Lattice QCD: B and D decays and mixing

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Outline

Heavy meson processes on the lattice

- Leptonic decays
- Semileptonic decays
- Neutral meson mixing

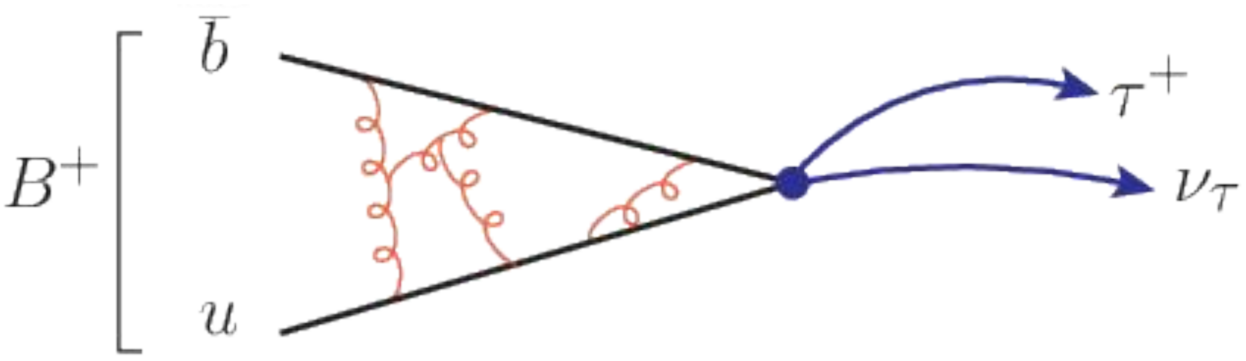
Systematic uncertainties

Selected recent results

Summary and outlook

Leptonic decays

Paradigm process:



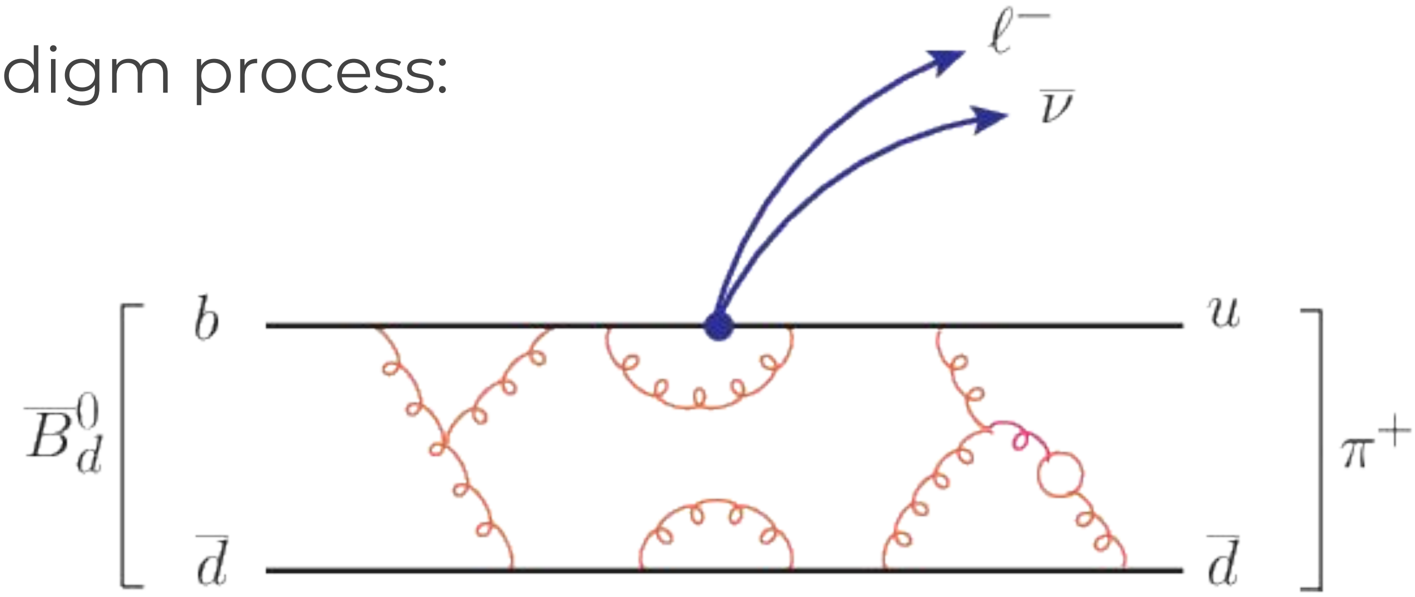
Branching fraction

$$\mathcal{B}(B \to \ell\nu_\ell) = \frac{m_B}{8\pi} G_F^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 |V_{ub}|^2 f_B^2$$

$$\langle 0|A_\mu|B_q(p)\rangle = i p_\mu f_B$$

Semileptonic decays

Paradigm process:



Differential decay rate

$$\frac{\mathrm{d}\Gamma(B_{(s)} \to X^{(*)}_{(s)})}{\mathrm{d}q^2} = \left[\sum_i C_i(V_{qb})\langle X^{(*)}_{(s)}|J_i|B_{(s)}\rangle\right]^2$$

$$X^{(*)}_{(s)} \in \{\pi, K, D, D_s, D^*, D^*_s\}$$

Pseudoscalar to pseudoscalar transitions

Differential decay rate

$$\frac{\mathrm{d}\Gamma(B_{(s)} \to X_{(s)})}{\mathrm{d}q^2} = \left[\sum_i C_i(V_{qb})\langle X_{(s)}|J_i|B_{(s)}\rangle\right]^2$$

Introduce form factors to parameterise QCD behavior

$$\langle X_{(s)}(p_{X_{(s)}})|V^\mu|B_{(s)}(p_{B_{(s)}})\rangle = f_0(q^2)\frac{M^2_{B_{(s)}} - M^2_{X_{(s)}}}{q^2}q^\mu + f_+(q^2)\left[p^\mu_{B_{(s)}} + p^\mu_{X_{(s)}} - \frac{M^2_{B_{(s)}} - M^2_{X_{(s)}}}{q^2}q^\mu\right]$$

So

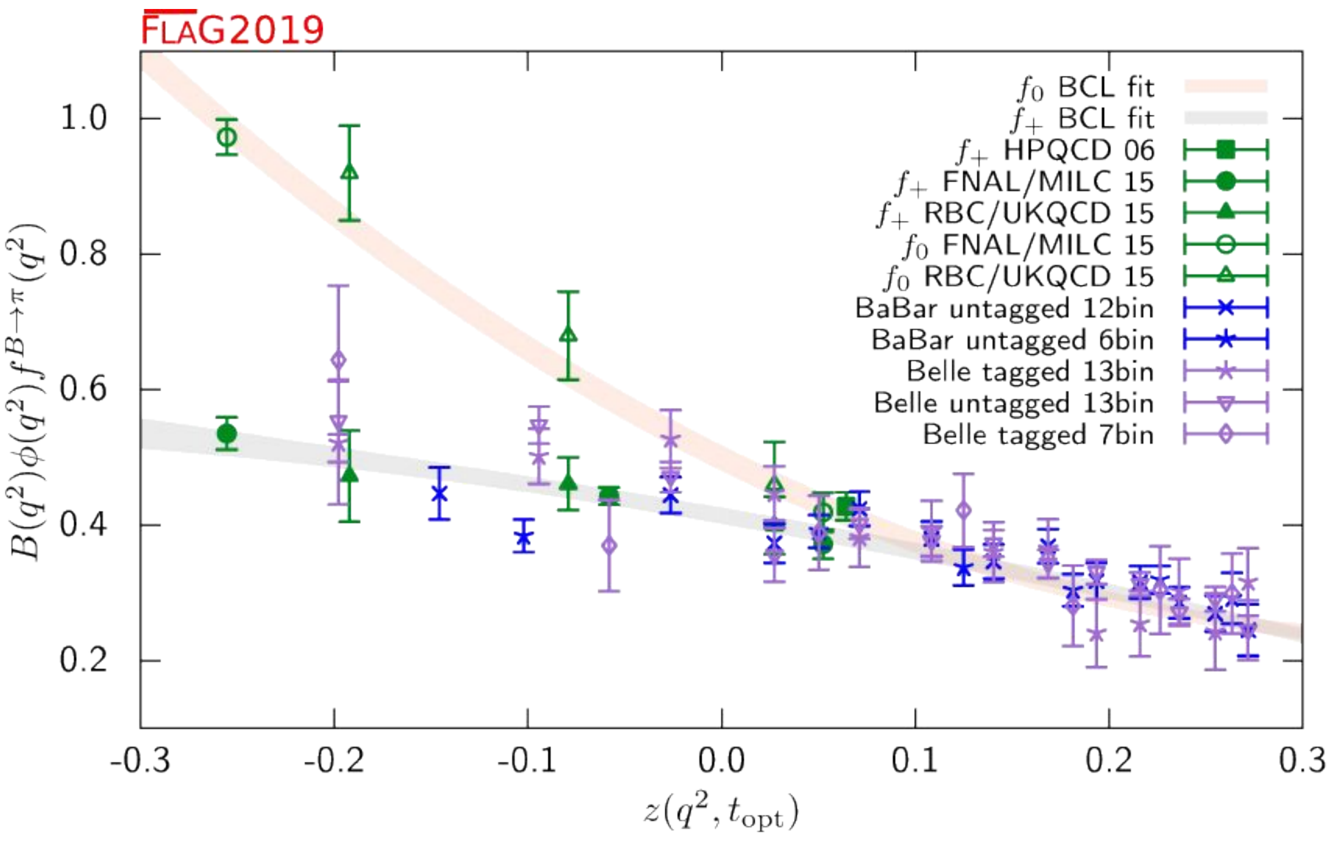
$$\frac{\mathrm{d}\Gamma(B_{(s)} \to X_{(s)})}{\mathrm{d}q^2} = \frac{G_F^2|V_{qb}|^2}{24\pi^3 M^2_{B_{(s)}}}\left(1 - \frac{m_\ell^2}{q^2}\right)^2 |\mathbf{p}_{X_{(s)}}| \times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M^2_{B_{(s)}}\mathbf{p}^2_{X_{(s)}}|f_+|^2 + \frac{3m_\ell^2}{8q^2}(M^2_{B_{(s)}} - M^2_{X_{(s)}})|f_0|^2\right]$$

$$q^\mu = p^\mu_{B_{(s)}} - p^\mu_{X_{(s)}}$$

Pseudoscalar to light pseudoscalar

For light leptons

$$\frac{d\Gamma(B_{(s)} \to X_{(s)})}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |\mathbf{p}_{X_{(s)}}|^3 |f_+|^2$$



Pseudoscalar to heavy pseudoscalar/vector

Differential branching fraction typically written

$$\frac{\mathrm{d}\Gamma(B_{(s)} \to D_{(s)})}{\mathrm{d}q^2} = \frac{G_F^2|V_{cb}|^2|\eta_{\mathrm{EW}}|^2}{48\pi^3} M_{D_{(s)}}^3 (M_{B_{(s)}} + M_{(D_{(s)})})^2 (\omega - 1)^{3/2} |\mathcal{G}(\omega)|^2$$

$$\frac{\mathrm{d}\Gamma(B_{(s)} \to D_{(s)}^{*})}{\mathrm{d}q^2} = \frac{G_F^2|V_{cb}|^2|\eta_{\mathrm{EW}}|^2}{4\pi^3} M_{D_{(s)}^*}^3 (M_{B_{(s)}} - M_{(D_{(s)}^*)})^2 (\omega - 1)^{1/2} \chi(\omega) |\mathcal{F}(\omega)|^2$$

$$\mathcal{F}(1) = h_{A_1}(1)$$

Here

$$\omega = v_{B_{(s)}}^{(*)} \cdot v_{D_{(s)}}^{(*)} \qquad v^\mu = \frac{p^\mu}{M}$$

