THERMODYNAMIC PARAMETERS OF A BOULWARE-DESER BLACK HOLE FROM FLUID-GRAVITY CORRESPONDENCE

Dylan Sutherland[1*], S. Shankaranarayanan[2], Saurya Das[1]

1 = University of Lethbridge
2 = Indian Institute of Technology Bombay
* = Speaker

PROBLEMS

- Many prospective theories of Quantum Gravity from several branches of Physics yet, test by which we can falsify them.
- The Schwarzchild Black Hole showed promise.
- Its failure must’ve been due to its universality.

FLUID GRAVITY CORRESPONDENCE

- First developed independently first by Damour then by Thorne et al, both in the early 1980’s.

- Damour showed that Einstein’s equations, projected on a Black Hole event horizon, took the form of the Navier-Stokes equation for a relativistic 2D fluid residing on the horizon.

FLUID/GRAVITY CORRESPONDENCE CONT.

When projected onto an event horizon surface, Einstein's equations take the form:

$$\frac{D\Pi_A}{dt} = -\frac{\partial}{\partial x^A}\left(\frac{\kappa}{8\pi}\right) + 2\frac{1}{16\pi}\sigma_{A;B}^{B} - \left(\frac{D-3}{D-2}\right)\frac{1}{8\pi}\frac{\partial\theta}{\partial x^A} - l^a T_{aA}$$

Compare with the Navier-Stokes Equation from fluid mechanics:

$$\rho\frac{\partial\vec{v}}{\partial t} + \rho(\vec{v}\cdot\nabla)\vec{v} = -\nabla P + \eta\nabla^2\vec{v} + \left(\zeta + \frac{\eta}{3}\right)\nabla(\nabla\cdot\vec{v})$$

As can be seen, the first equation shares the same form as the Navier-Stokes Equation for a fluid with the bulk viscosity as written in blue and with following constraints:

$$P = k_B\frac{T_H}{4} \qquad E = \left(\frac{D-2}{D-3}\right)\frac{A}{4}k_B T_H$$

OUR APPROACH

- The Damour-Navier-Stokes equation and the results from Thorne are a great start but they are strictly macroscopic and their application is limited.

- To improve upon this this, we suggest working from the Fluctuation Dissipation of the microscopic degrees of freedom. Which, if functional, would provide a wider range of applicability

CHARACTERISTICS OF THE FLUID

1. Black Hole event horizons seem to have a critical temperature due to its properties of universality.
2. For large horizon radii, the fluid degrees of freedom remain in their ground state.

Implies:

