An Introduction to Knot Theory from String Theory

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- **Knot Invariants and M-Theory I: Hitchin Equations, Chern-Simons Theory and Surface Operators,** **K.D, Veronica Errasti Diez, P. Ramadevi and Radu Tatar** **1608.05128.**
- **A Companion to Knot Invariants and M-Theory I: Proofs and Derivations,** **Veronica Errasti Diez,** **1702.07366**
- **Fivebranes and Knots,** **Edward Witten,** **1101.3216**
- **Electric Magnetic Duality and the Geometric Langland Programme,** **Anton Kapustin and Edward Witten,** **hep-th/0604151**
- **Knot Invariants and M-Theory II,** **K.D, Veronica Errasti Diez, K. Gopala Krishna, Rohit Jain, P. Ramadevi and Radu Tatar** **To appear**

Cast of characters

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Veronica Errasti Diez

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- **Discussions and conclusions**

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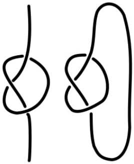

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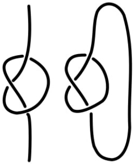

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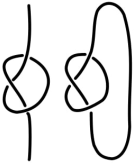

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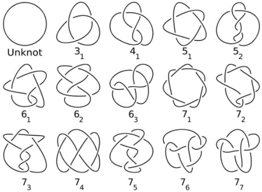

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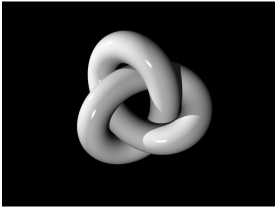

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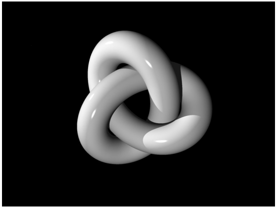

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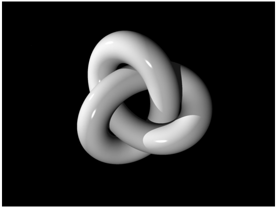

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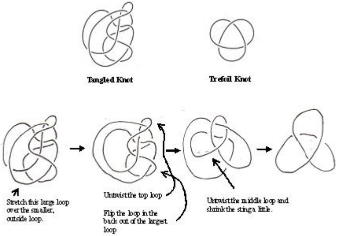

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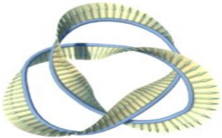

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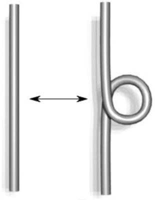

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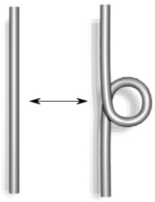

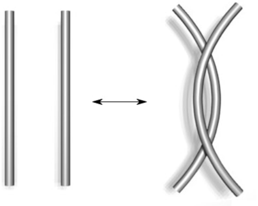

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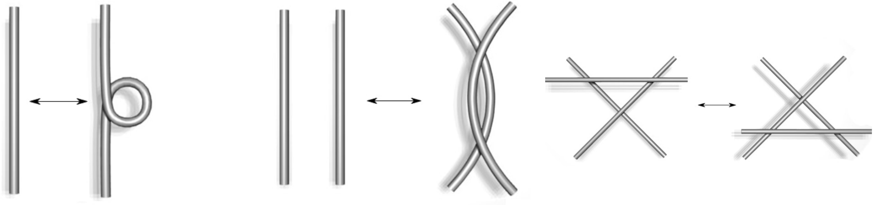

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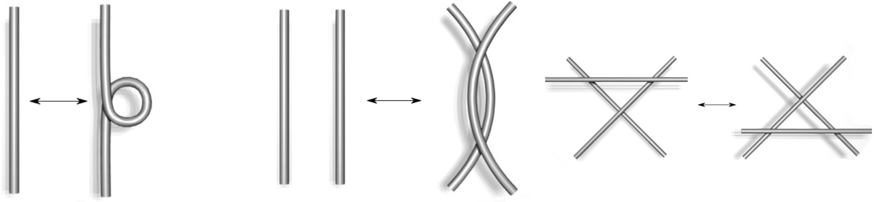

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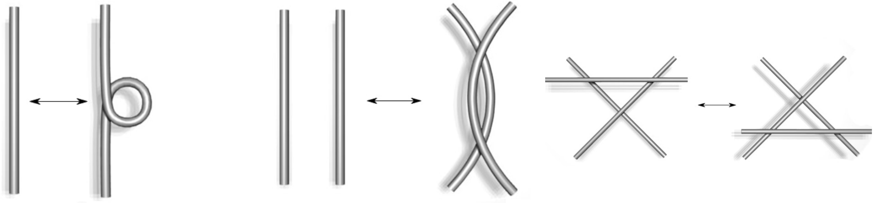