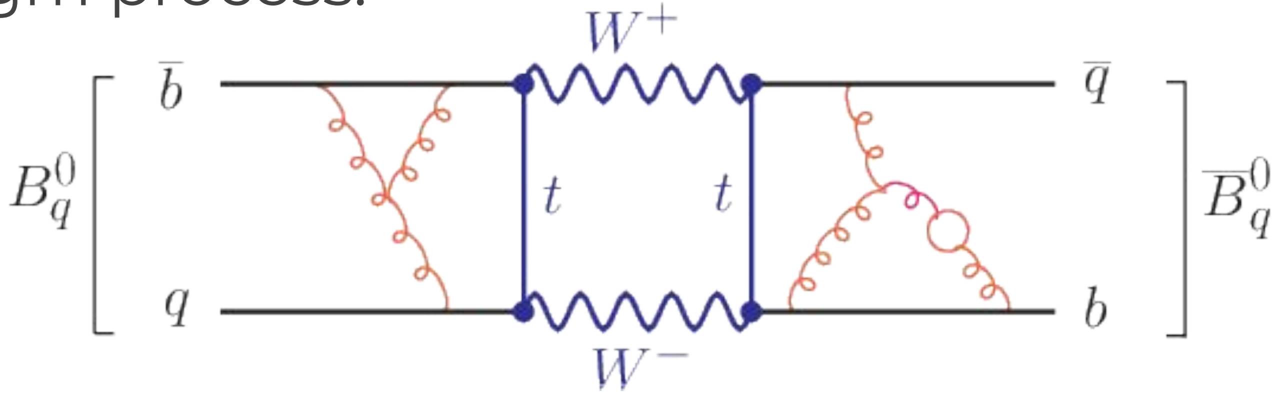
In general: form factor parameterisation just a choice

Many choices, but only a few independent ones

Physical form factors functions of single kinematic variable

Neutral meson mixing

Paradigm process:



Described by effective Hamiltonian

$$i\frac{\mathrm{d}}{\mathrm{d}t}\vec{B}_q = \left[M^q - \frac{i}{2}\Gamma^q\right]\vec{B}_q \qquad \vec{B}_q = \begin{pmatrix} B_q^0 \\ \overline{B}_q^0 \end{pmatrix}$$

Observables obtained from

$$\tau_{B_q} = \frac{1}{M_{11}^q}$$

$$\Delta m_q = m_H^q - m_L^q = 2|M_{21}^q|$$

$$\Delta\Gamma_q = \Gamma_H^q - \Gamma_L^q = -2|M_{21}^q|\,\mathrm{Re}\left(\frac{\Gamma_{21}^q}{M_{21}^q}\right)$$

Neutral meson mixing

Width difference

$$\Delta\Gamma_q = -2|M_{21}^q|\,\mathrm{Re}\left(\frac{\Gamma_{21}^q}{M_{21}^q}\right)$$

Related to matrix elements of dimension 6 operators

$$\Gamma_{21}^q = -\frac{G_F^2 m_b^2}{24\pi M_{B_q}} \sum_i C_i^q(\mu)\langle \overline{B}_q^0 | \mathcal{O}_i^q(\mu) | B_q^0 \rangle$$

Characterise matrix elements via “bag parameters”

$$\langle \overline{B}_q^0 | \mathcal{O}_i^q(\mu) | B_q^0 \rangle = \frac{8 m_{B_q}^2}{3} f_{B_q}^2 \widehat{B}_{B_s}(\mu)$$

SU(3) breaking ratio

$$\frac{\Delta m_s}{\Delta m_d} \propto \left|\frac{V_{ts}}{V_{td}}\right|^2 \xi^2 \qquad\qquad \xi = \frac{f_{B_s}\sqrt{\widehat{B}_{B_s}}}{f_{B_d}\sqrt{\widehat{B}_{B_d}}}$$

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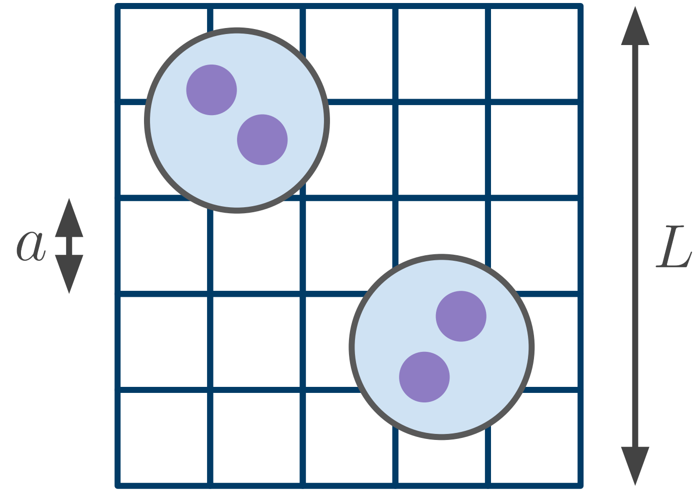
Systematic uncertainties on the lattice

In general, systematic uncertainties arise from:

- discretisation effects
- finite volume effects
- unphysical quark masses



But, for heavy meson processes in particular

- heavy quark effects
- renormalisation
- form factor parameterisation and kinematics

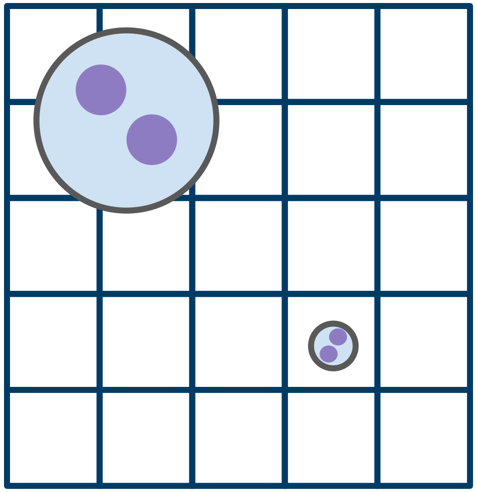
Quarks on the lattice

Some freedom in discretisation of the Dirac Lagrangian

Symanzik improvement reduces discretisation effects

- a^2 tadpole-improved staggered (AsqTad)
- highly improved staggered quarks (HISQ)
- twisted mass (TwM)
- domain-wall (DWF)

Cost increases as lattice spacing decreases



Lattice spacing too coarse to resolve heavy quarks

Relativistic quark actions have uncertainties ~ $(am_b)^n$

Heavy quarks on the lattice

Two approaches

1. Effective theories
   - heavy quark effective theory (HQET)
   - nonrelativistic QCD (NRQCD)

   or
   - relativistic heavy quarks (RHQ)

2. Relativistic actions extrapolated to physical b quark mass

Operator renormalisation

EFT heavy-light current operators require renormalisation

Three approaches

1. Nonperturbative schemes
   - PCAC relations
   - RI/(S)MOM-type schemes
2. Lattice perturbation theory
3. “Mostly nonperturbative”

Relativistic formulations, e.g. “heavy HISQ”, avoid this issue

N.B. Four-quark operators always require renormalisation

Kinematic extrapolation of form factors

Lattice calculations restricted to large momentum transfer

Two approaches

1. Model dependent, e.g.
   - Becirevic-Kaidalov — PLB 478 (2000) 417
   - Ball-Zwicky — PRD 71 (2005) 014015
   - Hill — PRD 73 (2006) 014012

2. Model independent z-parameterisation
   - Caprini, Lellouch, and Neubert (CLN) — NPB 530 (1998) 153
   - Boyd, Grinstein, and Lebed (BGL) — PRL 74 (1995) 4603
   - Bourrely, Caprini, and Lellouch (BCL) — PRD 79 (2009) 013008

$$f(q^2) = \frac{1}{B(q^2)\phi(q^2)} \sum_{n=0}^{\infty} a_n z(q^2)^n$$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_- - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_- - t_0}}$$

$$t_\pm = (M^2_{B_{(s)}} \pm M_{D^{(*)}_{(s)}})^2$$

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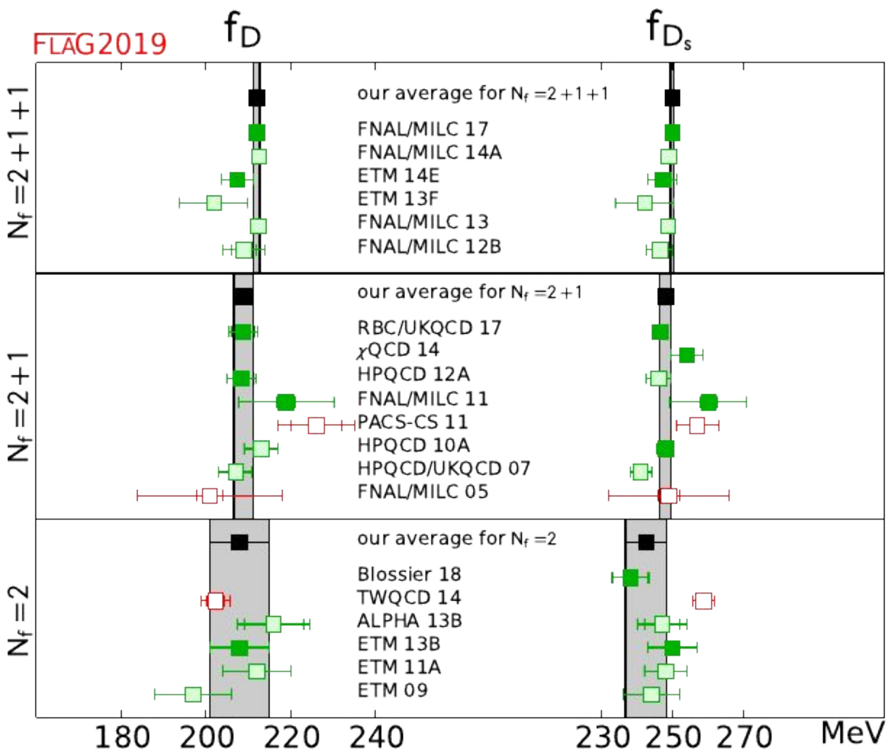
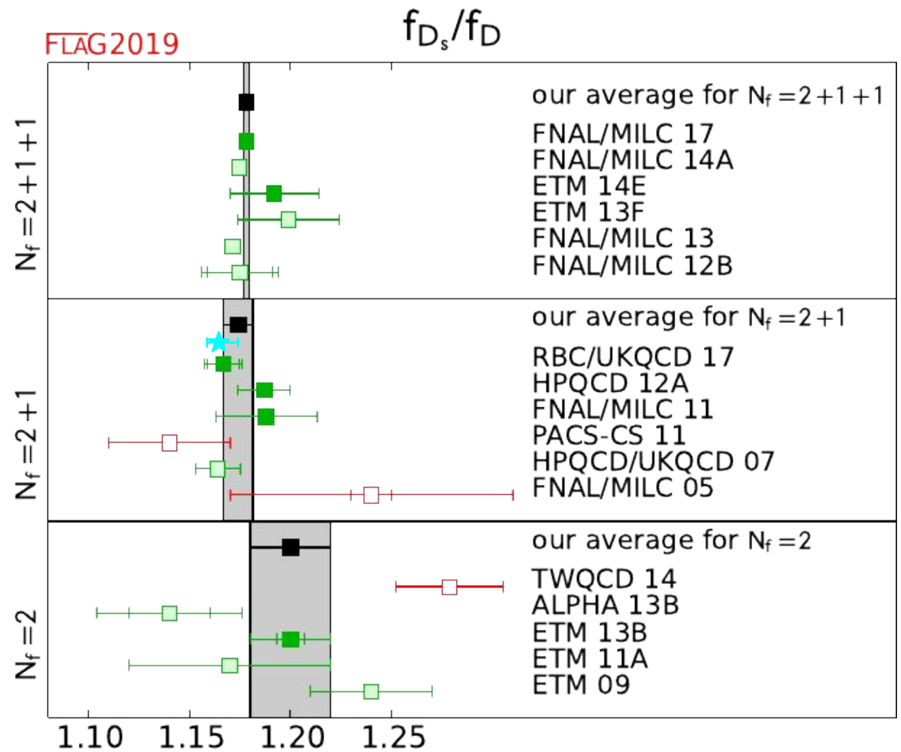
V_{cd} and V_{cs} : leptonic D decays