$$E = N_C \alpha_C T + N_N \alpha_N T$$

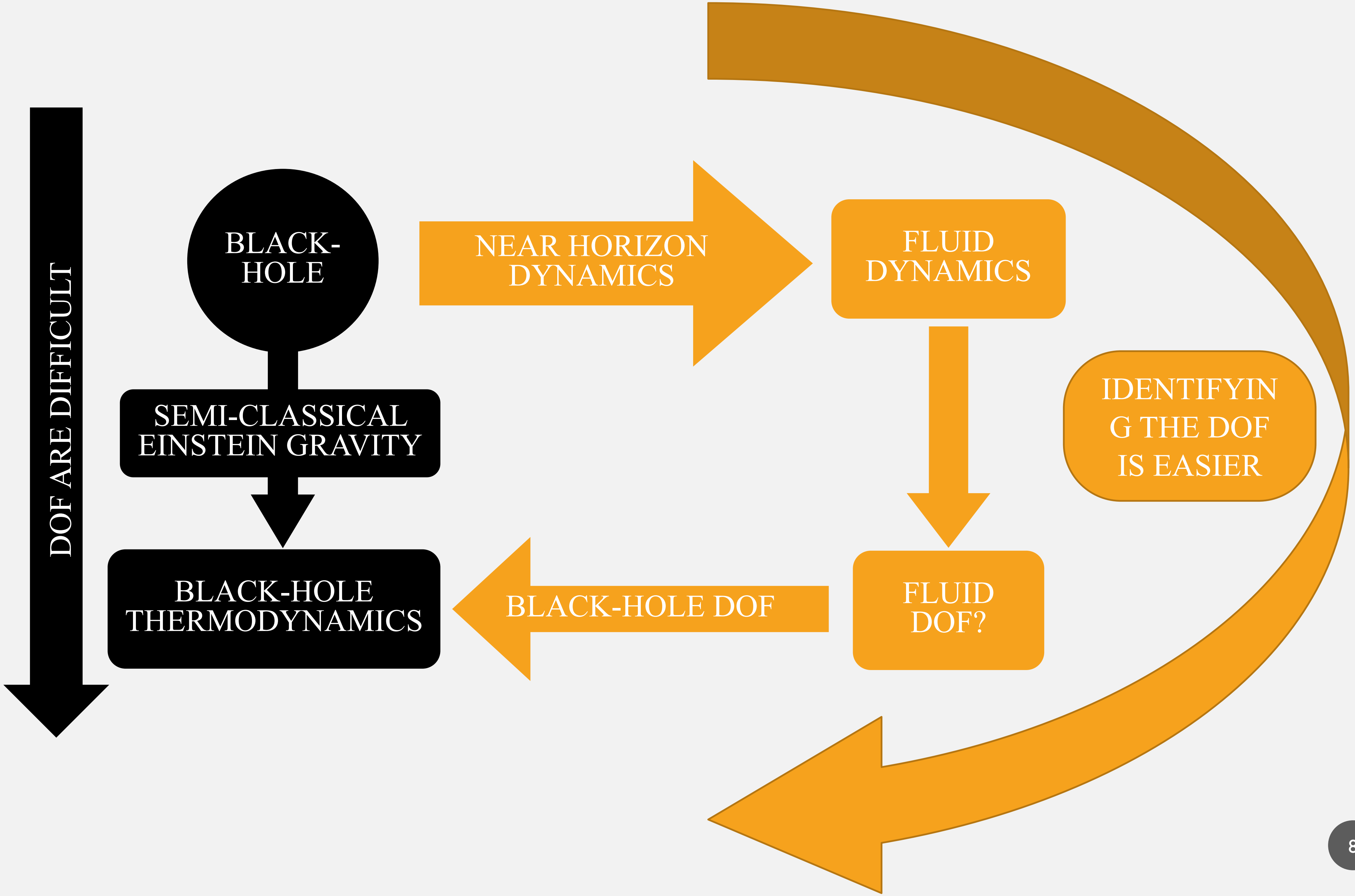
$$N_C \alpha_C + N_N \alpha_N = \frac{A}{3}$$

BOSE GASES

- Comprised of bosons with integer spin.
- Follows Bose-Einstein Statistics which governs gases comprised of non-interacting bosons.
- Forms a Bose-Einstein Condensate wherein all particles occupy the lowest energy level.

Examples:

- Photon Gas
- Helium-4


DOF ARE DIFFICULT
BLACK-HOLE
SEMI-CLASSICAL EINSTEIN GRAVITY
BLACK-HOLE THERMODYNAMICS
NEAR HORIZON DYNAMICS
FLUID DYNAMICS
FLUID DOF?
BLACK-HOLE DOF
IDENTIFYING THE DOF IS EASIER

GREEN-KUBO IN GENERAL RELATIVITY

$$\zeta = \left(\frac{1}{n}\right) \frac{1}{Ak_B T} \int_{-\infty}^{\infty} dt \sum_a \sum_b \langle J^{aa}(0) J^{bb}(t) \rangle$$

This yields:

$$\zeta = -\frac{P^2 A}{k_B T} \frac{\gamma^2 \beta^2}{\omega} \langle \delta N(0)^2 \rangle$$

LANCZOS-LOVELOCK GRAVITY

$$S_{LL} = \int d^6x[R + \lambda(R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma} + R_{\mu\nu\rho\sigma})]$$

GR $\qquad$ $extra$

Second Order Equations of Motion

THE BOULWARE-DESER BLACK HOLE

$$ds^2 = -V^2(r)dt^2 + V^{-2}(r)dr^2 + r^2 d\Omega_4^2$$

$$V^2(r) = 1 + \frac{r^2}{4\alpha} + \sigma \frac{r^2}{4\alpha}\sqrt{1 + \frac{16\alpha M}{r^4} + \frac{4\alpha\Lambda}{3}}$$

