Status of g-2 theory

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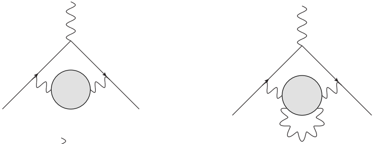
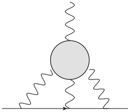
There is a tension of 3.7σ for the muon $a_\mu = (g_\mu - 2)/2$:

$$a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 27.4\,\underbrace{(2.7)}_{\rm HVP}\,\underbrace{(2.6)}_{\rm HLbL}\,\underbrace{(0.1)}_{\rm other}\,\underbrace{(6.3)}_{\rm EXP}\times 10^{-10}$$

HVP



HLbL



2019: $\delta a_\mu^{\rm EXP} \to 4.5 \times 10^{-10}$ (avg. of BNL/estimate of 2019 Fermilab result)
Targeted final uncertainty of Fermilab E989: $\delta a_\mu^{\rm EXP} \to 1.6 \times 10^{-10}$
$\Rightarrow$ by 2019 consolidate HVP/HLbL, over the next years uncertainties to O(1×10^{-10})

Also: In few years independent experimental result from J-PARC E34

There is also a tension of -2.4σ for the muon $a_e = (g_e - 2)/2$:

$$a_e^{\mathrm{EXP}} - a_e^{\mathrm{SM}} = -87\,\underbrace{(28)}_{\mathrm{EXP}}\,\underbrace{(23)}_{\alpha}\,\underbrace{(02)}_{\mathrm{SM}} \times 10^{-14}\,,$$

Gabrielse group 2008

Müller group 2018

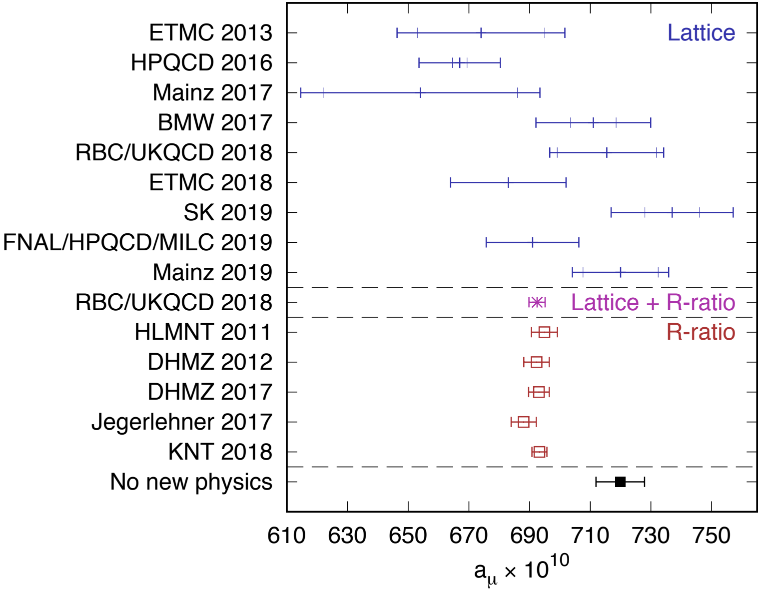
SM uncertainty far from dominant, however, check of five-loop QED calculation by Aoyama/Kinoshita/Nio is desirable (and a six-loop approximate answer?)

Possible future progress by lattice methods:

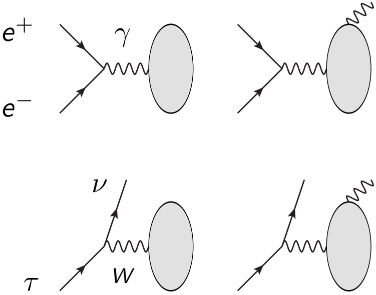
- Numerical Stochastic Perturbation Theory Burgio et al. 1998
- Diagrammatic Monte-Carlo Prokof'ev & B.V.Svistunov 1998

Status of hadronic vacuum polarization (HVP)

Status of HVP determinations



Dispersive method - Overview



$e^+e^- \to \mathrm{hadrons}(\gamma)$

$J_\mu = V_\mu^{I=1,I_3=0} + V_\mu^{I=0,I_3=0}$

$\tau \to \nu\mathrm{hadrons}(\gamma)$

$J_\mu = V_\mu^{I=1,I_3=\pm 1} - A_\mu^{I=1,I_3=\pm 1}$

Knowledge of isospin-breaking corrections and separation of vector and axial-vector components needed to use τ decay data.