$$\mathcal{B}(D_{(s)} \to \ell \nu_\ell) = \frac{G_F^2 |V_{cq}|^2 \tau_{D_{(s)}}}{8\pi} f_{D_{(s)}}^2 m_\ell^2 m_{D_{(s)}} \left(1 - \frac{m_\ell^2}{m_{D_{(s)}}^2}\right)^2 \qquad \langle 0 | A_{cq}^\mu | D_q(p) \rangle = i f_{D_q}\, p_{D_q}^\mu$$





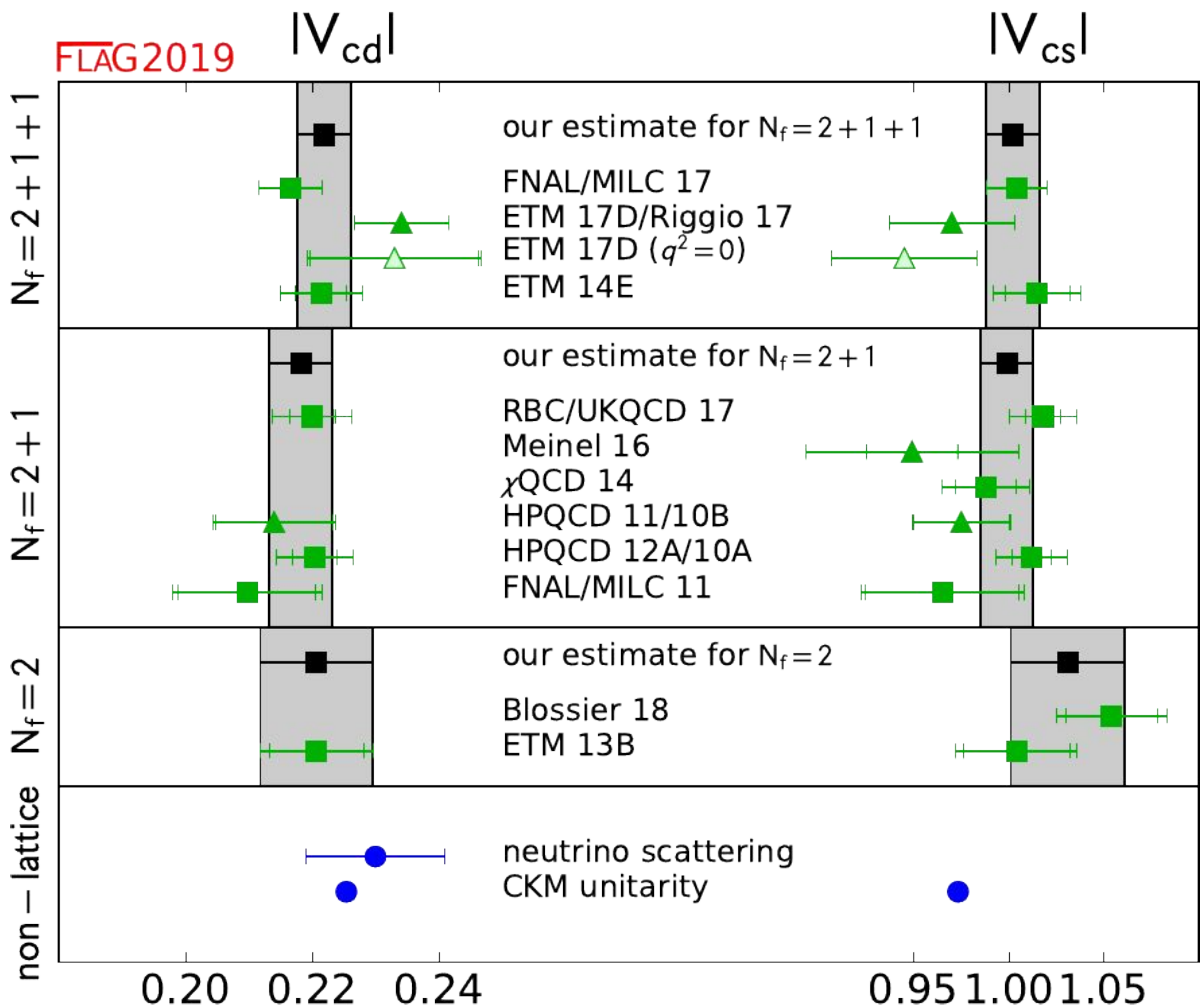
★ = Boyle et al., 1812.08791

FLAG 2019, 1902.08191

Preliminary 2+1 results from ALPHA at Lattice 2018

Bussone, 1812.05458

V_{cd} and V_{cs}



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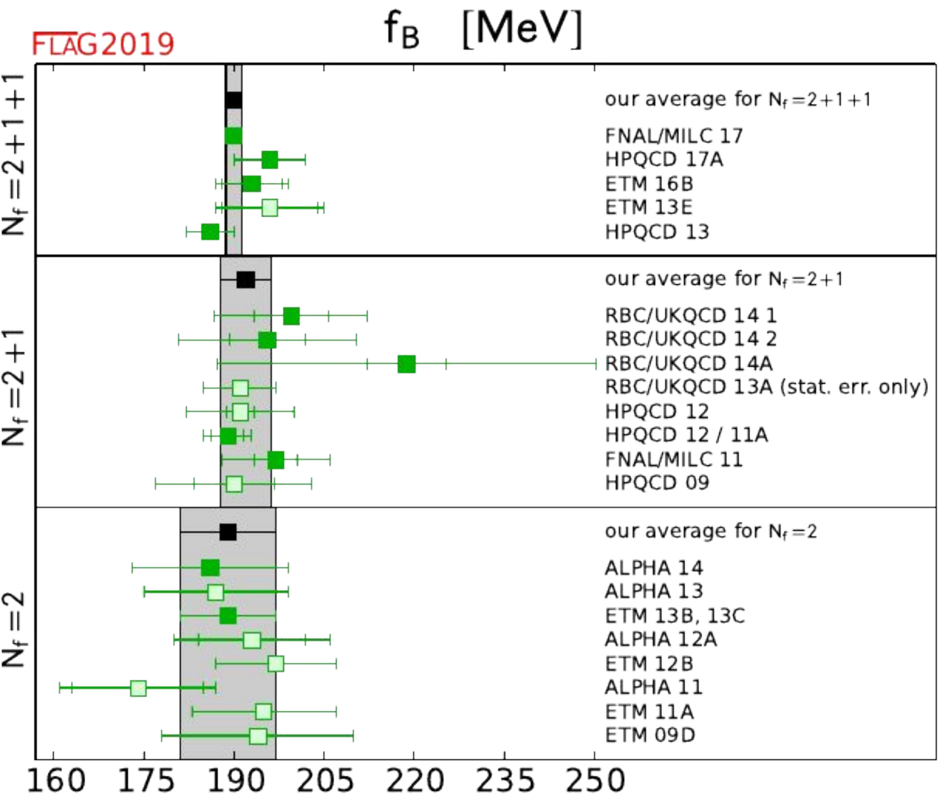
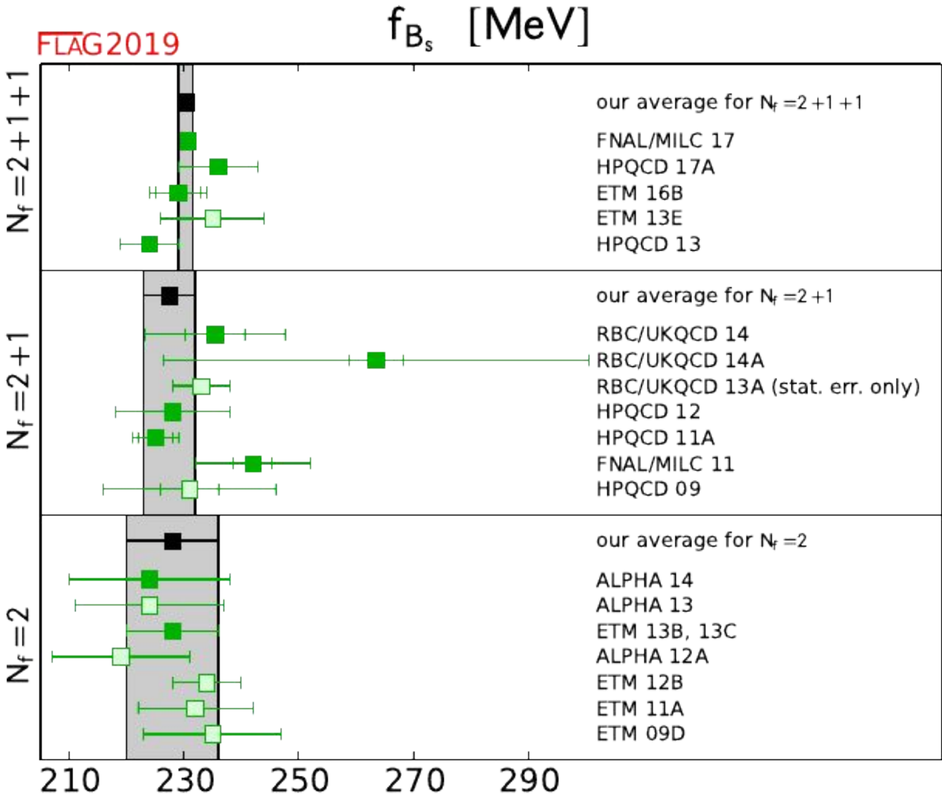
V_{ub} and V_{cb} : leptonic B decays

$$\Gamma(B \to \ell\nu_\ell) = \frac{m_B}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$