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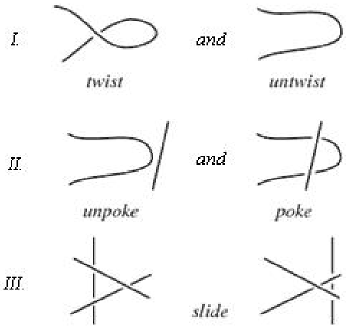

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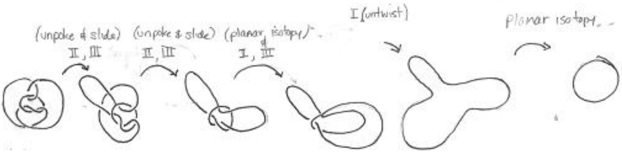

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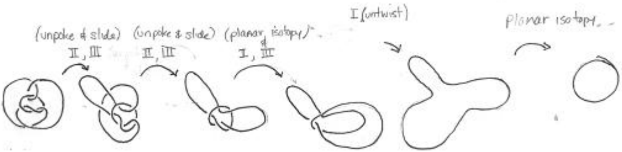

Thus using untwist, poke and slide moves, allows us to see the above simplification!

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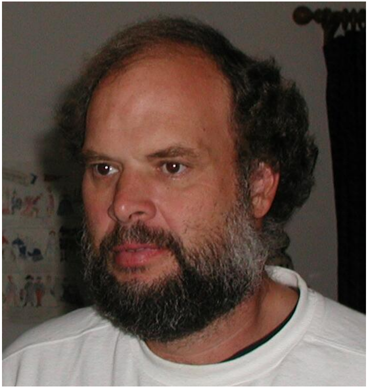

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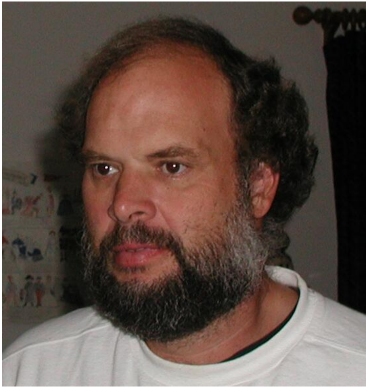

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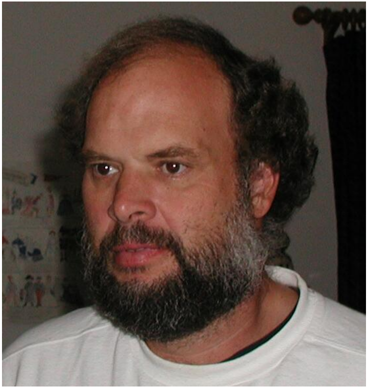

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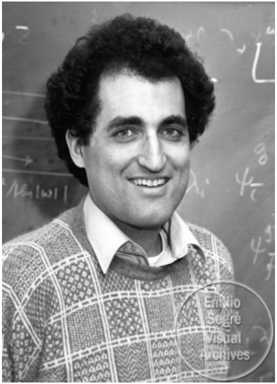

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$$J(\mathbf{K}, R, q) = \langle W(\mathbf{K}, R) \rangle$$

$$= \int \mathcal{D}A \exp\left[ik \int_{\mathbf{R}^3} \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)\right] \mathrm{Tr}_R P \exp\left(\oint_{\mathbf{K}} A\right)$$

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Similarly for the fundamental representation of $SO(N)$, we get the Kauffman polynomials.

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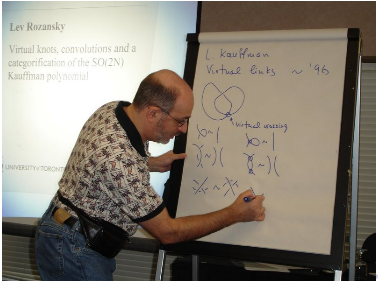

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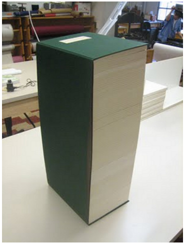

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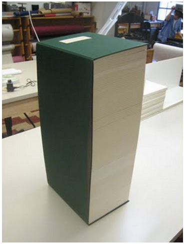

Despite the size, it is an immensely readable paper and discusses many interesting facets of S-duality related to the Euclideanized version of $\mathcal{N} = 4$ supersymmetric YM theory.