Can have both energy-scan and ISR setup.

Dispersive method - e^+e^- status

Recent results by Keshavarzi et al. 2018, Davier et al. 2017:

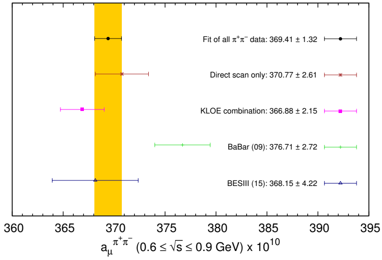
Channel	**This work (KNT18)**	**DHMZ17** [78]	**Difference**
Data based channels ($\sqrt{s} \leq 1.8$ GeV)			
$\pi^0\gamma$ (data + ChPT)	4.58 ± 0.10	4.29 ± 0.10	0.29
$\pi^+\pi^-$ (data + ChPT)	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$ (data + ChPT)	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
...			
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

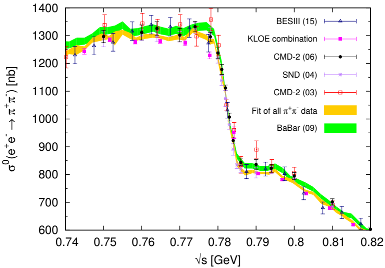
Good agreement for total, individual channels disagree to some degree. **Muon g-2 Theory Initiative workshops** recently held at Fermilab, KEK, UConn, and Mainz, intend to facilitate discussions and further understanding of these tensions. Whitepaper in preparation.

One difference: treatment of correlations, impactful in particular in case when not all experimental data agrees

Dispersive method - e^+e^- status

Tension in 2π experimental input. BaBar and KLOE central values differ by $\delta a_\mu = 9.8(3.5) \times 10^{-10}$, compare to quoted total uncertainties of dispersive results of order $\delta a_\mu = 3 \times 10^{-10}$.





Conflicting input limits the precision and reliability of the dispersive results.

Looking for more data and insight: energy-scans update from CMD-3 in Novosibirsk and ISR updates from KLOE2, BaBar, Belle, BESIII and BelleII. (For a BaBar update, see talk by M. Ebert, Tue 5:30pm.)

Dispersive method - τ status

Experiment	$a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10}) $2m_{\pi^\pm} - 0.36$ GeV	$0.36 - 1.8$ GeV
ALEPH	$9.80 \pm 0.40 \pm 0.05 \pm 0.07$	$501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	$9.82 \pm 0.13 \pm 0.04 \pm 0.07$	$506.4 \pm 1.9 \pm 2.2 \pm 1.9$

Davier et al. 2013: $a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau] = 516.2(3.5) \times 10^{-10}$ ($2m_\pi^\pm$ – 1.8 GeV)

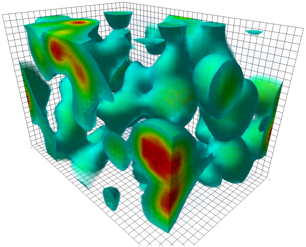
Compare to e^+e^-:

- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 507.1(2.6) \times 10^{-10}$ (DHMZ17, $2m_\pi^\pm$ – 1.8 GeV)
- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 503.7(2.0) \times 10^{-10}$ (KNT18, $2m_\pi^\pm$ – 1.937 GeV)

Here treatment of isospin-breaking to relate matrix elements of $V_\mu^{I=1,I_3=1}$ to $V_\mu^{I=1,I_3=0}$ crucial. Progress towards a first-principles calculation from LQCD+QED (arXiv:1811.00508).