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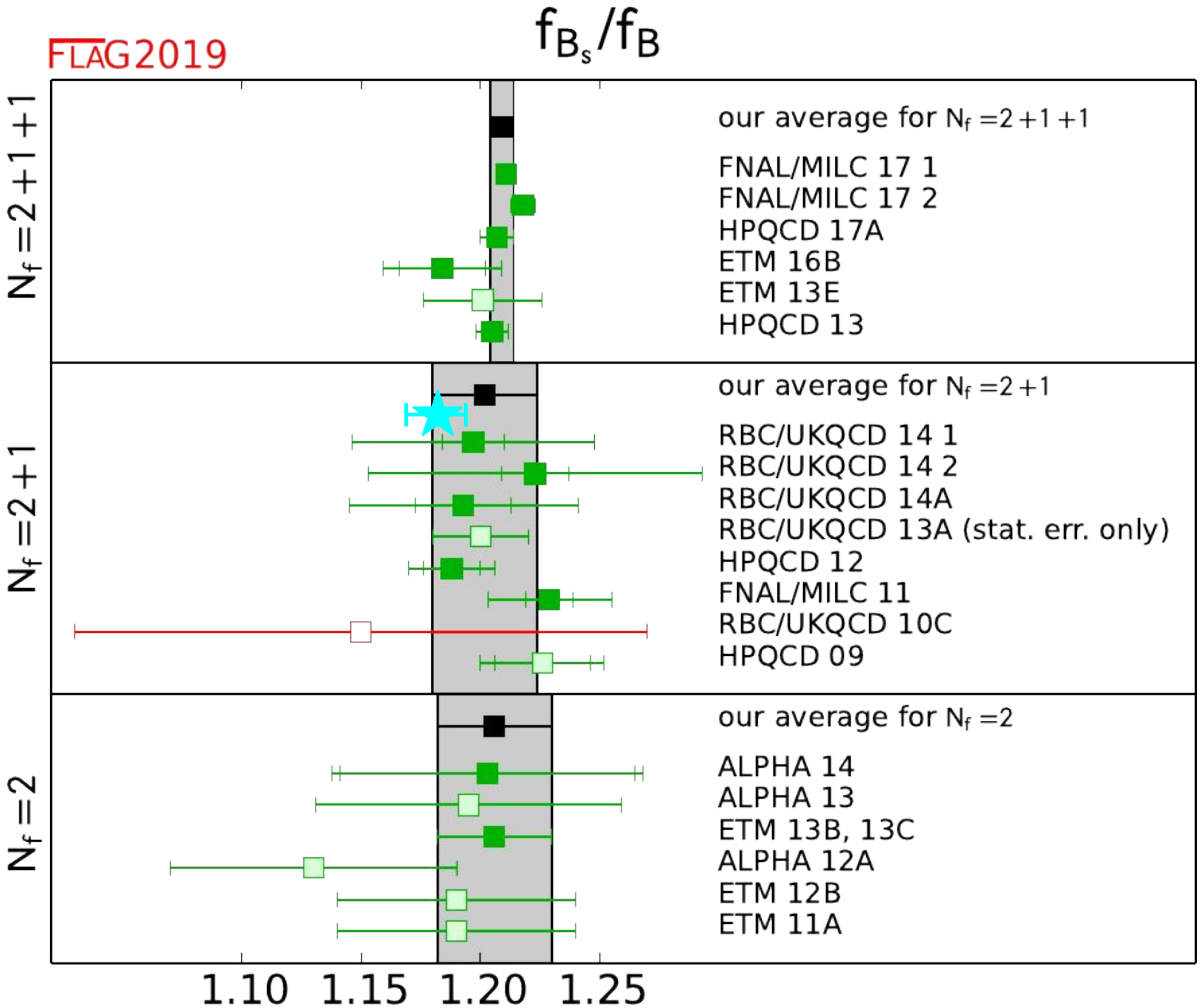
V_{ub} and V_{cb} : leptonic B decays



★ = Boyle et al., 1812.08791

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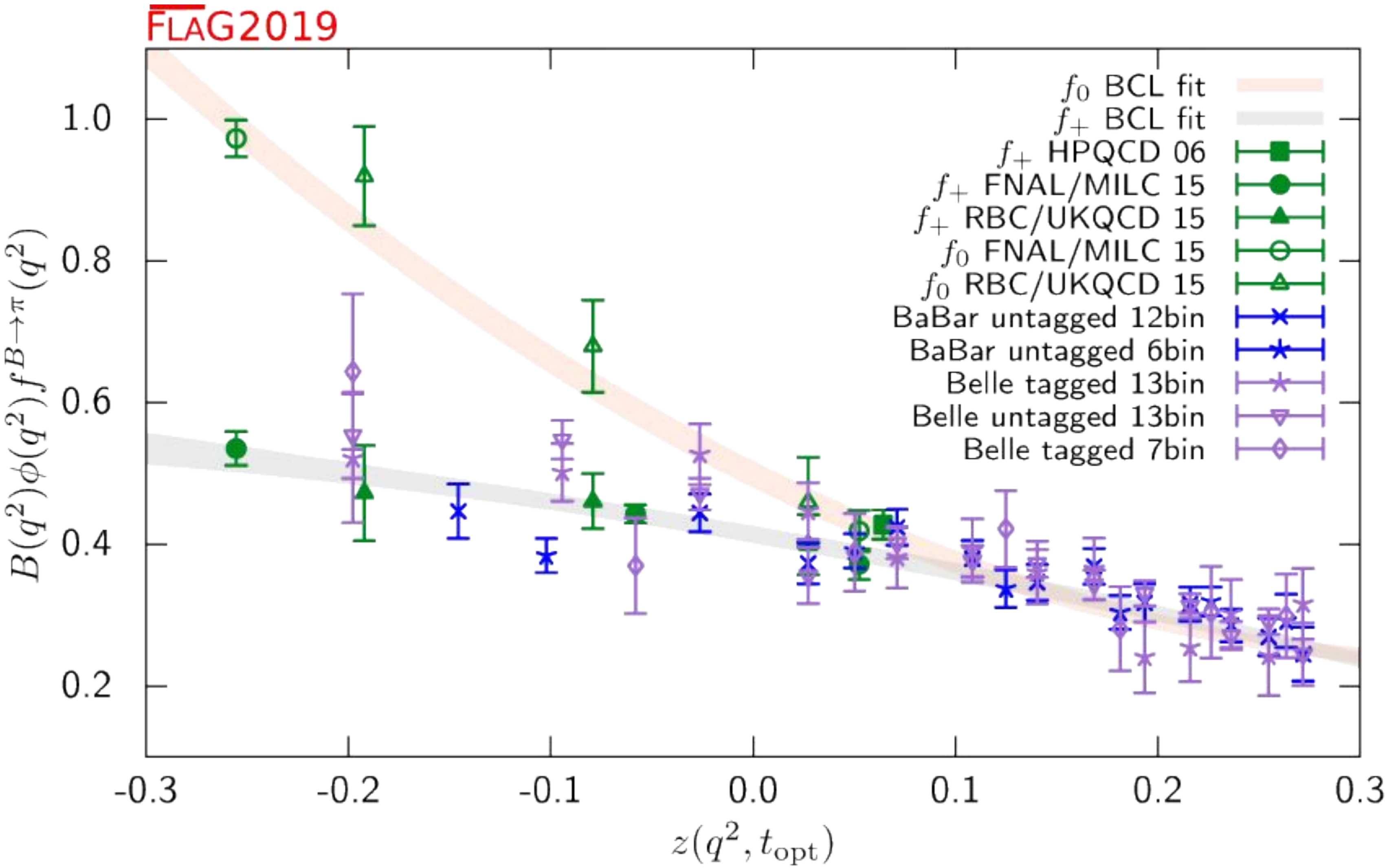
V_{ub} : semileptonic B to π decay



Preliminary 2+1 results from JLQCD at Lattice 2018

Colquhoun, 1811.00227

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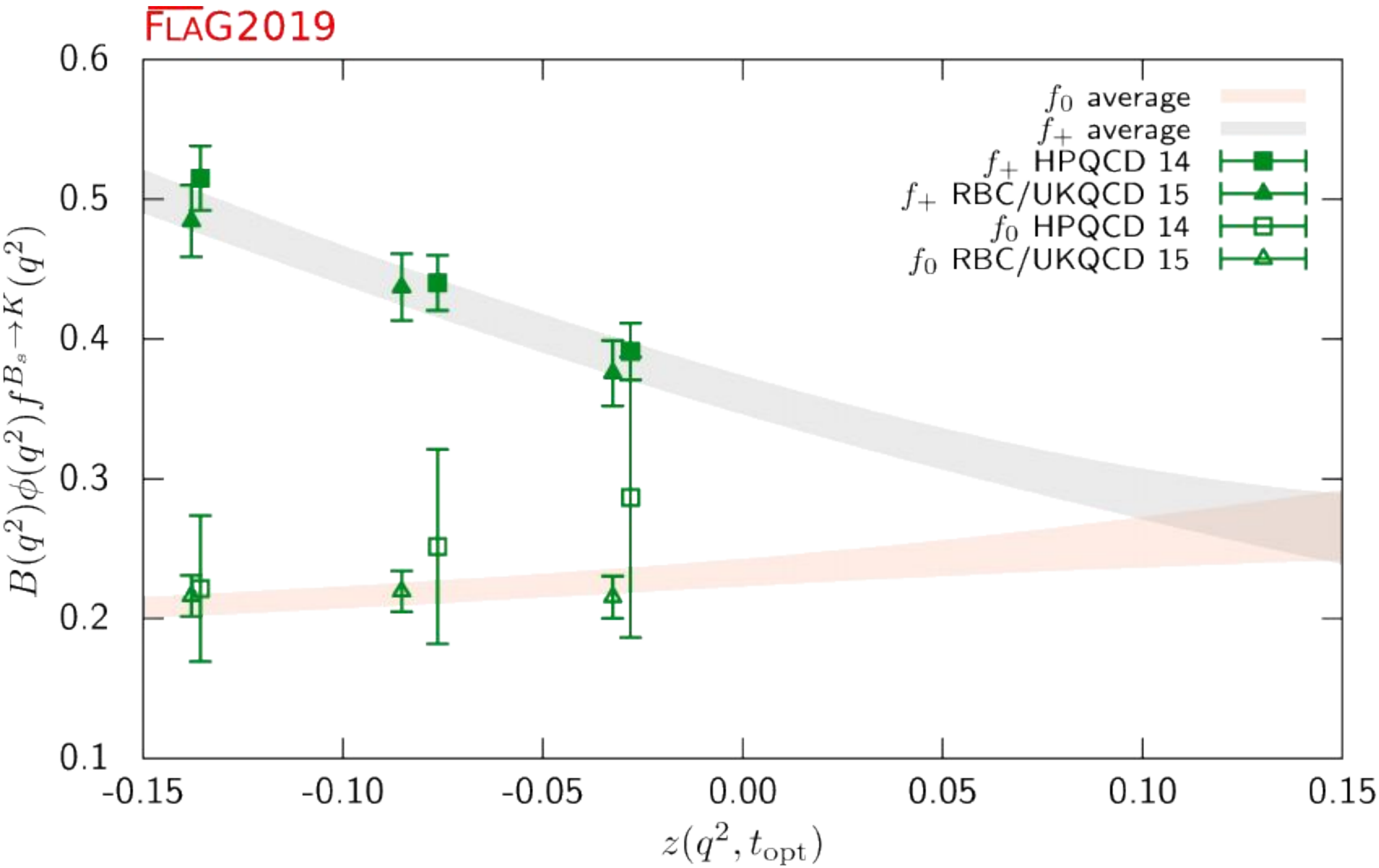
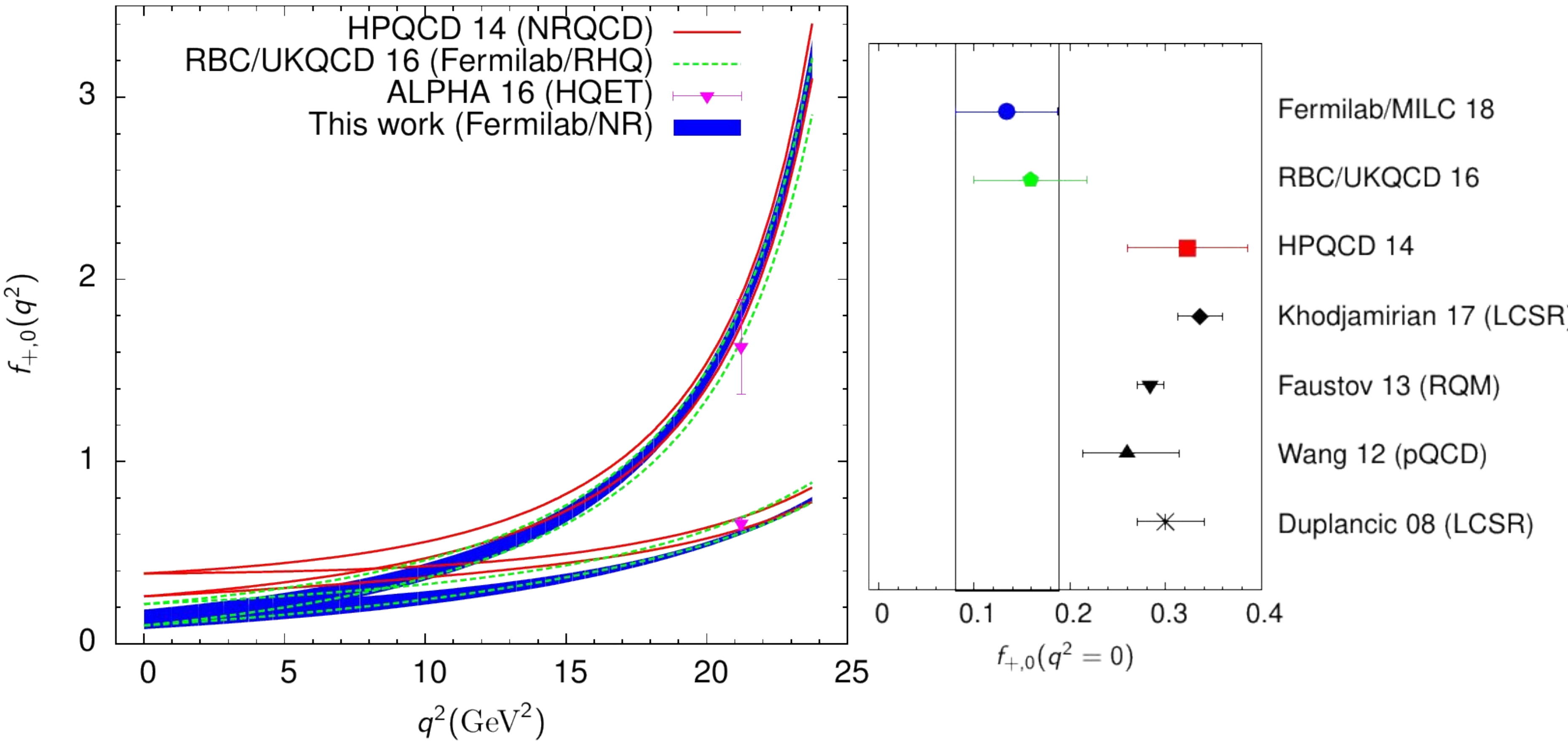
V_{ub} : semileptonic B_s to K decay



