Nuclear medium effects on structure functions

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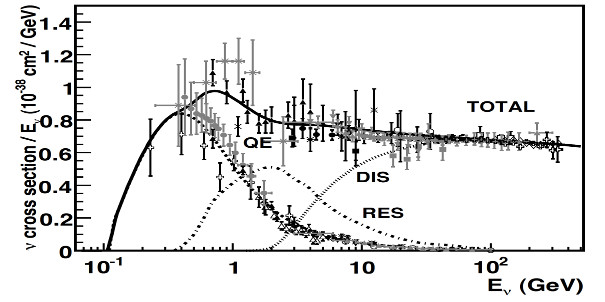
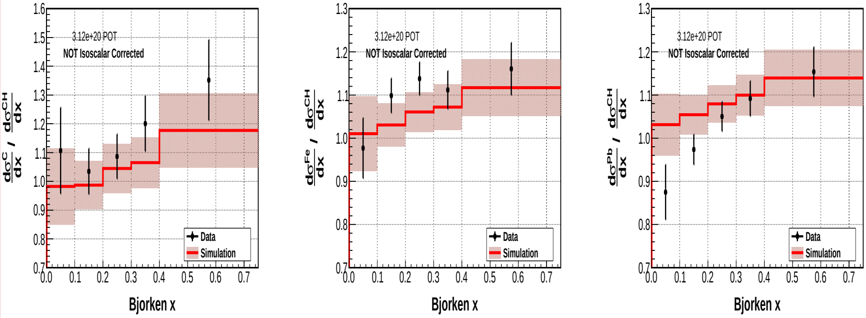
Few GeV region

Experiments like MINERνA, MicroBooNE, NOvA, DUNE....are using heavy nuclear targets. Neutrino energies involved in these experiments are of a few GeV.



It is important to understand nucleon dynamics and reduce the cross section uncertainty($\sim$20-25%) which is contributing to the systematic errors.

J. Mousseau et al. (MINERνA Collaboration) Phys. Rev. D 93, 071101(2016)



Deep Inelastic scattering(DIS)

General process for the deep inelastic scattering is

$$l(k)+N(p)\longrightarrow l(k')+X(p'), \quad l=e^{\pm},\mu^{\pm},\nu_l,\bar{\nu}_l,\ N=n,p$$

Kinematics(Nucleon in the rest frame)

$$Q^2=-q^2=-(k-k')^2=4EE'\sin^2\frac{\theta}{2}$$
$$M^2=p^2$$
$$\nu=p.q=M(E-E')$$
$$x=\frac{Q^2}{2M\nu}=\frac{Q^2}{2p.q}=\frac{Q^2}{2MEy}$$
$$y=\frac{p.q}{p.k}=1-\frac{E'}{E}$$
$$W^2=M^2+2p.q-Q^2$$
$$\frac{d^2\sigma^N}{d\Omega' dE'}=\frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}W^N_{\alpha\beta}$$

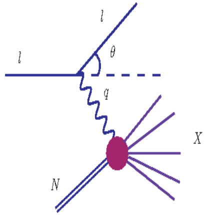


Figure: Deep Inelastic Scattering

In a nuclear medium for weak interactions the expression for the cross section is written as:

$$\frac{d^2\sigma^A}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2 - m_W^2}\right)^2 L^{\alpha\beta} W_{\alpha\beta}^A$$

$$\begin{aligned} W_{\alpha\beta}^A &= \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W_1^A + \frac{1}{M_A^2}\left(p_\alpha - \frac{p.q}{q^2}\, q_\alpha\right)\left(p_\beta - \frac{p.q}{q^2}\, q_\beta\right) W_2^A \\ &- \frac{i}{2M_A^2}\epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma W_3^A \end{aligned}$$