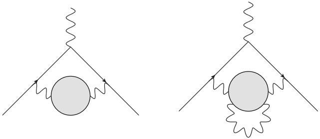
Lattice QCD

- Simulate QFT in terms of fundamental quarks and gluons (QCD) on a supercomputer with discretized four-dimensional space-time lattice
- Hadrons are emergent phenomena of statistical average over background gluon configurations to which quarks are coupled
- In this framework draw diagrams only with respect to quarks, photons, and leptons; gluons and their effects are generated by the statistical average.



Lattice QCD action density, Leinweber, CSSM, Adelaide, 2003

Euclidean Space Representation



Starting from the vector current $J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3}\sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x},t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the HVP diagrams.

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, charm, and bottom quark contributions.

Statistical variance of correlator

$$\langle J(t)J(0)\rangle$$

is itself a correlation function

$$\sigma^2(t) = \langle J(t)^2 J(0)^2\rangle - \langle J(t)J(0)\rangle^2 \,.$$

While $C(t) \propto e^{-m_\rho t}$ (vector channel), $\sigma^2(t) \propto e^{-m_\pi t}$ (pseudoscalar channel). Therefore signal-to-noise is exponentially bad for large t.

C(t) is, however, very precise for shorter Euclidean times t (on order of $1 - 2$ fm)

Lattice+R-ratio to replace part of $\pi\pi$ data (RBC/UKQCD 2018)

We therefore also consider a window method

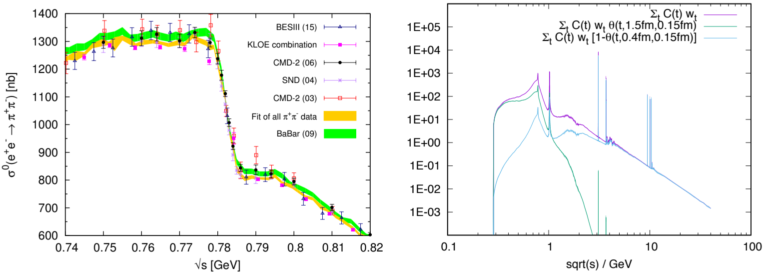
$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

with

$$\begin{aligned}
a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\
a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\
a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\
\Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]/2\,.
\end{aligned}$$

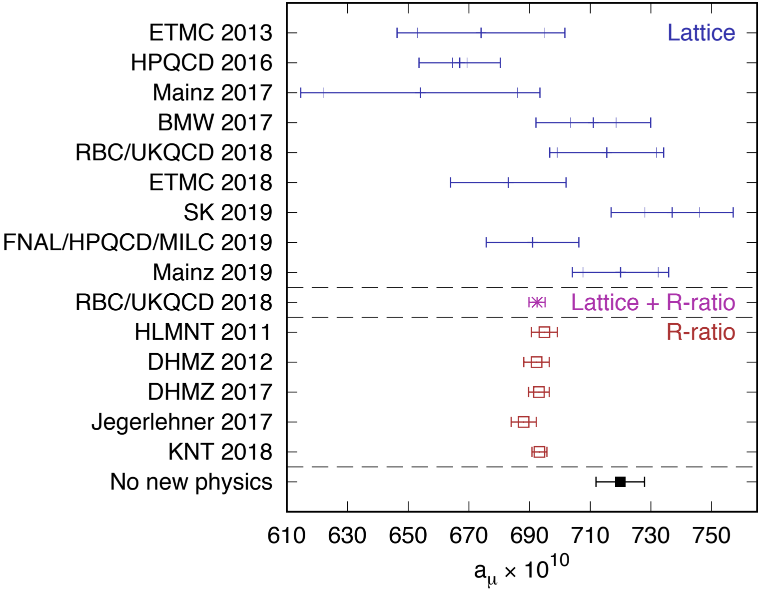
In this version of the calculation, we use $C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})$ to compute a_μ^{SD} and a_μ^{LD} and Lattice QCD+QED for a_μ^{W}.

How does this translate to the time-like region?



Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.