Preliminary 2+1 results from RBC/UKQCD at Lattice 2018

Flynn et al., 1903.02100

V_{ub} : semileptonic B_s to K decay



Bazavov et al., 1901.02561

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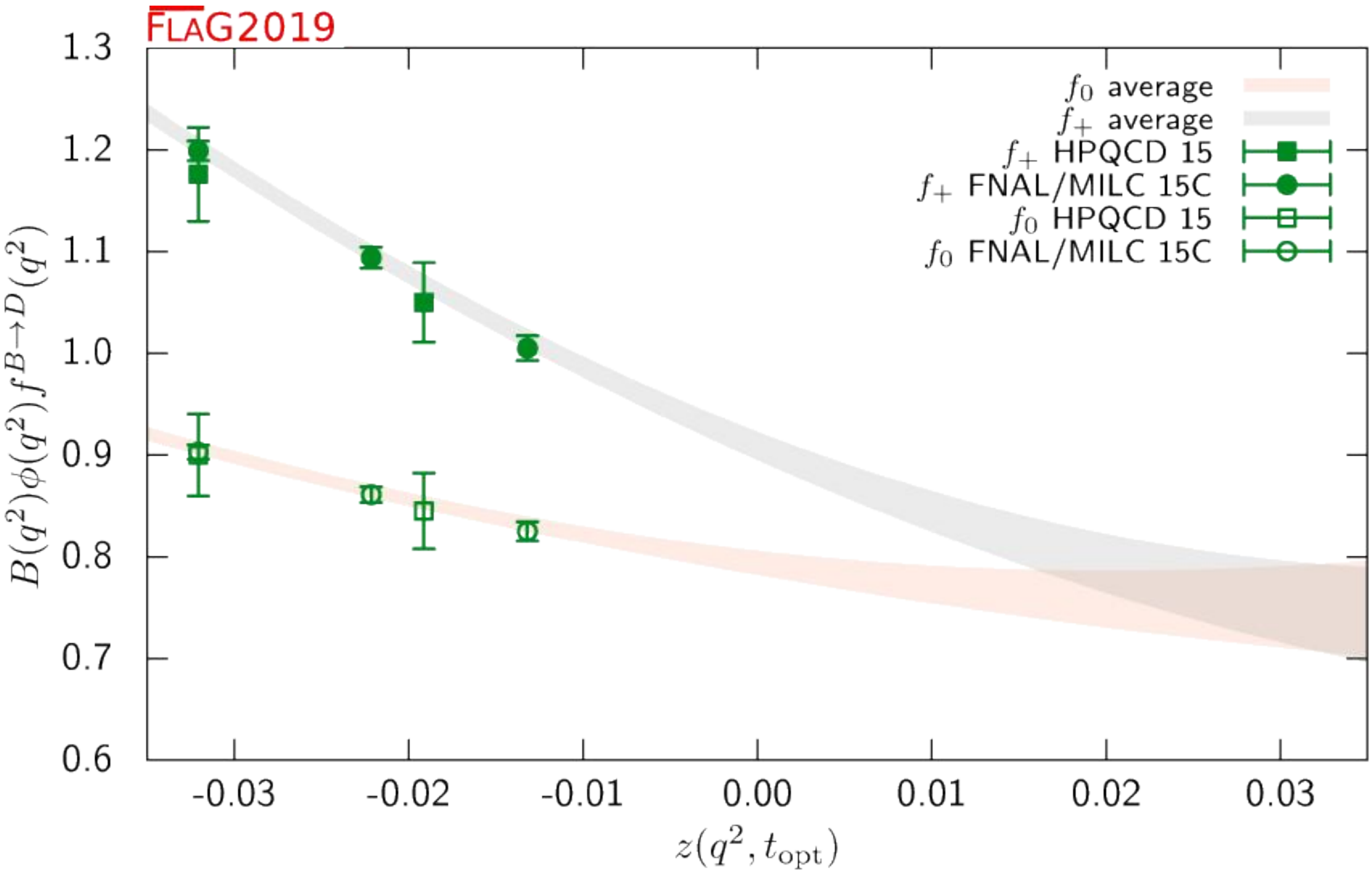
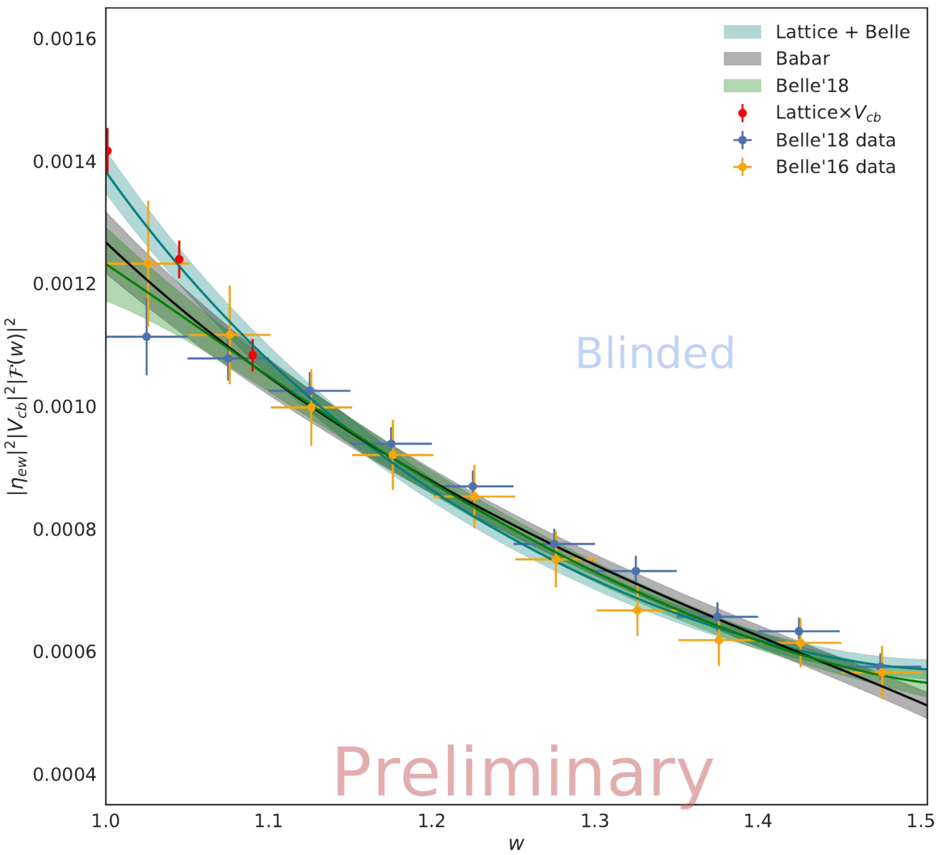
V_{cb} : semileptonic B to D decay

