The SJ Vacuum in de Sitter Spacetime

Theory Canada 14, Vancouver BC

Yasaman K. Yazdi

Work with Sumati Surya and Nomaan X
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UNIVERSITY OF ALBERTA

Outline

- Causal set theory
- The Sorkin-Johnston (SJ) vacuum of a free scalar field
- Applications and properties of the SJ vacuum: static spacetimes, entanglement entropy, etc.
- Vacuum states in de Sitter spacetime
- The SJ vacuum in de Sitter spacetime
- Conclusions and future directions

Causal Set Theory: Spacetime is Fundamentally Discrete [1]

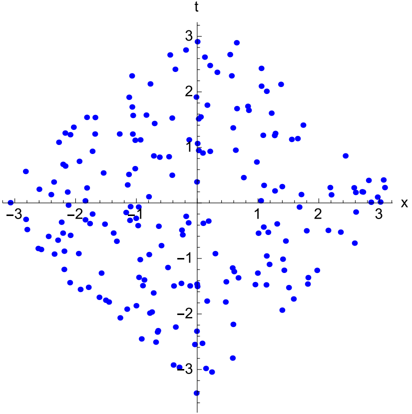
A **causal set** is a locally finite partially ordered set. It is a set $\mathcal{C}$ along with an ordering relation $\preceq$ that satisfy:

- It is reflexive: for all $X \in \mathcal{C}$, $X \preceq X$.
- It is antisymmetric: for all $X, Y \in \mathcal{C}$, $X \preceq Y \preceq X$ implies $X = Y$.
- It is transitive: for all $X, Y, Z \in \mathcal{C}$, $X \preceq Y \preceq Z$ implies $X \preceq Z$.
- And, it is locally finite: for all $X, Y \in \mathcal{C}$, $|I(X, Y)| < \infty$, where $|\cdot|$ denotes cardinality and $I(X, Y)$ is the causal interval defined by $I(X, Y) := \{Z \in \mathcal{C} | X \preceq Z \preceq Y\}$.

[1] Bombelli, L., Lee, J. H., Meyer, D. and Sorkin, R. D., 1987, Space-Time as a Causal Set, Phys. Rev. Lett. 59, 521.

Causal Set Theory

Sprinkling: generates a causal set from a given Lorentzian manifold $\mathcal{M}$, by placing points at random in $\mathcal{M}$ via a Poisson process with "density" ρ, such that $P(N) = \frac{(\rho V)^N}{N!} e^{-\rho V}$.



Lorentz invariant and non-local.

The Sorkin-Johnston Vacuum [2]

The covariant commutation relations are given by the Peierls bracket

$$[\hat{\phi}(x), \hat{\phi}(x')] = \mathrm{i}\Delta(x, x'), \tag{1}$$

where the Pauli-Jordan function is

$$\mathrm{i}\Delta(x, x') \equiv \mathrm{i}(G_R(x, x') - G_A(x, x')), \tag{2}$$

with $G_{R,A}(x, x')$ being the retarded and advanced Green functions.

$$\mathrm{Ker}(\hat{\Box} - m^2) = \overline{\mathrm{Im}(\hat{\Delta})}. \tag{3}$$

Thus the eigenvectors in the image of $\mathrm{i}\hat{\Delta}$ span the full solution space of the KG operator.

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[2] R.D. Sorkin, J. Phys. Conf. Ser. 306 (2011) 012017 [arXiv:1107.0698]. S. P. Johnston (2010) [arXiv:1010.5514].

$i\Delta$ is a self-adjoint operator on a bounded region of spacetime.

Write $i\Delta(x, x')$ in terms of its positive (u_k) and negative (v_k) eigenfunctions:

$$i\Delta(x, x') = \sum_k \left[\lambda_k u_k(x) u_k^\dagger(x') - \lambda_k v_k(x) v_k^\dagger(x')\right]. \qquad (4)$$

Restrict to positive eigenspace to get the Wightman or two-point function in the SJ vacuum:

$$W_{SJ}(x, x') \equiv \text{Pos}(i\Delta) = \sum_k \lambda_k u_k(x) u_k^\dagger(x'). \qquad (5)$$

Some Properties of the SJ Vacuum

- An observer independent vacuum which is unique.
- In static spacetimes, the SJ vacuum is the same one that is picked out by the timelike and hypersurface-orthogonal Killing vector.
- While not necessarily Hadamard itself, a family of Hadamard states can be constructed from it.
- Can be applied to both causal sets and continuum spacetimes.
- Prescription for fermions also exists.
- Is a pure state for a spacetime definition of entanglement entropy (while its restriction to a smaller region is not pure).

de Sitter invariant α-Vacua

α-vacua are a two-real-parameter family of dS invariant vacua. $\alpha = 0$ is special (Hadamard) and is called the Euclidean or Bunch-Davies vacuum[3].