Status of HVP determinations



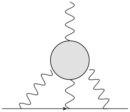
Conclusions and Outlook

- Target precision for HVP is of $O(1 \times 10^{-10})$ in a few years; for now consolidate error at $O(3 \times 10^{-10})$
- Dispersive result from $e^+e^- \rightarrow$ hadrons right now is at 3×10^{-10} but limited by experimental tensions
- Two-pion channel from DHMZ17, KNT18 (e^+e^-) and DHMYZ13 (τ) are scattered by 12.5×10^{-10}

  Experimental updates and first-principles calculation of isospin-breaking corrections desirable. Combination of dispersive and lattice results can in short term lessen dependence on contested experimental data.
- Lattice efforts by many groups, results at physical pion mass, QED, SIB corrections available. New methods to reduce statistical and systematic errors.
- By end of this year, first-principles lattice result could have error of $O(5 \times 10^{-10})$
- In a few years, new spacelike measurements from MUonE experiment (t-channel scattering) may be available

Status of hadronic light-by-light contribution (HLbL)

Current HLbL value is model estimate



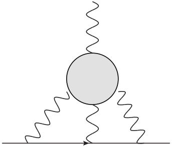
Contributions to $a_\mu^{\mathrm{HLbL}} \times 10^{10}$

	PdRV09	JN09	FJ17
π^0, η, η'	11.4(1.3)	9.9(1.6)	9.5(1.2)
π, K loops	-1.9(1.9)	-1.9(1.3)	-2.0(5)
axial-vector	1.5(1.0)	2.2(5)	0.8(3)
scalar	-0.7(7)	-0.7(2)	-0.6(1)
quark loops	0.2 (charm)	2.1(3)	2.2(4)
tensor			0.1(0)
NLO			0.3(2)
Total	10.5(4.9)	11.6(3.9)	10.3(2.9)
	10.5(2.6) (quadrature)		

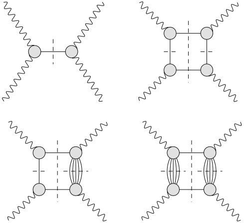
Potential double-counting and ad-hoc uncertainties

Two new avenues for a model-independent value for the HLbL

Dispersive analysis +
Experimental/lattice input



Direct lattice calculation



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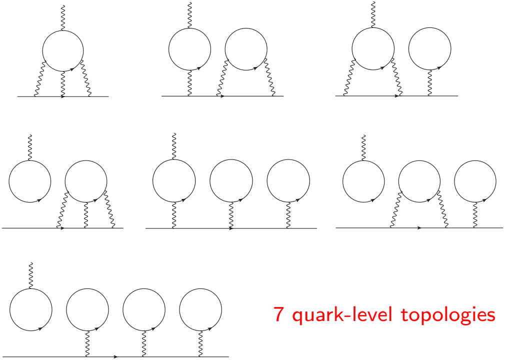
Truncation of cuts and states



7 quark-level topologies

Dispersive analysis

JHEP 1509 (2015) 074: Colangelo, Hoferichter, Procura, Stoffer

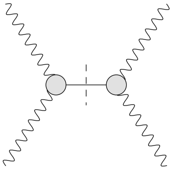
- Start with four-point function

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = -i \int d^4x\, d^4y\, d^4z\, e^{-i(q_1\cdot x+q_2\cdot y+q_3\cdot z)} \langle 0|T\{j^\mu_{\rm em}(x) j^\nu_{\rm em}(y) j^\lambda_{\rm em}(z) j^\sigma_{\rm em}(0)\}|0\rangle.$$

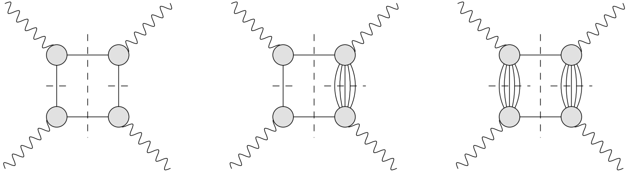
- A-priori 138 basic Lorentz structures (compare to 2 for HVP)
- Gauge invariance imposes 95 linear relations
- Special care needs to be taken (Tarrach) such that the resulting scalar functions are free of kinematic singularities that would complicate a dispersive discussion; a redundant basis satisfying this following Bardeen, Tung, and Tarrach with 54 elements can be chosen
- Crossing symmetry imposes additional constraints such that only **7** distinct structures remain