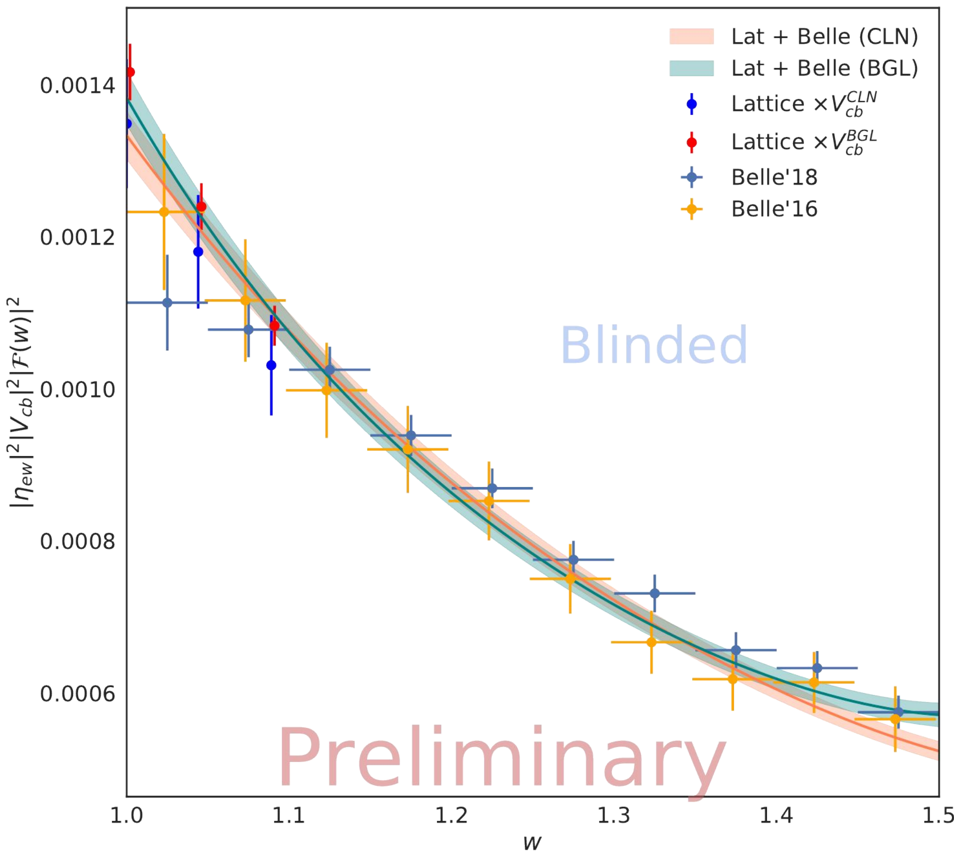


Preliminary 2+1 results from JLQCD at Lattice 2018

Kaneko et al., 1811.00794

V_{cb} : B to D* semileptonic decay





Vaquero et al., update of 1901.00216

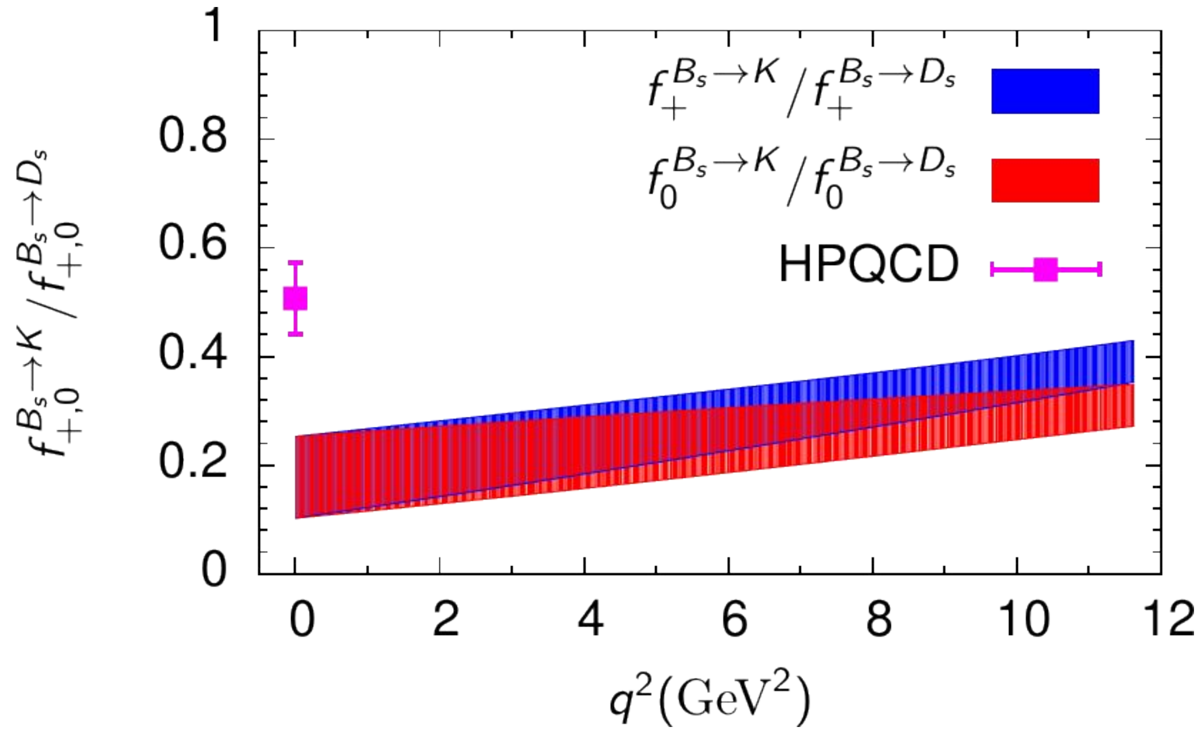
V_{ub}/V_{cb}: semileptonic B_s decays

Ratio of B_s to K and B_s to D_s proportional to V_{ub}/V_{cb}

First lattice calculation 2018 (HPQCD) CJM et al., PRD 98 (2018) 114509

Recent MILC/FNAL results in tension

Bazavov et al., 1901.02561

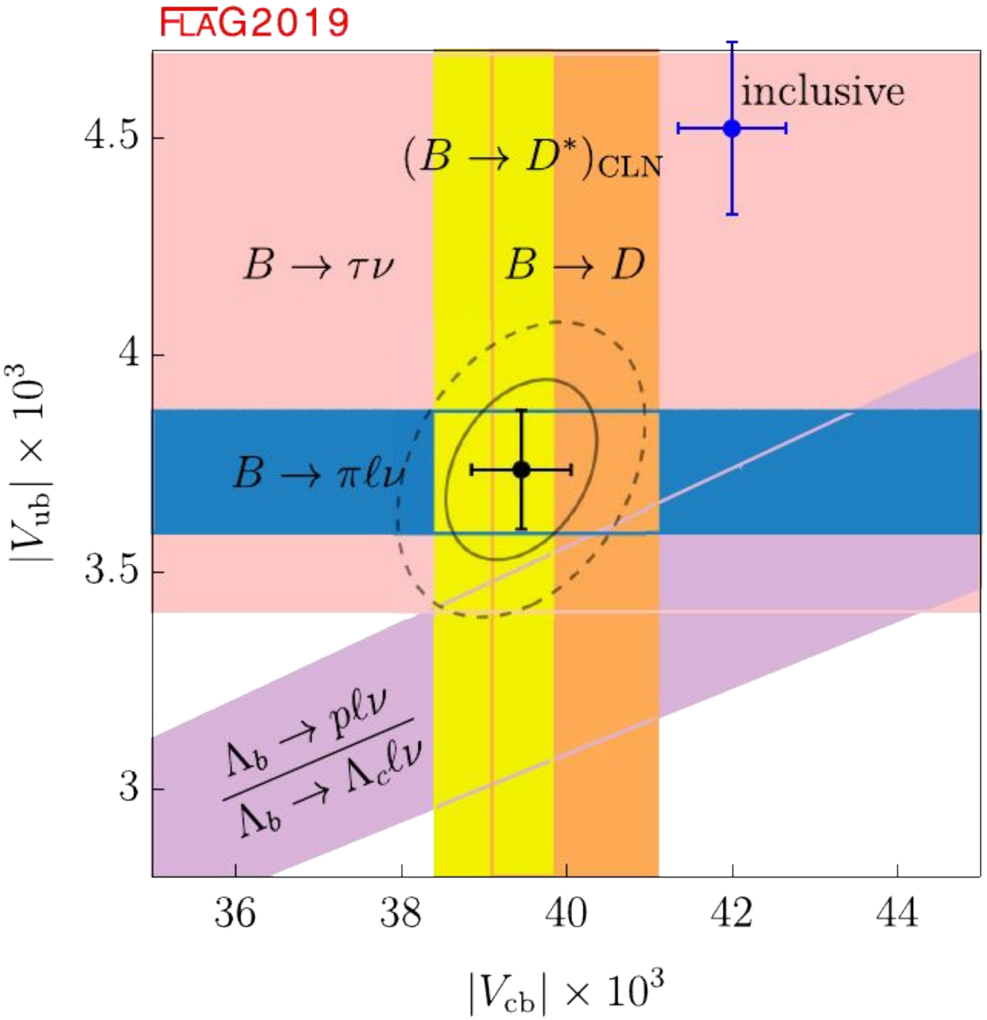


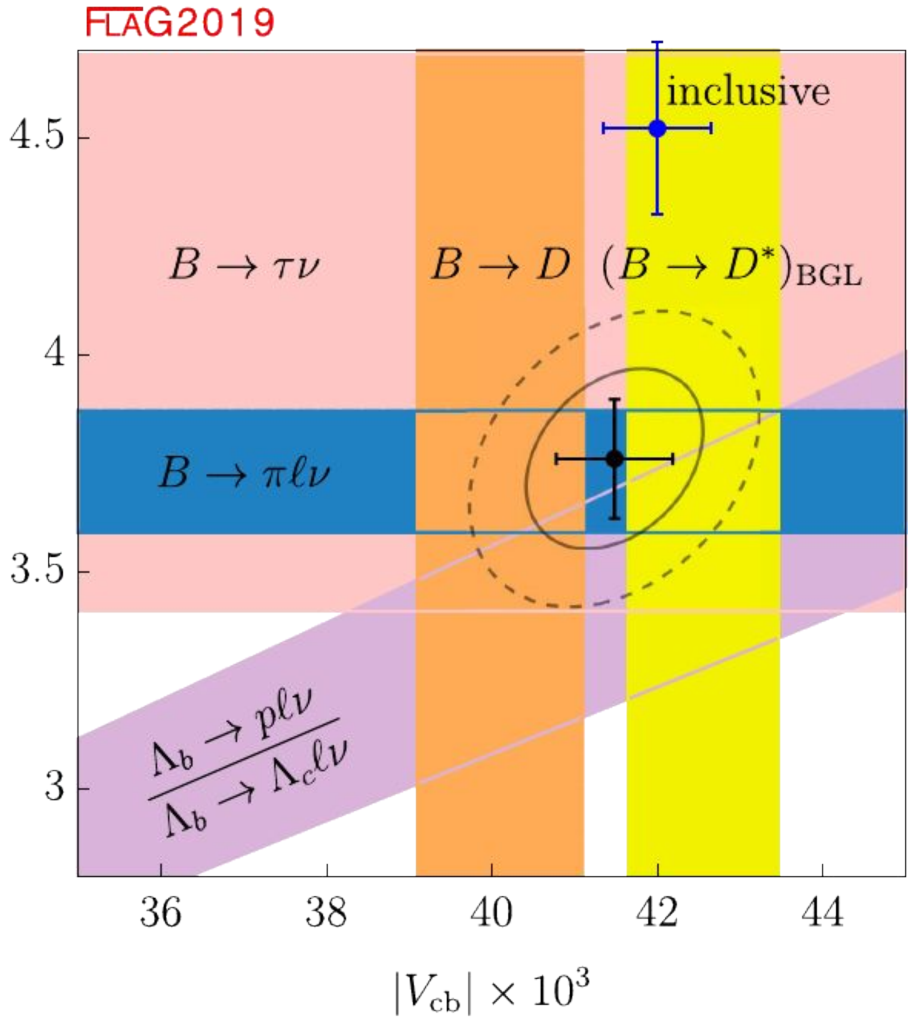
Preliminary 2+1+1 heavy HISQ results presented at Lattice 2018

McLean et al. (HPQCD), 1901.04979

McLean et al. (HPQCD) 1904.02046

V_{ub} and V_{cb}





Does not include latest Belle data

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V_{ub} and V_{cb}

Form factor parameterisation has been challenging

Considerable differences between BGL and CLN

Consistently applying the BGL parameterisation appeared to resolve the V_{cb} inclusive-exclusive tension, but new Belle data complicates this picture. No resolution for the tension in V_{ub}.