Our Interest

Thermodynamic Quantities of the Boulware-Deser Black Hole

Horizon Radius:
$$r_h = \left(\frac{\omega + \sqrt{\omega^2 + 4\lambda^3}}{2}\right)^{1/3} - \left(\frac{2\lambda^3}{\omega + \sqrt{\omega^2 + 4\lambda^2}}\right)^{1/3}$$

$$A = \frac{8}{3}\pi^2 r_h^4 \qquad \omega^{2/3} = \left(\frac{3\Omega_4^{1/4}}{8\pi^2}\right)^{2/3} A_{GR}^{1/2}$$

After a Taylor expansion in $\lambda/_{\omega^{2/3}}$, keeping 1st order terms:

$$A \approx A_{GR} - 4\lambda\Omega_4 \left(\frac{3\Omega_4^{1/4}}{8\pi^2}\right) A_{GR}^{1/2}$$

Area

Similarly,
$$E = E_{GR} + \frac{3\lambda}{2\pi}\Omega_4^{3/4} A^{1/4}$$
Energy

$$P = P_{GR} + \frac{6\lambda}{16\pi}\left(\frac{\Omega_4}{A}\right)^{-3/4}$$
Pressure

$$T \approx T_{GR} - \frac{15\lambda}{2\pi}\left(\frac{A}{\Omega_4}\right)^{-3/4}$$
Temperature

GREEN-KUBO IN LANCZOS-LOVELOCK

$$\zeta = \left(\frac{1}{n}\right)\frac{1}{Ak_BT}\int_{-\infty}^{\infty} dt \sum_a \sum_b \langle J^{aa}(0)J^{bb}(t)\rangle$$

This yields:

$$\zeta = -\frac{P^2A}{k_BT}\Big[\frac{\gamma_C^2\beta_C^2}{\omega_C}\langle \delta N_C(0)^2\rangle + \frac{\lambda^2\Gamma^2}{4}\frac{\gamma_N^2\beta_N^2}{\omega_N}\langle \delta N_N(0)^2\rangle\Big]$$

REVIEW

Black Hole Horizon ⟺ Perfect Fluid

Black Hole Thermodynamics ⟺ Fluid Mechanics

Microscopic DOF ⟺ Macroscopic Characteristics

We test this correspondence for higher dimensional Black Holes

CONCLUSION AND FUTURE WORK

- The horizon of a BD Black Hole seems lend itself to a fluid model, differing from the Schwarzchild Black Hole in that the fluid is not strictly a BEC; there are non-condensed elements.

- Of course, our job is not done we have yet to calculate the Bulk Viscosity exactly. We hope that this will provide an adequate test for Quantum Gravity theories.

ACKNOWLEDGEMENTS

We would like to thank Mitacs, the Indian Institute of Technology Bombay, the University of Lethbridge, and the Natural Sciences and Engineering Research Council

REFERENCES

1. Swastik Bhattacharya and S. Shankaranarayanan, Physical Review, D 93, 064030 (2016).
2. J. L. L´opez, Swastik Bhattacharya, and S. Shankaranarayanan, Physical Review, D 94, 024029 (2016).
3. S. Shankaranarayanan, Gravity and the Quantum, Fundamental Theories of Physics 187, (2017).

CALCULATING BULK VISCOSITY IN GR

$$\zeta = \left(\frac{1}{n}\right)\frac{1}{Ak_BT}\int_{-\infty}^{\infty} dt \sum_a \sum_b \langle J^{aa}(0)J^{bb}(t)\rangle$$

$$\zeta = \frac{1}{Ak_BT}\int_{\infty}^{-\infty} dt \Big\langle F_{Th}^{(a)}(t), F_{Th}^{(b)}(0)\Big\rangle \theta(-t)$$

With $F_{Th} = P\delta A(t)\theta(-t)$,

$$\zeta = \frac{P^2 A}{k_BT}\int_{\infty}^{-\infty} dt \Big\langle \delta A_{GR}(t), \delta A_{GR}(0)\Big\rangle \theta(-t)$$

CALCULATING BULK VISCOSITY IN GR CONT.