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Many questions now arise: What set-up are we talking about? How do we distinguish the knots using instanton numbers? Where is the topological field theory? Why on earth would solutions of certain differential equations have anything to do with knot polynomials?

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This doesn't entail the full Khovanov homology, but is a step towards that direction.

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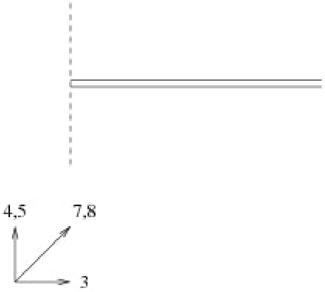

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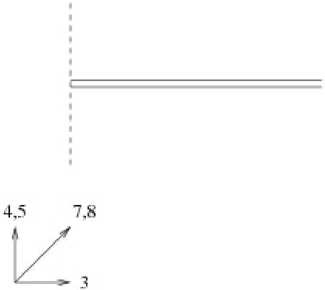

The dotted lines being the NS5-brane and the solid lines are the D3-branes. The intersection is three-dimensional i.e along (x_0, x_1, x_2) directions in Euclidean space.

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You might ask what's the big deal here? While without doing any computations one might have predicted the boundary 3d theory to be of the Chern-Simons kind, but the subtlety is that the gauge field that appears in S_b is not the Chern-Simons gauge field $\mathcal{A}$!

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Note that under twisting, the $\mathcal{N} = 4$ scalar fields action gets a contribution from the intersection region in such a way so as to tag along with the gauge field $\mathcal{A}$ to give us precisely a Chern-Simons action S_b! And that's the miracle!

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The answer turns out to be yes, by dualizing the Witten's set-up to M-theory. Once we insert another parallel NS5-brane at the other end of the D3-branes and dualize this to M-theory, the branes disappear and are converted to geometry in M-theory!

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The story is very detailed, but thankfully straightforward.

After the dust settles, the four-dimensional Hamiltonian is easy to write down. This is given by

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which, as described in [11], can be made by picking the three scalar fields in $\overrightarrow{X}$ and one scalar field from $\overrightarrow{Y}$ (which we take here as φ_3). This means the complex σ field of [11], for our case will become:

$$\sigma \equiv \mathcal{A}_r + i\mathcal{A}_{\phi_1}. \tag{3.157}$$

The Gauss law constraint and the identification of the scalar fields will lead us to compute the Hamiltonian from the total effective action (3.153). Isolating the same scalar $\mathcal{A}_3$, the expression for the Hamiltonian, for the case when $c_2 = 0$ in (3.153), can be expressed as sum of squares of various terms in the following way:

$$\begin{aligned}
\mathcal{H} = \int d^3x \ \mathrm{Tr}\Bigg\{ & \sum_{\alpha=1}^{2} \frac{c_1}{v_3}\left(\sqrt{c_{11}}\mathcal{F}_{\alpha 0} - \sqrt{c_{\alpha 3}}\mathcal{D}_\alpha \mathcal{A}_3\right)^2 + \frac{c_1}{v_3}\left(\sqrt{c_{12}}\mathcal{F}_{\psi 0} - \sqrt{c_{\psi 3}}\mathcal{D}_\psi \mathcal{A}_3\right)^2 \\
& + \frac{c_1}{v_3}\left(\sqrt{c_{0r}}\mathcal{D}_0\mathcal{A}_r - i\sqrt{a_2}[\mathcal{A}_3, \mathcal{A}_r]\right)^2 + \frac{c_1}{v_3}\left(\sqrt{c_{0\phi_1}}\mathcal{D}_0\mathcal{A}_{\phi_1} - i\sqrt{a_4}[\mathcal{A}_3, \mathcal{A}_{\phi_1}]\right)^2 \\
& + \frac{c_1}{v_3}\left(s^{(1)}c_{\psi r}(\mathcal{D}_\psi\mathcal{A}_r)^2 + s^{(2)}c_{\psi\phi_1}(\mathcal{D}_\psi\mathcal{A}_{\phi_1})^2 + t^{(1)}c_{\beta r}(\mathcal{D}_\beta\mathcal{A}_r)^2 + t^{(2)}c_{\beta\phi_1}(\mathcal{D}_\beta\mathcal{A}_{\phi_1})^2\right) \\
& + \sum_{k=1}^{3}\left(\sqrt{b_{0k}}\mathcal{D}_0\varphi_k - i\sqrt{c_{3k}}[\mathcal{A}_3, \varphi_k]\right)^2 + \frac{c_1 c_{03}}{v_3}\left(\mathcal{D}_0\mathcal{A}_3\right)^2 + \sum_{\alpha,\beta=1}^{2}\Bigg(\sqrt{\frac{c_1 c_{11}}{2v_3}}\mathcal{F}_{\alpha\beta} \\
& + \sqrt{\frac{c_1 c_{\psi r}}{v_3}}s^{(1)}_{\alpha\beta}\epsilon_{\alpha\beta\psi r}\mathcal{D}_\psi\mathcal{A}_r + \sqrt{\frac{c_1 c_{\psi\phi_1}}{v_3}}s^{(2)}_{\alpha\beta}\epsilon_{\alpha\beta\psi\phi_1}\mathcal{D}_\psi\mathcal{A}_{\phi_1} + \sum_{\delta=1}^{3}\sum_{k=1}^{3}\sqrt{b_{\delta k}}\epsilon_{\alpha\beta}\cdot m^{(1)}_{\delta k}\mathcal{D}_\delta\varphi_k \\
& - \sum_{k,l} i g^{(1)}_{\alpha\beta kl}\sqrt{d_{kl}}\,[\varphi_k, \varphi_l] - \sum_{k=1}^{3} i\Big(g^{(2)}_{\alpha\beta k}\sqrt{c_{rk}}\,[\mathcal{A}_r, \varphi_k] + g^{(3)}_{\alpha\beta k}\sqrt{c_{\phi_1 k}}\,[\mathcal{A}_{\phi_1}, \varphi_k]\Big) \\
& - i g^{(4)}_{\alpha\beta}\sqrt{\frac{c_1 a_1}{v_3}}\,[\mathcal{A}_r, \mathcal{A}_{\phi_1}]\Bigg)^2 + \frac{(\mathbf{Q_E} + \mathbf{Q_M})\,\delta^3 x}{\dim\, G} + \sum_{\alpha=1}^{2}\Bigg(\sqrt{\frac{c_1 c_{12}}{2v_3}}\mathcal{F}_{\alpha\psi} + \sqrt{\frac{c_1 c_{\beta r}}{v_3}}t^{(1)}_\alpha\epsilon_{\alpha\psi\beta r}\mathcal{D}_\beta\mathcal{A}_r \\
& + \sqrt{\frac{c_1 c_{\beta\phi_1}}{v_3}}t^{(2)}_\alpha\epsilon_{\alpha\psi\beta\phi_1}\mathcal{D}_\beta\mathcal{A}_{\phi_1} + \sum_{\delta=1}^{3}\sum_{k=1}^{3}\sqrt{b_{\delta k}}\epsilon_{\alpha\psi}\cdot m^{(2)}_{\delta k}\mathcal{D}_\delta\varphi_k - \sum_{k,l} i h^{(1)}_{\alpha\psi kl}\sqrt{d_{kl}}\,[\varphi_k, \varphi_l] \\
& - \sum_{k=1}^{3} i\Big(h^{(2)}_{\alpha\psi k}\sqrt{c_{rk}}\,[\mathcal{A}_r, \varphi_k] + h^{(3)}_{\alpha\psi k}\sqrt{c_{\phi_1 k}}\,[\mathcal{A}_{\phi_1}, \varphi_k]\Big) - i h^{(4)}_{\alpha\psi}\sqrt{\frac{c_1 a_1}{v_3}}\,[\mathcal{A}_r, \mathcal{A}_{\phi_1}]\Bigg)^2 \\
& + \sum_{k,l} q^{(1)}_{kl} d_{kl}\ [\varphi_k, \varphi_l]^2 + \sum_{k=1}^{3}\sum_{\gamma=2}^{3} q^{(\gamma)}_k c_{y_\gamma k}\ [\mathcal{A}_{y_\gamma}, \varphi_k]^2 + \frac{q^{(4)}c_1 a_1}{v_3}\,[\mathcal{A}_r, \mathcal{A}_{\phi_1}]^2\Bigg\},
\end{aligned} \tag{3.158}$$

where $\mathbf{Q_E}$ and $\mathbf{Q_M}$ are the electric and the magnetic charges respectively, which will be determined later; $\dim\, G$ is the dimension of the group; and $\delta \equiv (\alpha, \psi)$, $(y_2, y_3) \equiv (r, \phi_1)$. Most of coefficients appearing in (3.158) have been determined