Organizing principle: systematic cuts and state truncation

- Estimate of truncation of this procedure is crucial and still being developed; ideas to use lattice for this are being explored (RBC 2018)
- Dominant contributions from pion-pole (needs $\pi \to \gamma^*\gamma^*$ form factors)



- next leading contribution from two-pion states (box topologies)



Recent results

- PRD94(2016)074507 (Mainz): Pion-pole contribution
$a_\mu^{\pi-pole} = 6.50(83) \times 10^{-10}$ using a model parametrization of the $\pi \to \gamma^*\gamma^*$ form factor constrained by lattice data

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}^{\mathrm{LMD+V}}(q_1^2,q_2^2) = \frac{\widetilde{h}_0\, q_1^2 q_2^2 (q_1^2+q_2^2) + \widetilde{h}_1 (q_1^2+q_2^2)^2 + \widetilde{h}_2\, q_1^2 q_2^2 + \widetilde{h}_5\, M_{V_1}^2 M_{V_2}^2\,(q_1^2+q_2^2) + \alpha\, M_{V_1}^4 M_{V_2}^4}{(M_{V_1}^2-q_1^2)(M_{V_2}^2-q_1^2)(M_{V_1}^2-q_2^2)(M_{V_2}^2-q_2^2)}$$

- JHEP1704(2017)161 (Colangelo et al.): Pion-box plus S-wave rescattering
$a_\mu^{\pi-box} + a_\mu^{\pi\pi,\pi-pole\ LHC,J=0} = -2.4(1) \times 10^{-10}$

- PRL121(2018)112002 (Hoferichter et al.); 1808.04823: Pion-pole contribution
$a_\mu^{\pi-pole} = 6.26(30) \times 10^{-10}$ reconstructing $\pi \to \gamma^*\gamma^*$ form factor from $e^+e^- \to 3\pi$, $e^+e^-\pi^0$ and $\pi^0 \to \gamma\gamma$ width

Combining these results one finds: $a_\mu^{\pi-pole} + a_\mu^{\pi-box} + a_\mu^{\pi\pi} = 3.9(3) \times 10^{-10}$

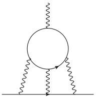
ikely dominant missing terms: η, η' pole: $O(3 \times 10^{-10})$

Compare to Glasgow consensus of $a_\mu^{\mathrm{HLbL}} = 10.5(2.6) \times 10^{-10}$ which also models contributions of heavier states and includes a matching with an high-energy quark picture. Control of truncation error very important.

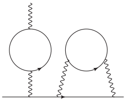
Direct lattice calculation

7 quark-level topologies of direct lattice calculation

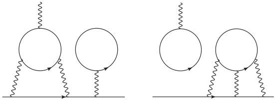
Hierarchy imposed by QED charges of dominant up- and down-quark contribution



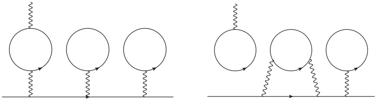
$Q_u^4 + Q_d^4 = 17/81$



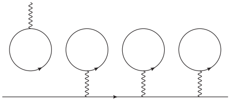
$(Q_u^2 + Q_d^2)^2 = 25/81$



$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$



$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$

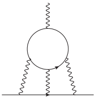


$(Q_u + Q_d)^4 = 1/81$

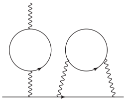
Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