Local Density Approximation

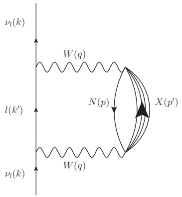
In the local density approximation reaction takes place at a point r, lying inside a volume d^3r with local density $\rho_p(r)$ and $\rho_n(r)$ corresponding to the proton and neutron densities

$$\begin{aligned} \rho_p(r) &= \frac{Z}{A}\rho(r) \\ \rho_n(r) &= \frac{A-Z}{A}\rho(r) \end{aligned}$$

Fermi momentum of the nucleon is

$$\begin{aligned} p_{F_p} &= (3\pi^2\rho_p(\vec{r}))^{1/3} \\ p_{F_n} &= (3\pi^2\rho_n(\vec{r}))^{1/3} \end{aligned}$$

$$d\sigma = \frac{-2m}{E_l(\mathbf{k})} Im\Sigma(k) \frac{E_l(\mathbf{k})}{|\mathbf{k}|} d^3r,$$



ν *self energy* $\Sigma(k)$:

$$\Sigma(k) = (-i)\frac{G_F}{\sqrt{2}}\frac{4}{m_\nu}\int\frac{d^4k'}{(2\pi)^4}\frac{1}{k'^2 - m_l^2 + i\epsilon}\left(\frac{m_W}{q^2 - m_W^2}\right)^2 L_{\alpha\beta}\Pi^{\alpha\beta}(q)$$

$\Pi^{\alpha\beta}(q)$ is the W self-energy:

$$-i\Pi^{\alpha\beta}(q) = (-)\int\frac{d^4p}{(2\pi)^4}iG(p)\sum_X\sum_{s_p,s_i}\prod_{i=1}^{n}\int\frac{d^4p'_i}{(2\pi)^4}\prod_l iG_l(p'_l)\prod_j iD_j(p'_j)$$
$$\left(\frac{-G_F m_W^2}{\sqrt{2}}\right)\langle X|J^\alpha|N\rangle\langle X|J^\beta|N\rangle^*(2\pi)^4\delta^4(q+p-\Sigma_{i=1}^n p'_i)$$

Relativistic nucleon propagator in the nuclear medium:

$$G(p^0,\mathbf{p}) = \frac{M}{E(\mathbf{p})} \sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p}) \left[\int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,\mathbf{p})}{p^0-\omega-i\epsilon} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega,\mathbf{p})}{p^0-\omega+i\epsilon} \right]$$

for $p^0 \leq \mu$

$$S_h(p^0,\mathbf{p}) = \frac{1}{\pi} \frac{\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p})}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma(p^0,\mathbf{p}))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p}))^2}$$

for $p^0 > \mu$

$$S_p(p^0,\mathbf{p}) = -\frac{1}{\pi} \frac{\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p})}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma(p^0,\mathbf{p}))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p}))^2}$$

P.Fernandez de Cordoba and E. Oset, PRC 46, 1697(1992)

RECALL

Neutrino self energy $\Sigma(k)$ in the nuclear medium:

$$\Sigma(k) = \quad (-i)\frac{G_F}{\sqrt{2}}\frac{4}{m_\nu}\int\frac{d^4k'}{(2\pi)^4}\frac{1}{k'^2 - m_l^2 + i\epsilon}\left(\frac{m_W}{q^2 - m_W^2}\right)^2 L_{\alpha\beta}\;\Pi^{\alpha\beta}(q)\,.$$

Scattering cross section: $d\sigma = -\frac{2m_\nu}{|\mathbf{k}|}\,\mathrm{Im}\,\Sigma\, d^3r\,.$

Differential scattering cross section for $\nu(\bar{\nu}) - A$ interaction:

$$\frac{d^2\sigma}{d\Omega' dE'} = -\frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2 - m_W^2}\right)^2 L^{\alpha\beta}\int d^3r \mathrm{Im}\Pi^{\alpha\beta}(q)\,.$$

$\nu(\bar{\nu})$-N DCX: $\frac{d^2\sigma^N}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}W^N_{\alpha\beta}$

$\nu(\bar{\nu})$-A DCX: $\frac{d^2\sigma^A}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}W^A_{\alpha\beta}$

$$W^A_{\alpha\beta} = -\int d^3r \mathrm{Im}\Pi_{\alpha\beta}(q)$$

Nuclear hadronic tensor:

It is written as a convolution of nucleonic hadronic tensor with the hole spectral function

$$W^{A}_{\alpha\beta} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 \frac{M}{E(\mathbf{p})} S_h(p^0, \mathbf{p}, \rho(r)) W^{N}_{\alpha\beta}(p, q)$$

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$$\begin{aligned} W^A_{\alpha\beta} &= \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W^A_1 + \frac{1}{M_A^2}\left(p_\alpha - \frac{p.q}{q^2}\, q_\alpha\right)\left(p_\beta - \frac{p.q}{q^2}\, q_\beta\right) W^A_2 \\ &- \frac{i}{2M_A^2}\epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma W^A_3 \end{aligned}$$

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Weak Nuclear Structure Function

$$F_1^A(x_A) = 4AM \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(\mathbf{r})) \left[\frac{F_1^N(x_N)}{M} + \frac{1}{M^2} {p_x}^2 \frac{F_2^N(x_N)}{\nu}\right]$$

$$F_2^A(x_A) = 2 \sum_{p,n} \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h^{p,n}(p^0, \mathbf{p}, \rho_{p,n}(\mathbf{r})) F_2^N(x_N) C$$

$$C = \left[\frac{Q^2}{q_z^2}\left(\frac{p^2 - p_z^2}{2M^2}\right) + \frac{(p.q)^2}{M^2\nu^2}\left(\frac{p_z\, Q^2}{p.q q_z} + 1\right)^2 \frac{q_0 M}{p_0\, q_0 - p_z\, q_z}\right]$$

$$F_3^A(x_A, Q^2) = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(\mathbf{r})) \frac{p^0\gamma - p_z}{(p^0 - p_z\gamma)\gamma} F_3^N(x_N)$$

Additional Nuclear Effects

π and ρ mesons contribution to the nuclear structure function "Significant at low-x and mid-x". Probability of interaction of mediating boson with the meson cloud increases.

$$F^A_{1,\pi}(x_\pi) = -6AM \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\, \delta ImD(p)\, 2m_\pi \times \left[\frac{F_{1\pi}(x_\pi)}{m_\pi} + \frac{|\mathbf{p}|^2 - p_z^2}{2(p_0\, q_0 - p_z q_z)} \frac{F_{2\pi}(x_\pi)}{m_\pi} \right]$$

$$F^A_{2,\pi}(x_\pi) = -6 \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\, \delta ImD(p)\, 2m_\pi \frac{m_\pi}{p_0 - p_z\, \gamma} C_1 F_{2\pi}(x_\pi)$$

$$C_1 = \frac{Q^2}{q_z^2} \left(\frac{|\mathbf{p}|^2 - p_z^2}{2m_\pi^2} \right) + \frac{(p_0 - p_z\, \gamma)^2}{m_\pi^2} \left(\frac{p_z\, Q^2}{(p_0 - p_z\, \gamma) q_0 q_z} + 1 \right)^2$$

Shadowing and antishadowing effects"Significant at low-x and low-Q^2"

The shadowing and antishadowing of nuclear structure functions is due respectively to the destructive and constructive interference of amplitudes arising from the multiple-scattering of quarks in the nucleus.

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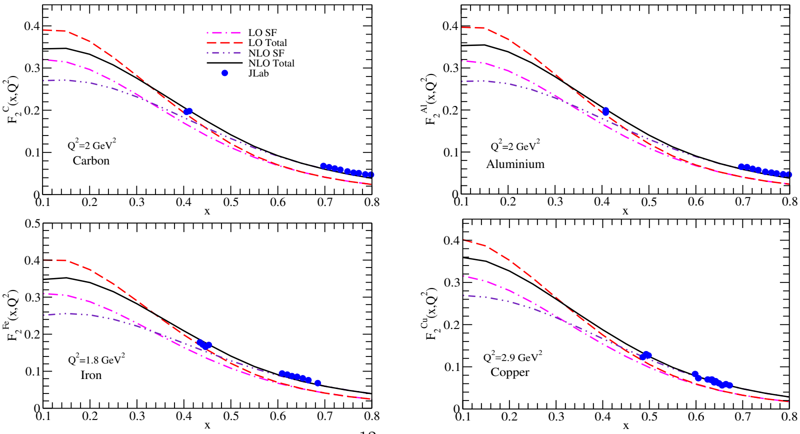
The shadowing and antishadowing of nuclear structure functions is due respectively to the destructive and constructive interference of amplitudes arising from the multiple-scattering of quarks in the nucleus.

