A NEW (τ AND LATTICE-BASED) DETERMINATION OF $|V_{us}|$

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CONTEXT (Pre-2016)

- The $> 3\,\sigma$ low inclusive FB τ FESR $|V_{us}|$ puzzle

$\vert V_{us}\vert$	Source
0.2258(9)(?)	3-family unitarity, HT14 $\vert V_{ud}\vert$
$0.2231(4)_{exp}(7)_{latt}$	$K_{\ell 3}$, 2+1+1 lattice $f_+(0)$
$0.2250(4)_{exp}(9)_{latt}$	$\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$, lattice f_K/f_π
$0.2176(19)_{exp}(10?)_{th}$	Inclusive FB kinematic wt τ FESR (Passemar CKM14)

- τ result 3.6σ low: lepton flavor non-universality??

  [Recall $\sim 4\sigma$ $R\left[D^{(*)}\right]$ discrepancy c.f. SM]

BASICS: HADRONIC τ DECAYS IN THE SM

- $R_{ij;V/A} \equiv \Gamma[\tau \to \nu_\tau \, \text{hadrons}_{ij;V/A}(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$

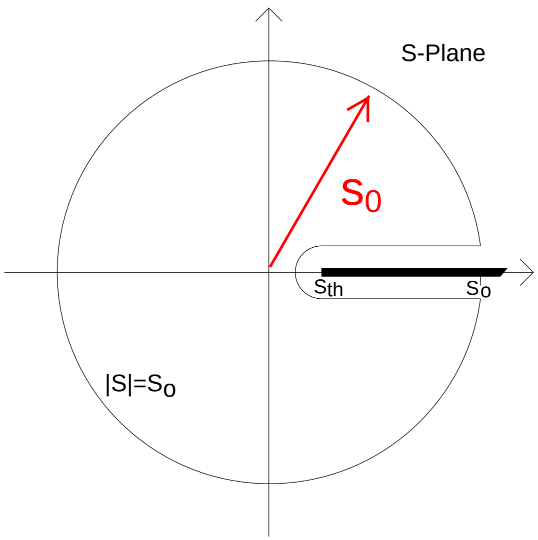
- With $y_\tau \equiv s/m_\tau^2$, flavor ij decays in SM [Tsai PRD4 (1971) 2821]

$$\frac{dR_{ij;V+A}}{ds} = \frac{12\pi^2 \, |V_{ij}|^2 S_{EW}}{m_\tau^2} \, [1 - y_\tau]^2 \, \tilde{\rho}_{ij;V+A}(s)$$

$$\tilde{\rho}_{ij;V+A}(s) \equiv \left[(1 + 2y_\tau) \, \rho^{(J=1)}_{ij;V+A}(s) + \rho^{(J=0)}_{ij;V+A}(s) \right]$$

$$\text{kinematic weight} : \, w_\tau(y) = (1 - y)^2 (1 + 2y)$$

- Finite energy sum rules (FESRs) (Cauchy's theorem)



$$\int_{s_{th}}^{s_0} ds\, w(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\Pi(s)$$

$expt'l\ data$ $\qquad\qquad$ OPE

- $s_0 \to \infty$: generalized dispersion relation

- $R^w_{ij;V/A}(s_0)$: re-weighted $R_{ij;V/A}$ analogue

$$R^w_{ij;V/A}(s_0) \sim \int_{th}^{s_0} ds\, \frac{dR_{ij;V/A}}{ds}\, \frac{w(s/s_0)}{w_\tau(s/m_\tau^2)}$$

- FB differences: $\delta R^w(s_0) \equiv \frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - \frac{R^w_{us;V+A}(s_0)}{|V_{us}|^2}$