The Wightman function for the Euclidean vacuum in d is given by

$$W_E(x,y) = \frac{\Gamma[h_+]\Gamma[h_-]}{(4\pi)^{d/2}\ell^2\Gamma[\frac{d}{2}]}\,{}_2F_1\left(h_+, h_-, \tfrac{d}{2}; \tfrac{1+Z(x,y)+i\epsilon\,\mathrm{sign}(x^0-y^0)}{2}\right)$$

where $Z(x,y) = \eta_{AB}X^A(x)X^B(y)$, $h_\pm = \frac{d-1}{2} \pm \nu$, $\nu = \ell\sqrt{\left(\frac{d-1}{2\ell}\right)^2 - m^2}$, and ${}_2F_1$ is a hypergeometric function.

It is usually said that there is no known de Sitter invariant Fock vacuum for the massless, minimally coupled theory.

[3] Also known by other names.

Results: 2d massless & massive, $ds^2 = \frac{1}{\cos^2 \tilde{T}} \left(-d\tilde{T}^2 + d\Omega^2_{d-1} \right)$

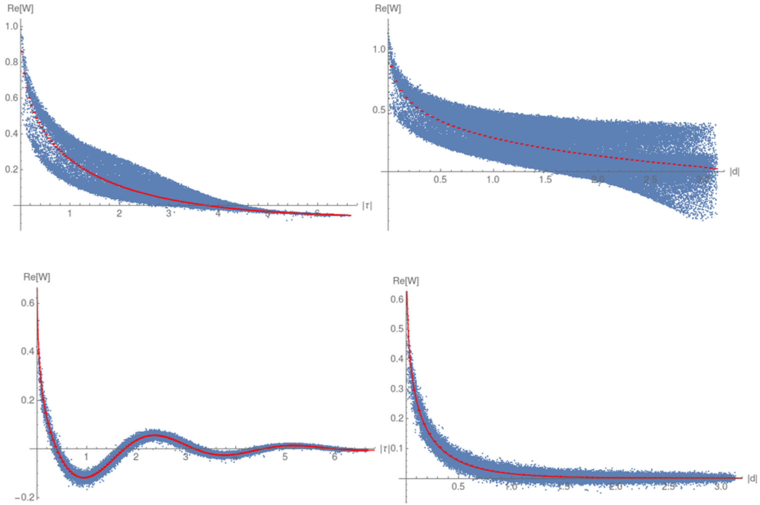

Figure: Upper: massless scatter plot with mean values in red. Lower: m=2.3 scatter plot with W_E in red. Left: causal. Right: spacelike. $T = \tilde{T}_{max} = 1.5$

Results: 4d massless & massive

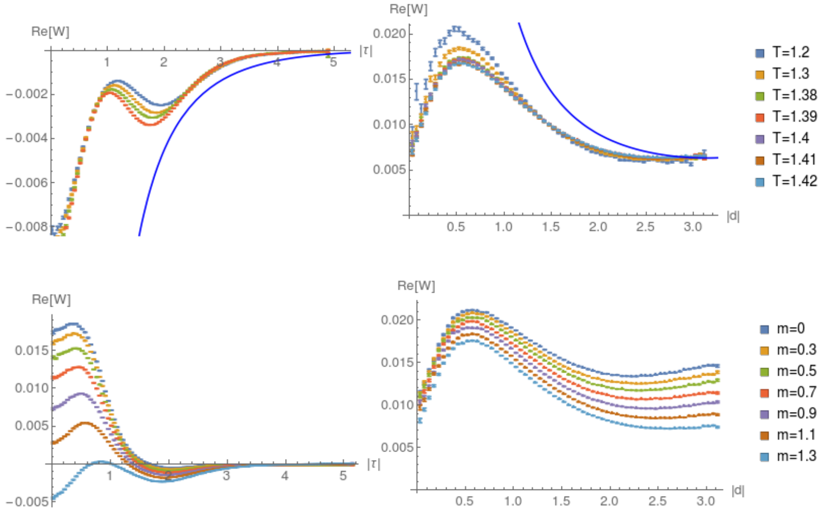

Figure: Upper: m=1.41 mean values with W_E in blue. Lower: T=1.42 mean values. Left: causal. Right: spacelike

Our work strongly suggests that the SJ state is an altogether new de Sitter invariant vacuum in 4d.

- Analytic understanding of the SJ vacuum, perhaps in a corner of the parameter space.
- Spacetime entanglement entropy for de Sitter horizons.
- Early universe phenomenology. Extract observational consequences.