For the shadowing and antishadowing effects, Glauber-Gribov multiple scattering model has been used following the works of Kulagin and Petti. PRD76(2007)094033.

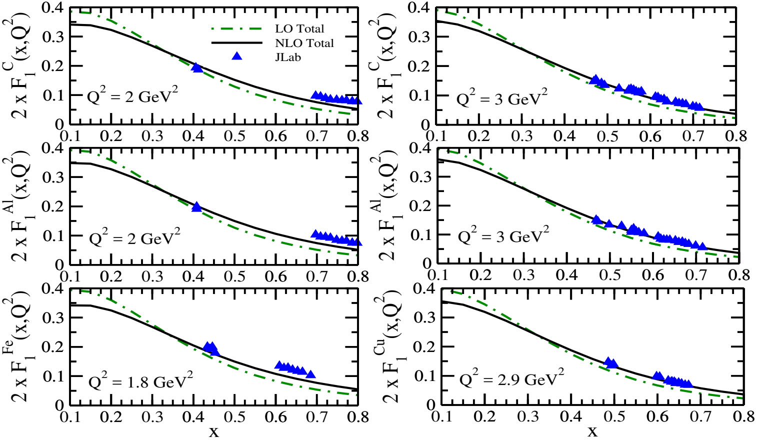
$F_{2A}^{EM}(x,Q^2)$ vs x(Nucl. Phys. A **943**, 58 (2015))



- At LO(SF→Full): $\sim$ 15% increase at low x in ^{12}C, and difference vanishes at high x.
- At NLO: Results at low x get suppressed while at high x results get enhanced compared to LO results.

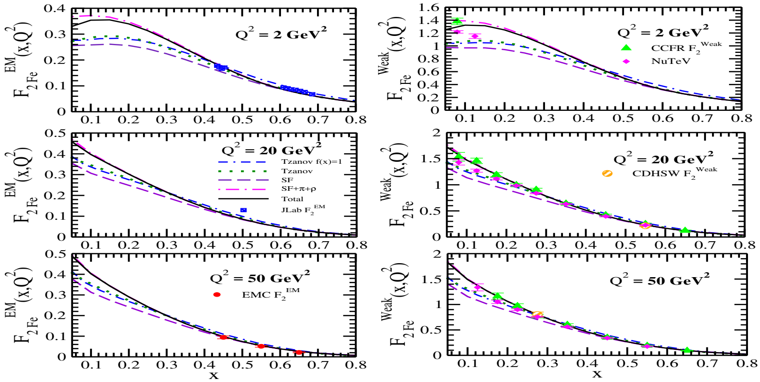
- NME depends on 'A'