- FB FESR: OPE for $\delta R^w(s_0)$, input $|V_{ud}| \Rightarrow$

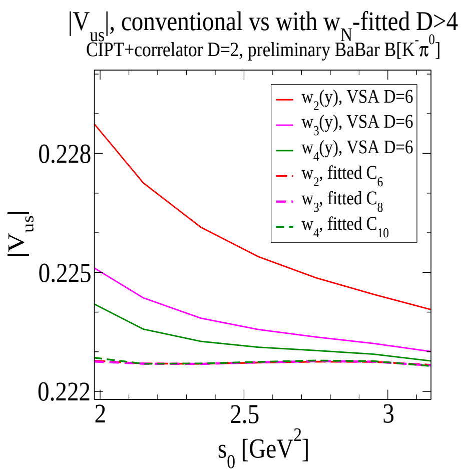
$$|V_{us}| = \sqrt{\frac{R^w_{us;V+A}(s_0)}{\frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - [\delta R^w(s_0)]^{OPE}}}$$

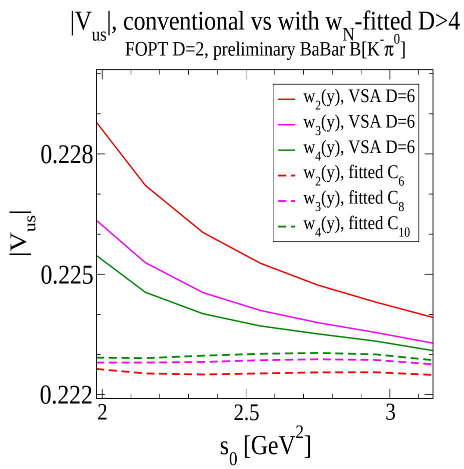
Self-consistency: $|V_{us}|$ independent of s_0, w

- **The conventional implementation** [Gamiz et al. JHEP03(2003)060]
  - $s_0 = m_\tau^2$, $w = w_\tau$ only ($\Rightarrow$ OPE up to $D = 8$)
  - ASSUME $D > 4$ OPE small
  - Variable w, s_0 tests $\Rightarrow$ self-consistency problems

- **An alternate implementation** [HLMZ17, arXiv:1702.01767]
  - No $D > 4$ assumptions: effective condensates $C_{D>4}$ from fits to data (variable s_0 **required**)
  - 3-loop-truncated FOPT $D = 2$, standard $D = 2 + 4$ error estimates [from comparison to lattice]
  - w- and s_0-stability tests

- **The alternate FB FESR implementation results**
  - $|V_{us}| = 0.2229(22)_{exp}(4)_{th}$
  - Resolves long-standing low τ $|V_{us}|$ puzzle
  - Unphysical s_0-, $w(y)$-dependence problems solved





  - $|V_{us}|$ increased by ~ 0.0020 with fitted $C_{D>4}$

- Similar increase from newer us BF normalizations

- Error budget, 3-weight, Adametz $B[K^-\pi^0\nu_\tau]$, 3-loop-truncated FOPT $D=2$ fit

Source	$\delta\vert V_{us}\vert$ (w_2 FESR)	$\delta\vert V_{us}\vert$ (w_3 FESR)	$\delta\vert V_{us}\vert$ (w_4 FESR)
$\delta\alpha_s$	0.00001	0.00004	0.00004
$\delta m_s(2\ GeV)$	0.00017	0.00019	0.00019
$\delta\langle m_s\bar{s}s\rangle$	0.00035	0.00035	0.00035
$\delta(long\ corr)$	0.00009	0.00009	0.00009
ud exp	0.00027	0.00028	0.00028
us exp	0.00226	0.00227	0.00227

- us spectral integrals dominant current error source

- Limitation for near-term improvement: $\sim$ 25% higher-multiplicity "residual mode" contribution error (sub-% us spectral integral error to be fully competitive)

Relative exclusive mode $R^{w}_{us:V+A}$ contributions

Wt	s_0 $[GeV^2]$	K	$K\pi$	$K\pi\pi$ (B-factory)	Residual
w_2	2.15	0.496	0.426	0.062	0.010
	3.15	0.360	0.414	0.162	0.065
w_3	2.15	0.461	0.446	0.073	0.019
	3.15	0.331	0.415	0.182	0.074
w_4	2.15	0.441	0.456	0.082	0.021
	3.15	0.314	0.411	0.194	0.081