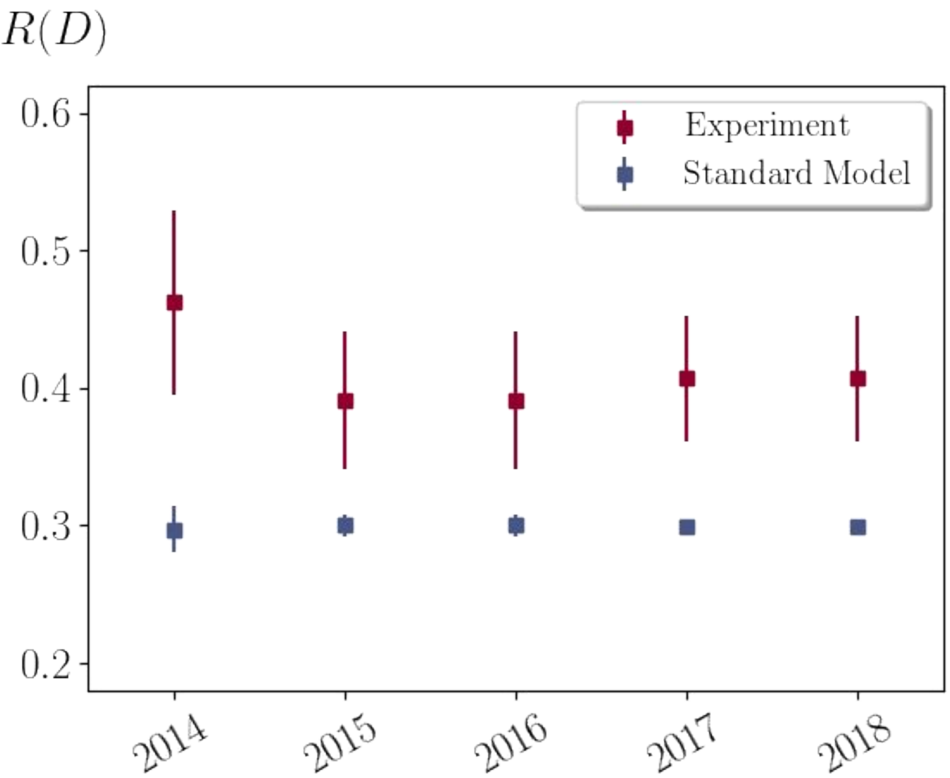
Attempts at systematic comparisons relevant to lattice data appear to prefer BCL to BGL, but do not include latest data.

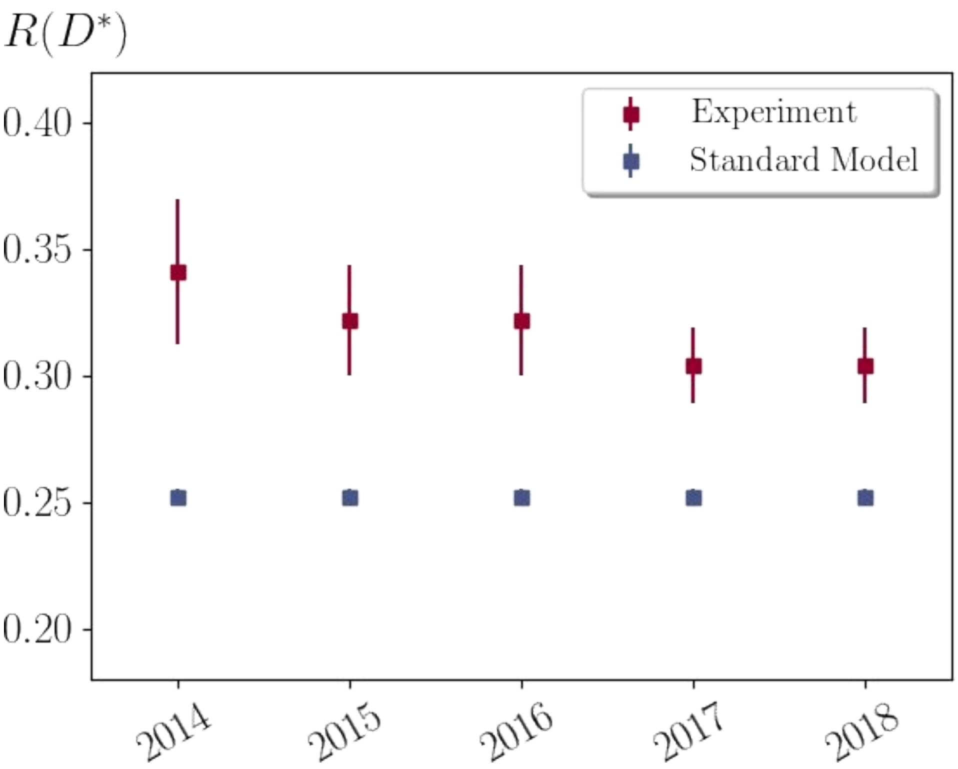
Gustafson & Meurice, 1807.10257

Branching fraction ratios

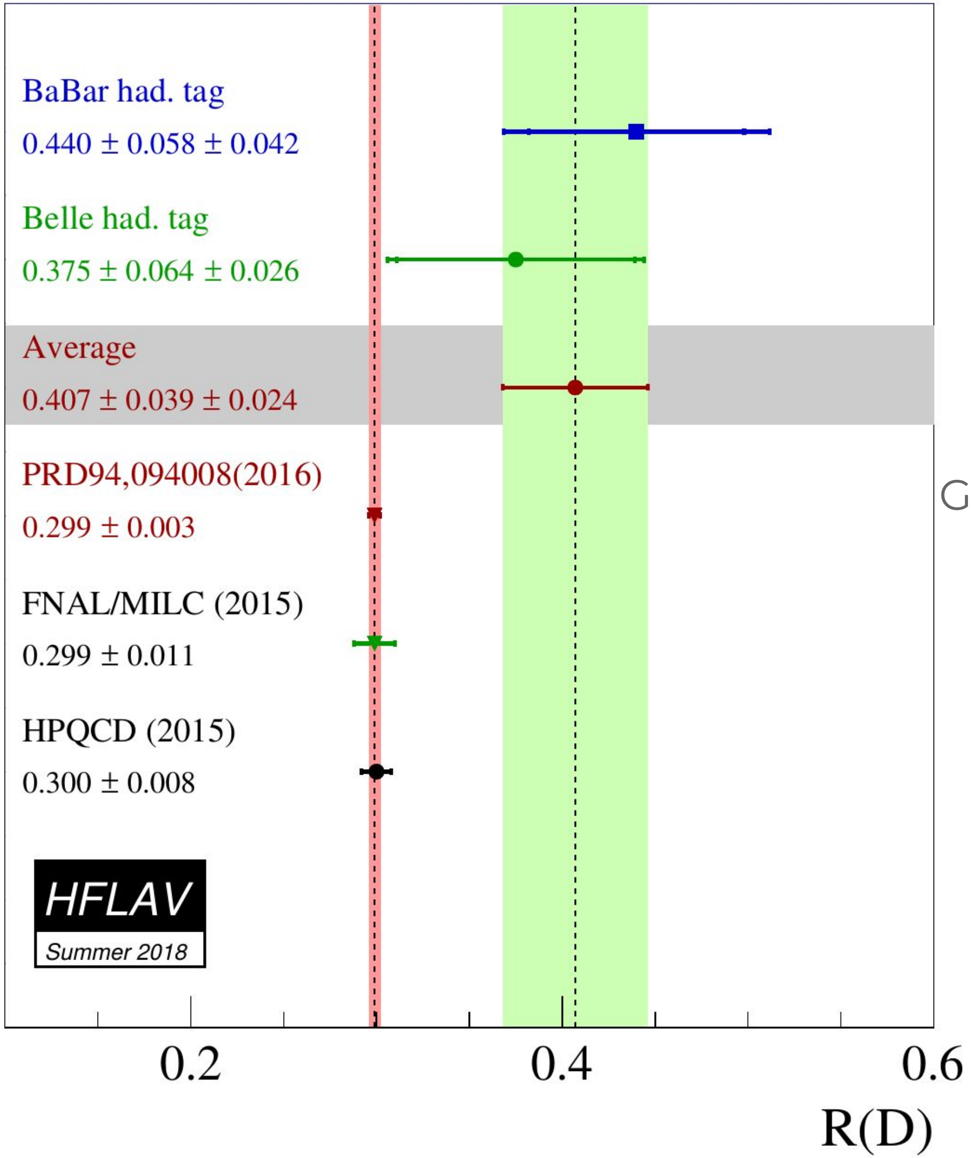
Tension in branching fraction ratios

$$R(D_{(s)}^{(*)}) = \frac{\mathcal{B}(B_{(s)} \to D_{(s)}^{(*)}\tau\overline{\nu}_\tau)}{\mathcal{B}(B_{(s)} \to D_{(s)}^{(*)}\ell\overline{\nu}_\ell)}$$





Branching fraction ratios



$$R(D_{(s)}) = \frac{\mathcal{B}(D_{(s)} \to \tau\nu)}{\mathcal{B}(D_{(s)} \to \ell\nu)}$$

$$R(D) = 0.299(3)$$

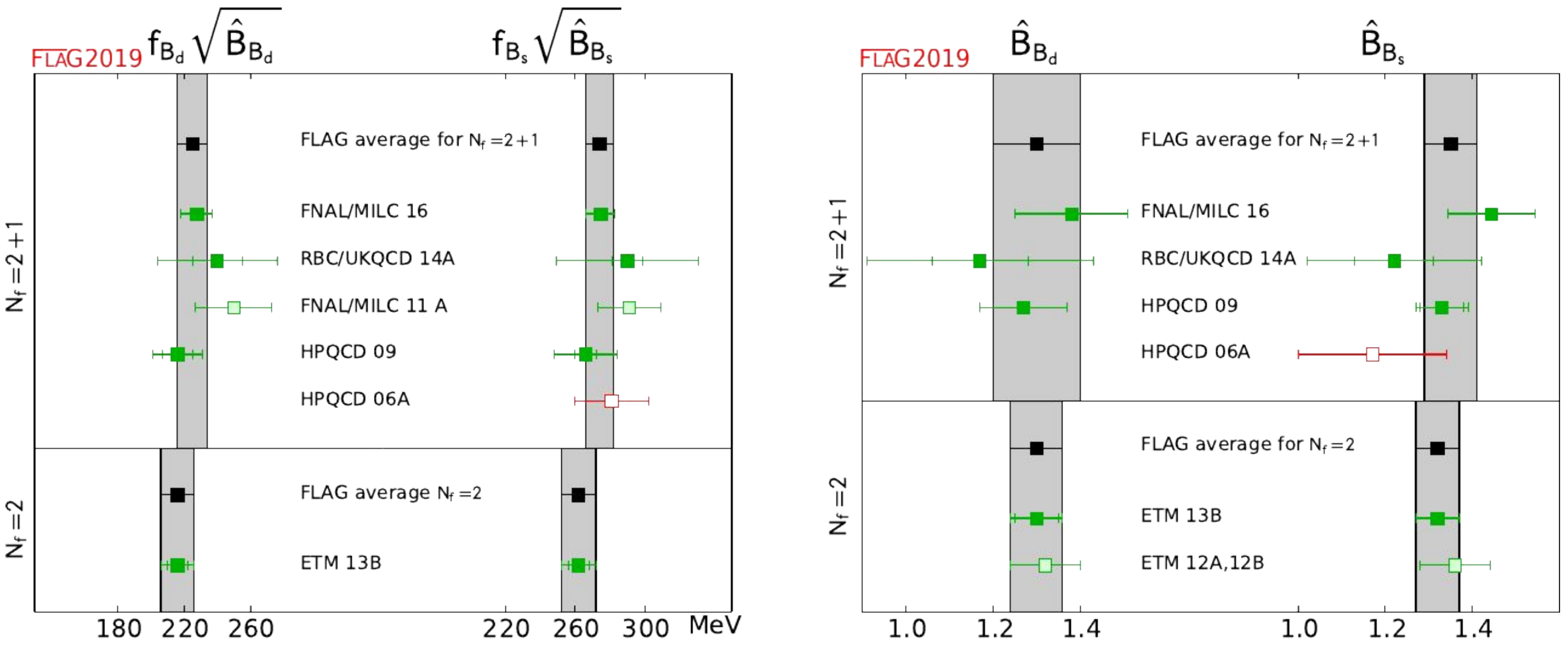
Gambino and Bigi., PRD 94 (2016) 094008

$$R(D_s) = 0.314(6)$$

CJM et al., PRD 95 (2017) 114506

Neutral meson mixing

$$\langle \overline{B}^0_q | \mathcal{O}^q_i(\mu) | B^0_q \rangle = \frac{8 m^2_{B_q}}{3} f^2_{B_q} \widehat{B}_{B_s}(\mu)$$