$2xF_{1A}(x,Q^2)$ *vs x(Nucl. Phys. A* **943**, *58 (2015))*

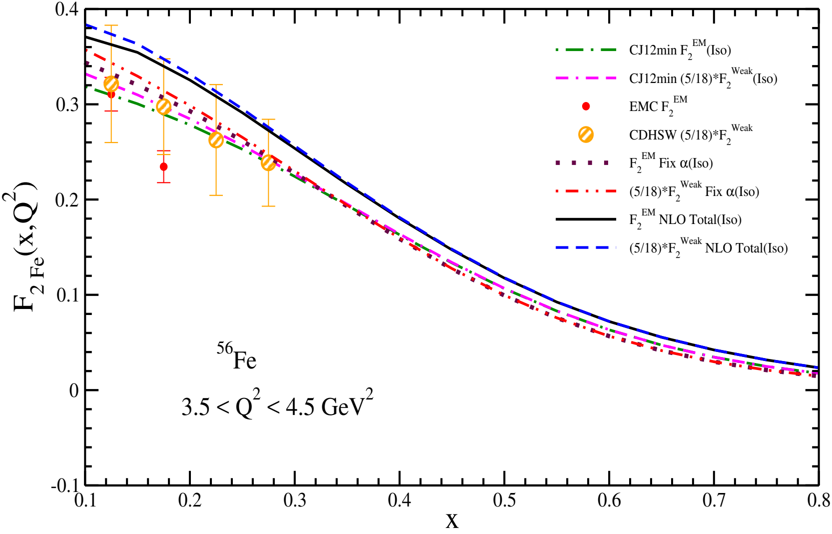


- Qualitatively similar in nature to that found in $F_{2A}^{EM}(x,Q^2)$.
- Quantitatively some variation, specially in low x region.

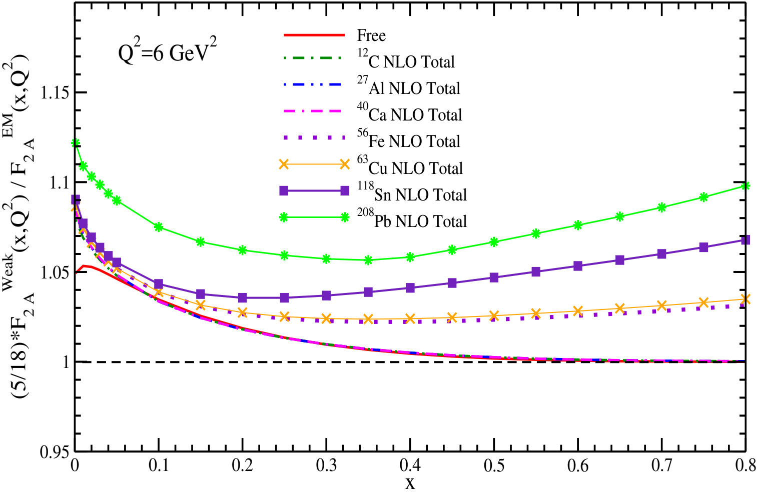
$F_{2A}^{EM}(x,Q^2)$ and $F_{2A}^{Weak}(x,Q^2)$ (Nucl. Phys. A **955**, 58 (2016))



- Free→SF: Reduction of $\sim 8\%$ at $x = 0.1$; $\sim 18\%$ at $x = 0.4$; $\sim 3\%$ at $x = 0.7$.
- SF→ $\pi\&\rho$: Increase in results at low and mid values of x i.e $\sim 30\%$ at x=0.1; 15% at x=0.4.
- shadowing effects reduces results at low x i.e $\sim 10\%$ at x=0.05 and $\sim 5\%$ at x=0.1.


CJ12min F_2^{EM}(Iso)
CJ12min (5/18)*F_2^{Weak}(Iso)
EMC F_2^{EM}
CDHSW (5/18)*F_2^{Weak}
F_2^{EM} Fix α(Iso)
(5/18)*F_2^{Weak} Fix α(Iso)
F_2^{EM} NLO Total(Iso)
(5/18)*F_2^{Weak} NLO Total(Iso)
^{56}Fe
$3.5 < Q^2 < 4.5$ GeV^2
$F_{2\,Fe}(x,Q^2)$
x
0.4
0.3
0.2
0.1
0
-0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

$\frac{\frac{5}{18}F_{2A}^{Weak}(x,Q^2)}{F_{2A}^{EM}(x,Q^2)}$ *vs x(Nucl. Phys. A* **955**, *58 (2016))*



$R_A(x,Q^2) = \frac{F_{LA}(x,Q^2)}{2xF_{1A}(x,Q^2)}$ *(arXiv:1705.09903)*

$$F_{LA}(x,Q^2) = \left(1 + \frac{4M_N^2 x^2}{Q^2}\right) F_{2A}(x,Q^2) - 2xF_{1A}(x,Q^2)$$