7 quark-level topologies of direct lattice calculation

Hierarchy imposed by QED charges of dominant up- and down-quark contribution

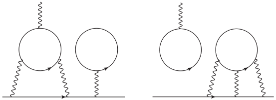


$Q_u^4 + Q_d^4 = 17/81$

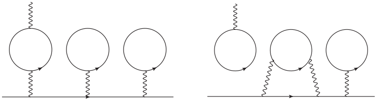


$(Q_u^2 + Q_d^2)^2 = 25/81$

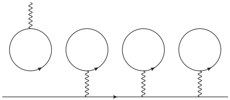
Dominant diagrams in top row: connected and leading disconnected diagram



$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$



$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$



$(Q_u + Q_d)^4 = 1/81$

Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

Finite-volume and infinite-volume formulations

- a_μ^{HLbL} in finite-volume QCD and QED:
  - PRD93(2016)014503 (RBC/UKQCD): Connected diagram with $m_\pi = 171$ MeV; $a_\mu^{\mathrm{HLbL}} = 13.21(68) \times 10^{-10}$
  - PRL118(2017)022005 (RBC/UKQCD): Connected and leading disconnected diagram with $m_\pi = 139$ MeV; $a_\mu^{\mathrm{HLbL}} = 5.35(1.35) \times 10^{-10}$ (potentially large finite-volume systematics)

  Strategy: extrapolate away $1/L^n$ ($n \geq 2$) errors

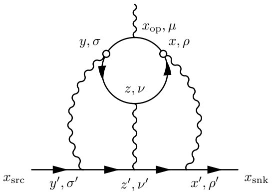
- a_μ^{HLbL} in finite-volume QCD and infinite-volume QED:
  - Method proposed and successfully tested against the lepton-loop analytic result: arXiv:1510.08384 (Mainz), arXiv:1609.08454 (Mainz)
  - Similar method plus subtraction scheme to reduce systematic errors; successfully tested against lepton-loop analytic result: PRD96(2017)034515 (RBC/UKQCD)

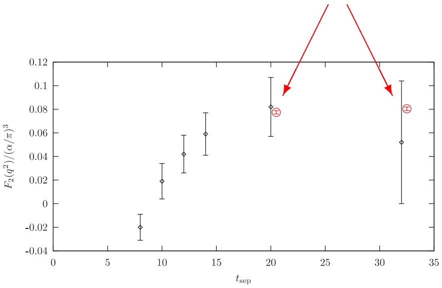
  Strategy: FV errors exponentially suppressed but still may be significant, effect on noise?

At heavy pion mass of $m_\pi \approx 300$ MeV, both groups have successfully cross-checked the connected contribution (g-2 Theory Initiative Whitepaper)

PRD93(2015)014503 (Blum, Christ, Hayakawa, Izubuchi, Jin, and CL):

New sampling strategy with 10x reduced noise for same cost (red versus black):





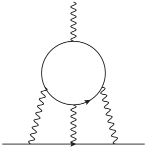
Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

PRL118(2016)022005 (Blum, Christ, Hayakawa, Izubuchi, Jin, Jung, and CL):

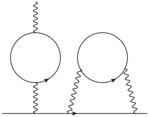
- Calculation at physical pion mass with finite-volume QED prescription ($\mathrm{QED_L}$) at single lattice cutoff of $a^{-1} = 1.73$ GeV and lattice size $L = 5.5$ fm.

- Connected diagram:



$$a_\mu^{\mathrm{cHLbL}} = 11.6(0.96) \times 10^{-10}$$

- Leading disconnected diagram:

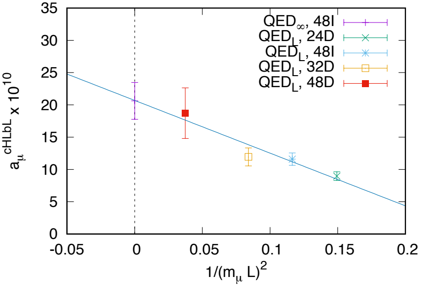


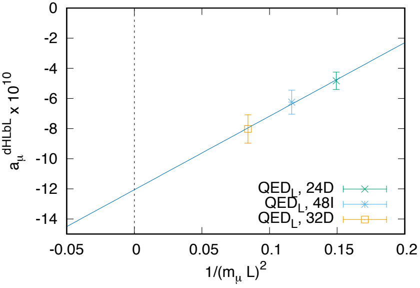
$$a_\mu^{\mathrm{dHLbL}} = -6.25(0.80) \times 10^{-10}$$

