it is a doublet under $SU(2)_L$ and has hypercharge $Y = \frac{1}{2}$. The Higgs potential is

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \,, \tag{1}$$

with $\mu^2 < 0$, such that electroweak symmetry is broken. By a gauge transformation, we can write the vev of $H(x)$ as

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} . \tag{2}$$

(a) It is customary to write the $SU(2)_L$ generators as $T^a = \sigma^a/2$, where $a = 1, 2, 3$ and σ^a are the Pauli matrices. Show that $\langle H \rangle$ is invariant under the generator $Q = T^3 + Y$. That is, this implies that although $SU(2)_L \times U(1)_Y$ is broken, there is a residual $U(1)$ gauge symmetry that remains unbroken, which corresponds to electromagnetism. (Note: you can think of Y as proportional to the 2×2 identity matrix when acting on $SU(2)_L$ doublets.)

(b) Considering fluctuations around the vacuum, the Higgs field can be written in unitary gauge as

$$H(x) = \begin{pmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{pmatrix} . \tag{3}$$

Expand the Higgs potential $V(H)$ in terms of h in the following form

$$V(h) = \frac{m_h^2}{2} h^2 + \kappa h^3 + \frac{\lambda}{4} h^4 \,. \tag{4}$$

Determine the cubic coupling κ in terms of m_h and λ. (The fact that the three parameters are not independent is the memory of the original symmetry that was broken.)

(c) The covariant derivative term for the Higgs field is

$$|D_\mu H|^2 \tag{5}$$

where the covariant derivative is

$$D_\mu = \partial_\mu + i g W_\mu + i g' Y B_\mu \,, \tag{6}$$

where we have a shorthand

$$W_\mu = \sum_{a=1}^{3} T^a W_\mu^a \tag{7}$$

for the 2×2 matrix gauge field. First, let's consider the electromagnetic charges Q of the gauge fields. Trivially, B_μ must have $Q = 0$.[1] Next, we'll consider the W_μ^a fields. Recall the tranformation rule of a nonabelian gauge field:

$$W_\mu \to U W_\mu U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger \,. \tag{8}$$

For simplicity, consider an infinitesimal tranformation with the generator Q, i.e., $U = 1 + iQ\alpha$, where α is a small parameter. Then Eq. (8) becomes

$$W_\mu \to W_\mu + i\alpha[Q, W_\mu] + iQ\partial_\mu \alpha \,. \tag{9}$$

Using this equation, show that the linear combinations

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right) \tag{10}$$

are eigenstates of Q with charges $Q = \pm 1$.

(d) Using the result from part (c), let's find the W boson mass term. The mass term for a charged vector boson is of the form

$$\mathcal{L} = m_W^2 W_\mu^+ W^{-\mu} \,. \tag{11}$$

Expand $|D_\mu H|^2$, keeping only terms involving $W_\mu^{1,2}$ to find m_W in terms of g and v (i.e., you may set $W_\mu^3 = B_\mu = h = 0$).

(e) Next, let's find the massed of the neutral gauge bosons by expanding $|D_\mu H|^2$ and keeping only W_μ^3 and B_μ (i.e., you may set $W_\mu^{1,2} = h = 0$). You should find a mass term of the form

$$\frac{v^2}{8}\left(B_\mu, W_\mu^3\right)\begin{pmatrix} g'^2 & -gg' \\ -gg' & g^2 \end{pmatrix}\begin{pmatrix} B^\mu \\ W^{3\mu} \end{pmatrix} . \tag{12}$$

Diagonalize the mass matrix in terms of rotated fields

$$Z_\mu = \cos\theta_W\, W_\mu^3 - \sin\theta_W B_\mu \,, \qquad A_\mu = \sin\theta_W\, W_\mu^3 + \cos\theta_W B_\mu \tag{13}$$

where θ_W is known as the weak mixing angle. (We often abbreviate $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$.) Determine the mass eigenstates and $\tan\theta_W$ in terms of g, g', and v. Answer: You should find $\tan\theta_W = g'/g$, $m_Z = m_W/c_W$, while the A gauge field is massless, as we expect for the photon.

(f) Expand out $|D_\mu H|^2$ fully in terms of the fields we have derived: h, $W_\mu^\pm$, Z, and A. Determine the Feynman rules of the Higgs boson h couplings to gauge bosons in terms of their masses and the vev v. (You should find that the photon does not couple to the Higgs boson h, but this is only true at tree-level, as we will see later.)

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[1] The abelian gauge field B_μ does not carry its own charge ($Y = 0$), nor does B_μ transform under $SU(2)_L$, so its $T^3 = 0$ also.

(f) Show that the covariant derivative for a field with $SU(2)_L \times U(1)_Y$ quantum numbers, given in Eq. (6), may be expressed as

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) - i\frac{g}{c_W} Z_\mu (T^3 - Q s_W^2) - ieQA_\mu\,. \tag{14}$$

The photon field A_μ couples to our electric charge operator Q which remains an unbroken gauge symmetry.