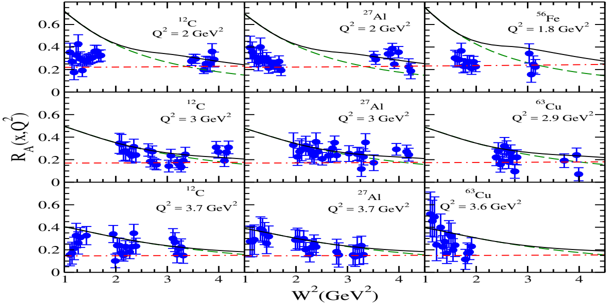


Figure: Spectral function:long dashed line, the full model:solid line, free nucleon case using the parameterization of Whitlow et al. :double dashed-dotted and experimental data of the JLab.

$R_A(x,Q^2) = \frac{F_{LA}(x,Q^2)}{2xF_{1A}(x,Q^2)}$ (arXiv:1705.09903)

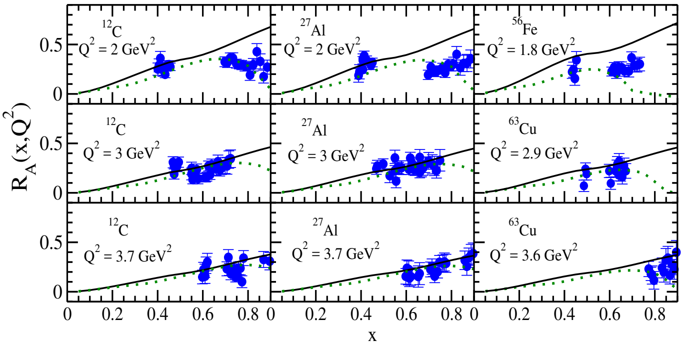
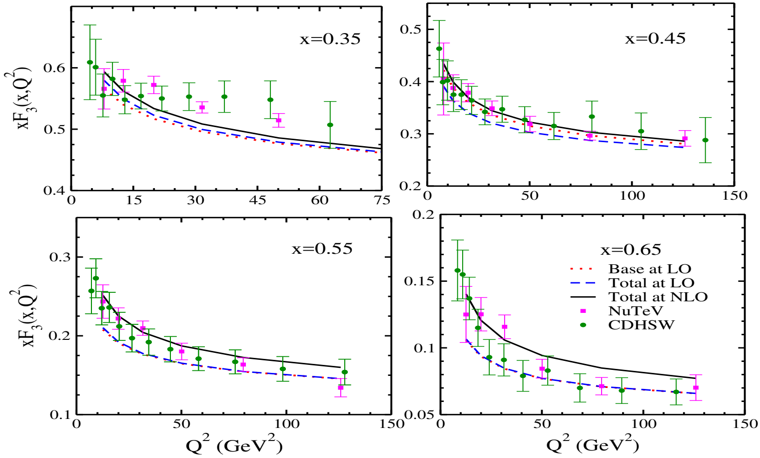


Figure: The results are obtained using the full model **(i)** without any kinematical cut on CM energy(solid line) and **(ii)** with a kinematical cut on CM energy $W > 1.4\ GeV$ (dotted line). The results are compared with the experimental data of the JLab(bold circles).

Phys. Rev. C **84**, *054610 (2011)*



1. We have studied nuclear medium effects in electromagnetic and weak nuclear structure functions.

2. For the nuclear medium effects, we took into account Fermi motion, nuclear binding, nucleon correlations, effect of meson degrees of freedom, and shadowing effects. The calculations are performed both at LO and NLO.

3. Nuclear medium effects are not same for electromagnetic and weak nuclear structure functions.

4. Nuclear medium effects are A dependent.

5. The use of DIS formalism to calculate the contribution of $R_A(x, Q^2)$ in the region of low W^2 and low Q^2 is not suitable.


thank
you . . .