Using the constraints shown earlier,

$$\zeta = \frac{P^2 A}{k_B T} \int_{\infty}^{-\infty} dt \gamma_C^2 \beta_C^2 \langle \delta N_C(t), \delta N_C(0) \rangle \theta(-t)$$

Finally, taking a long wavelength limit,

$$\zeta = -\frac{P^2 A}{k_B T} \frac{\gamma^2 \beta^2}{\omega} \langle \delta N(0)^2 \rangle$$

DAMOUR-NAVIER-STOKES EQUATION

$$R_{ab} - \frac{1}{2}g_{ab} + \Lambda g_{ab} = 8\pi T_{ab}$$

Geodesic Equation: $l^a \nabla_a l_b = \kappa l_b$

With coordinates: $l = \partial_t + v^A \partial_A \qquad l^a = (1,\, v^A,\, 0)$

Thus, the metric becomes:

$$ds^2 = q_{AB}(dx^A - v^A dt)(dx^B - v^B dt)$$

$$q_{ab} = g_{ab} + l_a k_a + l_b k_a$$

DAMOUR-NAVIER-STOKES EQUATION

The contracted Codazzi Equation can be written:

$$R_{mn} l^m q_a^n = \left(\frac{1}{2} g_{ab} - \Lambda g_{ab} + 8\pi T_{ab} \right) l^m q_a^n$$

Which becomes:

$$8\pi T_{mA} l^m = (\partial_0 + v^B \partial_B)\omega_A + \partial_B \sigma_A^B - \partial_A \kappa - \frac{D-3}{D-2} \partial_A \theta$$

Equivalently:

$$\frac{D\Pi_A}{dt} = -\frac{\partial}{\partial x^A}\left(\frac{\kappa}{8\pi}\right) + 2\frac{1}{16\pi}\sigma_{A;B}^B - \left(\frac{D-3}{D-2}\right)\frac{1}{8\pi}\frac{\partial \theta}{\partial x^A} - l^a T_{aA}$$

CALCULATING BULK VISCOSITY IN LL

$$\zeta = \left(\frac{1}{n}\right)\frac{1}{Ak_BT}\int_{-\infty}^{\infty} dt \sum_a \sum_b \langle J^{aa}(0)J^{bb}(t)\rangle$$

$$\zeta = \frac{1}{Ak_BT}\int_{\infty}^{-\infty} dt \Big\langle F_{Th}^{(a)}(t), F_{Th}^{(b)}(0)\Big\rangle \theta(-t)$$

With $F_{Th} = P\delta A(t)\theta(-t)$,

$$\zeta = \frac{P^2A}{k_BT}\int_{\infty}^{-\infty} dt \Big[\Big\langle \delta A_{GR}(t), \delta A_{GR}(0)\Big\rangle \theta(-t) + \frac{\lambda^2\Gamma^2}{4}\Big\langle A_{GR}^{-1/2}\delta A_{GR}(t), A_{GR}^{-1/2}\delta A_{GR}(0)\Big\rangle \theta(-t)\Big]$$

CALCULATING BULK VISCOSITY IN LL CONT.

Using the constraints shown earlier,

$$\zeta = \frac{P^2 A}{k_B T} \int_{\infty}^{-\infty} dt \Big[\gamma_C^2 \beta_C^2 \langle \delta N_C(t), \delta N_C(0) \rangle \theta(-t) + \frac{\lambda^2 \Gamma^2}{4} \gamma_N^2 \beta_N^2 \langle \delta N_N(t), \delta N_N(0) \rangle \theta(-t) \Big]$$

Finally, taking a long wavelength limit,

$$\zeta = \frac{P^2 A}{k_B T} \Big[\frac{\gamma_C^2 \beta_C^2}{\omega_C} \langle \delta N_C(0)^2 \rangle + \frac{\lambda^2 \Gamma^2}{4} \frac{\gamma_N^2 \beta_N^2}{\omega_N} \langle \delta N_N(0)^2 \rangle \Big]$$