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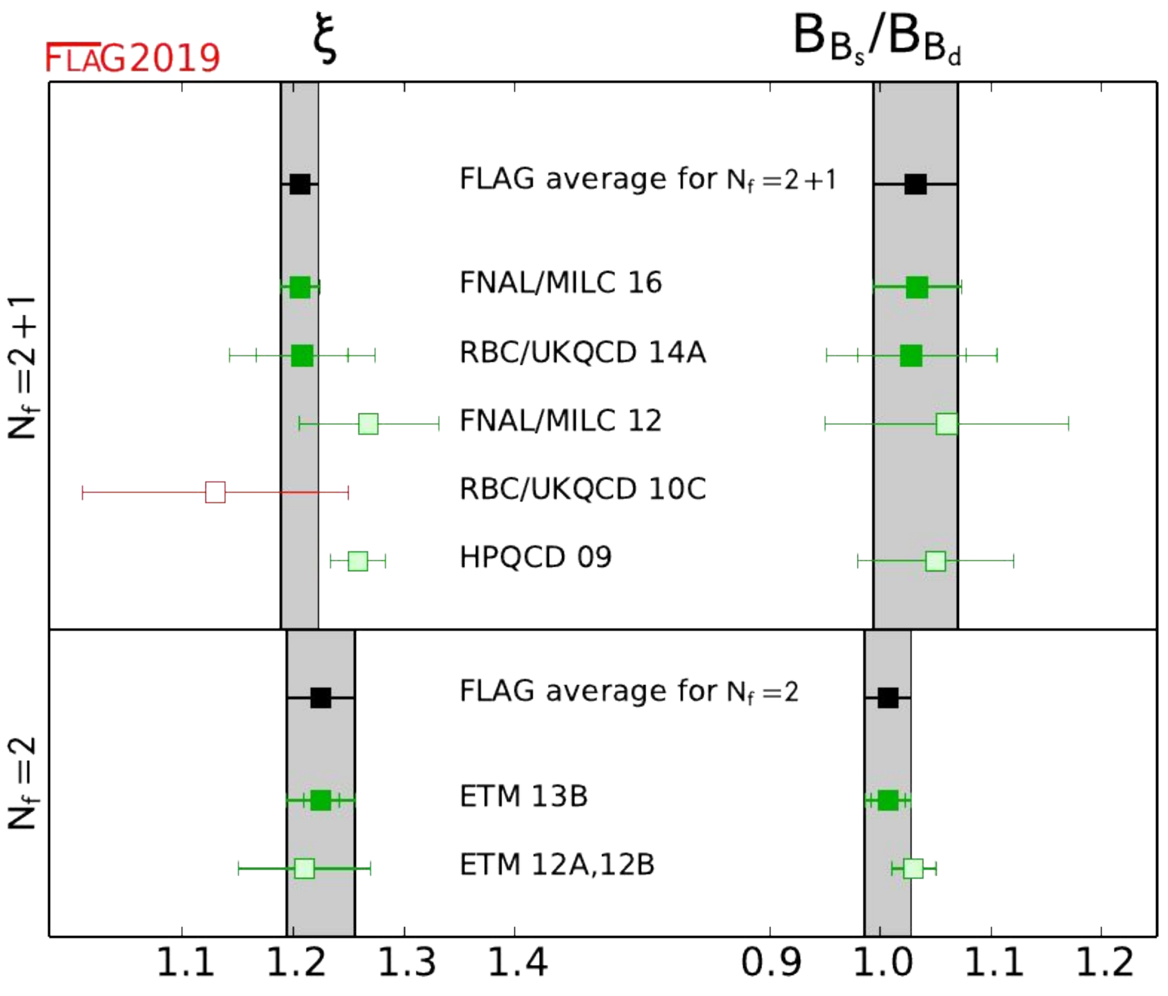
Neutral meson mixing



$$\xi = \frac{f_{B_s}\sqrt{\widehat{B}_{B_s}}}{f_{B_d}\sqrt{\widehat{B}_{B_d}}}$$

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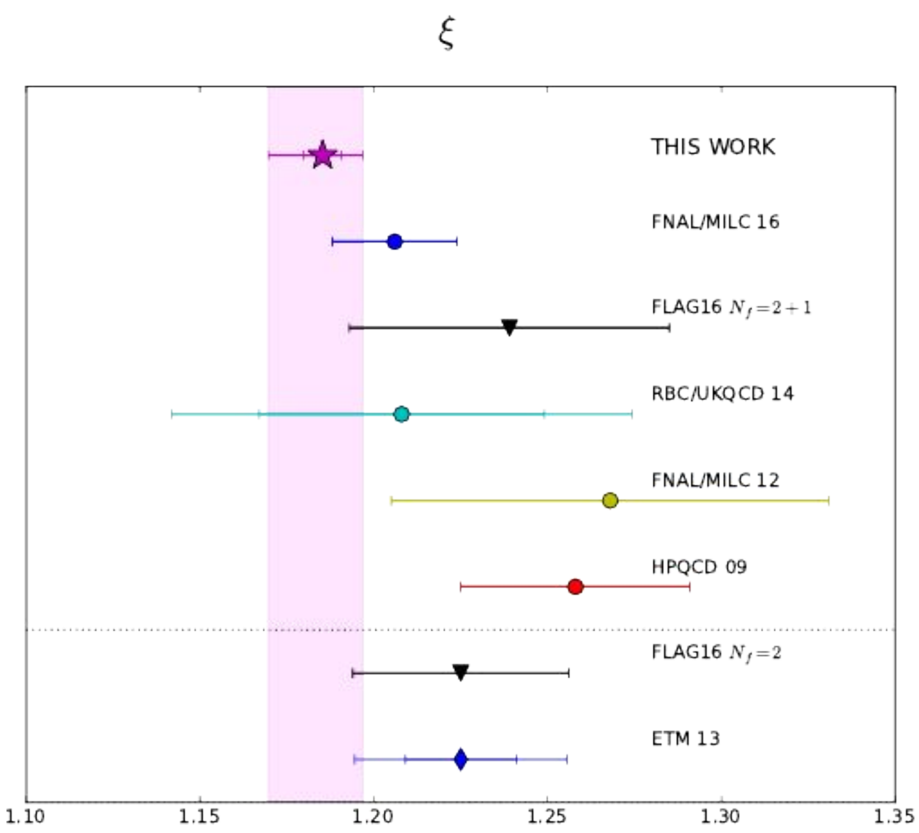
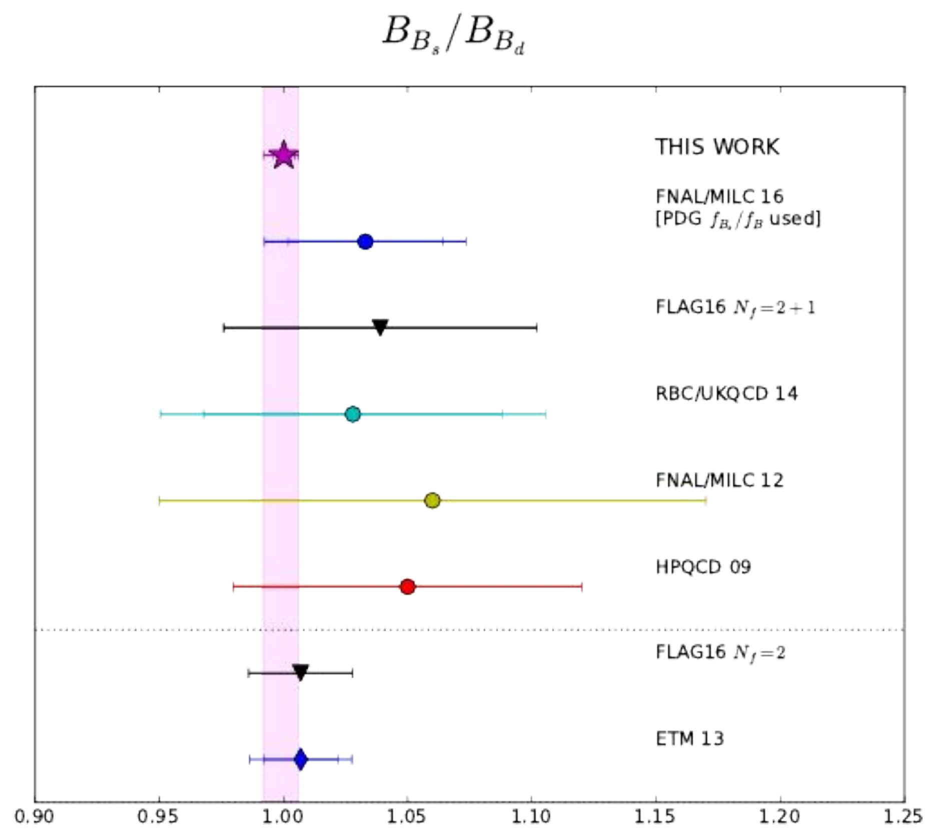
Neutral meson mixing





$$|V_{cd}/V_{cs}| = 0.2148\,(56)_{\rm exp}\,\left({}^{+22}_{-10}\right)_{\rm lat}$$

$$|V_{td}/V_{ts}| = 0.20181\,(41)_{\rm exp}\,\left({}^{+197}_{-266}\right)_{\rm lat}$$

Boyle et al., 1812.08791

Summary

- Decay constants
  - 0.3-0.5% errors for D decays
  - 0.6-0.7% errors for B decays

- Form factor parameterisation challenging and tensions in
  - V_{cb}
  - V_{ub}
  - $R(D^{(*)})$

- Neutral meson mixing still the most challenging
  - Little recent activity for D meson mixing
  - 5-10% errors for B meson bag parameters
  - No published state-of-the-art (2+1+1) results

Outlook

- Next few years will see many more lattice results

- Heavy HISQ (and DWF) results very promising
  - allows entirely nonperturbative current renormalisation
  - should facilitate sub-1% precision

- Anticipating exp. results, B_s decays a real growth industry
  - but further progress really requires experimental data

- Form factor parameterisation remains central issue for V_{qb}

- Moving beyond ~03.-0.5% precision will require
  - isospin breaking effects
  - QED effects

Thank you

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