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$$\mathcal{D}_0\mathcal{A}_3 = 0, \quad \left(\sqrt{b_{0k}} - \sqrt{c_{3k}}\right)^2 [\mathcal{A}_3, \varphi_k]^2 = 0$$
$$\left(\sqrt{c_{11}} - \sqrt{c_{\alpha 3}}\right)^2 (\mathcal{D}_\alpha \mathcal{A}_3)^2 = 0, \quad \left(\sqrt{c_{12}} - \sqrt{c_{\psi 3}}\right)^2 (\mathcal{D}_\psi \mathcal{A}_3)^2 = 0$$
$$\left(\sqrt{c_{0r}} - \sqrt{a_2}\right)^2 [\mathcal{A}_3, \mathcal{A}_r]^2 = 0, \quad \left(\sqrt{c_{0\phi_1}} - \sqrt{a_4}\right)^2 [\mathcal{A}_3, \mathcal{A}_{\phi_1}]^2 = 0.$$

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$$c_{11}(\theta) = R_3 \ \sec\ \theta \int_0^\infty dr\ e^{2\phi_0} \sqrt{\frac{F_1 \widetilde{F}_2 F_3}{\widetilde{F}_2 - F_3}}\ \ln\left|\frac{\sqrt{\widetilde{F}_2} + \sqrt{\widetilde{F}_2 - F_3}}{\sqrt{\widetilde{F}_2} - \sqrt{\widetilde{F}_2 - F_3}}\right|$$

$$c_{\alpha 3}(\theta) = R_3 \ \sec\ \theta \int_0^\infty dr\ \frac{e^{2\phi_0}}{H_2} \sqrt{\frac{F_1 \widetilde{F}_2 F_3}{\widetilde{F}_2 - F_3}}\ \ln\left|\frac{\sqrt{\widetilde{F}_2} + \sqrt{\widetilde{F}_2 - F_3}}{\sqrt{\widetilde{F}_2} - \sqrt{\widetilde{F}_2 - F_3}}\right|.$$

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They are equal if and only if $H_2 = 1$, where $H_2 \equiv H_2(\widetilde{F}_1, F_2, F_3, F_4)$ is another warp factor.

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One can easily check that they are identical if and only of $H_2 = 1$. In fact one may check all the minimizing equations and find similar conclusion! Thus these equations are exactly solved with $H_2 = 1$!

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However the challenge is to get the boundary topological theory after twisting. Can we get this right?

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$$\begin{aligned} S_{bnd} &= k\int_{\mathbf{W}} \mathrm{Tr}\left(\mathcal{A}\wedge d\mathcal{A} + \frac{2i}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right) \\ &+ \int_{\mathbf{W}} \mathrm{Tr}\left\{2d_1\mathcal{F}\wedge\phi + \frac{2i}{3}\left(\frac{d_1^3}{k^2}\right)\phi\wedge\phi\wedge\phi + \left(\frac{d_1^2}{k}\right)\phi\wedge d_{\mathcal{A}}\phi\right\} \end{aligned}$$

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&= k \int_{\mathbf{W}} \mathrm{Tr}\left\{ \left[\mathcal{A} + \left(\frac{d_1}{k}\right)\phi\right] \wedge d\left[\mathcal{A} + \left(\frac{d_1}{k}\right)\phi\right] \right. \\
&+ \left. \frac{2i}{3}\left[\mathcal{A} + \left(\frac{d_1}{k}\right)\phi\right] \wedge \left[\mathcal{A} + \left(\frac{d_1}{k}\right)\phi\right] \wedge \left[\mathcal{A} + \left(\frac{d_1}{k}\right)\phi\right] \right\},
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where k and d_1 are determined from the warp-factors F_i appearing in our M-theory set-up discussed earlier.

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Conclusion and discussion

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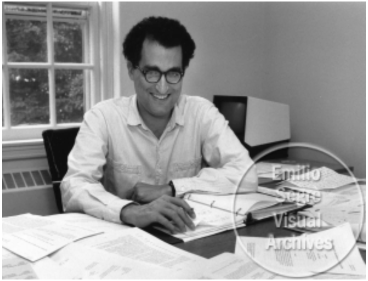

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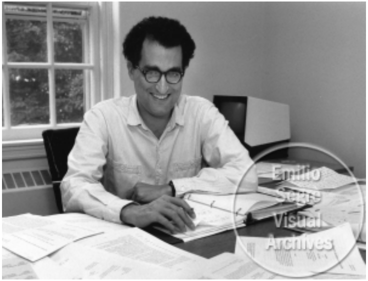

which has lead us to make further connections to the geometric Langland programme, Khovanov-Rozanski homology, opers and conformal blocks (that we did not discuss here).

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