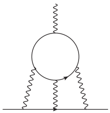
- Large cancellation expected from pion-pole-dominance considerations is realized: $a_\mu^{\mathrm{HLbL}} = a_\mu^{\mathrm{cHLbL}} + a_\mu^{\mathrm{dHLbL}} = 5.35(1.35) \times 10^{-10}$

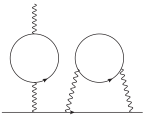
Potentially large systematics due to finite-volume QED!

Preliminary results for infinite-volume extrapolation



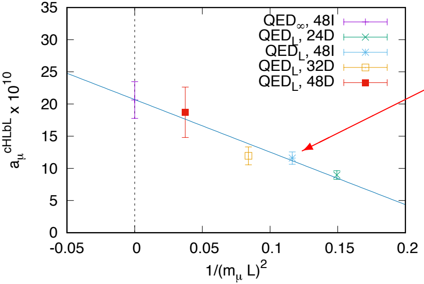


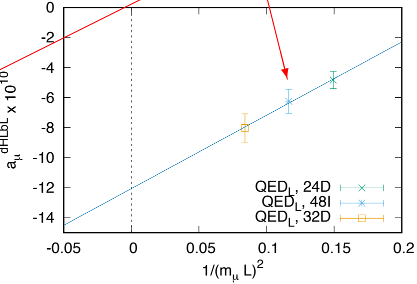


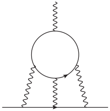


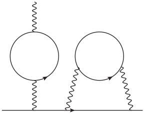
Preliminary results for infinite-volume extrapolation

Data used for finite-volume result in PRL118(2016)022005









Roadmap to complete first-principles light-by-light calculation

- Calculation of connected plus leading disconnected diagram at physical pion mass completed
- Infinite-volume extrapolation done (to be published)
- Discretization errors are now controlled for (four different lattice spacings over two different actions, to be published)
- Calculation of sub-leading disconnected diagrams, starting with 3-1 topology first results
- Crosscheck of dispersive versus lattice (see, e.g., arXiv:1712.00421) desirable

Summary

Summary

- By end of 2019, experimental uncertainty may be reduced to $O(5 \times 10^{-10})$ level
- By end of 2019, lattice QCD+QED results for HVP and HLbL with $O(5 \times 10^{-10})$ errors are likely available
- Combination of lattice+dispersive methods may reduce dependence on conflicting input data ($\pi\pi$) and help estimate truncation errors for dispersive HLbL
- Dispersive HVP awaits updates for $\pi\pi$ channel
- Extensive checks within dispersive results are currently being performed as part of g-2 Theory Initiative
- g-2 Theory Initiative Whitepaper to be released before Fermilab E989 result

Backup

Editors' Suggestion

Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

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We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is $a_\mu^{\mathrm{HVP\ LO}} = 715.4(18.7) \times 10^{-10}$. By supplementing lattice data for very short and long distances with R-ratio data, we significantly improve the precision to $a_\mu^{\mathrm{HVP\ LO}} = 692.5(2.7) \times 10^{-10}$. This is the currently most precise determination of $a_\mu^{\mathrm{HVP\ LO}}$.

This method allows us to reduce HVP uncertainty over next years to $\delta a_\mu^{\mathrm{LO\ HVP}} \sim 1 \times 10^{-10}$, below Fermilab E989 uncertainty

Computing resources

The RBC/UKQCD $g-2$ project has used on the order of 10^9 core hours (100k years on a single core) on the Mira supercomputer at Argonne, USQCD clusters at JLab and BNL, the BNL CSI KNL cluster, and the Oakforest and Hokusai supercomputers in Japan.

We have processed on the order of 5 petabytes of QCD data related to this project.

10 PFLOPS

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Next generation of runs on Summit in preparation

200 PFLOPS