A LATTICE+τ-BASED ALTERNATIVE

- Work with J. Hudspith, R. Lewis (York) and T. Izubuchi, H. Ohki, C. Lehner + $\cdots$ (RBC/UKQCD)

- Basic idea: generalized dispersion relations for products of combination $\tilde{\Pi}$ of $J = 0, 1$ us V+A polarizations with weights having poles at Euclidean Q^2

  - $\tilde{\Pi}(Q^2)$: polarization sum with spectral function $\tilde{\rho}(s)$ (experimental $dR_{us;V+A}/ds$)

  - Theory: Lattice us 2-point function data (no OPE)

  - Weights tunable, allow suppression of larger-error, higher-multiplicity us spectral contributions

More on the lattice-inclusive us τ approach

- $|V_{us}|^2\,\tilde{\rho}_{us;V+A}(s)$ from experimental $dR_{us;V+A}/ds$

$$\tilde{\rho}_{us;V+A}(s) \equiv \left(1+2\frac{s}{m_\tau^2}\right)\rho^{(J=1)}_{us;V+A}(s) + \rho^{(J=0)}_{us;V+A}(s)$$

(no continuum us $J=0$ subtraction required)

- Associated (kinematic-singularity-free) polarization

$$\tilde{\Pi}_{us;V+A}(Q^2) \equiv \left(1-2\frac{Q^2}{m_\tau^2}\right)\Pi^{(J=1)}_{us;V+A}(Q^2) + \Pi^{(J=0)}_{us;V+A}(Q^2)$$

- $\tilde{\rho}_{us;V+A}(s) \sim s$ as $s \to \infty$

- For weights $w_N(s) \equiv \frac{1}{\prod_{k=1}^{N}(s+Q_k^2)}$, $Q_k^2 > 0$, $N \geq 3$, obtain convergent, unsubtracted 'dispersion relation'

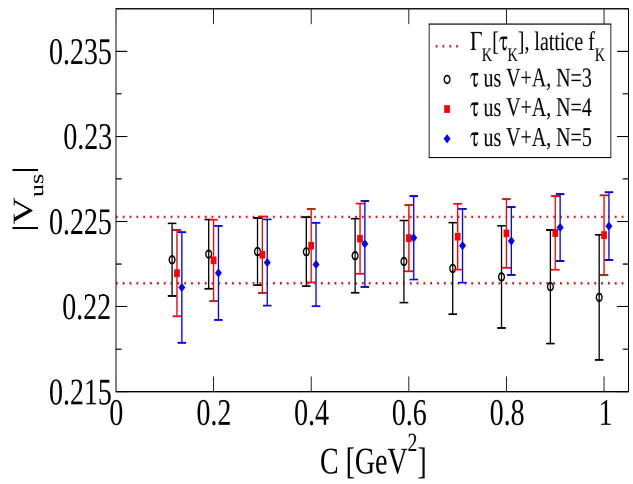
$$\int_{th}^{\infty} ds\, w_N(s)\, \tilde{\rho}_{us;V+A}(s) = \sum_{k=1}^{N} \frac{\tilde{\Pi}_{us;V+A}(Q_k^2)}{\prod_{j\neq k}\left(Q_j^2 - Q_k^2\right)}$$

  - Lattice data for $\tilde{\Pi}_{us;V+A}(Q_k^2)$ on RHS
  - LHS from experimental $dR_{us;V+A}/ds$, up to $|V_{us}|^2$
  - $w_N(s)$: rapid fall-off if all $Q_k^2 < 1\ GeV^2$
    $\Rightarrow$ K, $K\pi$ **dominate LHS, near-endpoint multi-particle,** $s > m_\tau^2$ **contributions strongly suppressed**
  - Optimization: increasing $\{Q_k^2\}$ decreases RHS lattice error, increases LHS experimental error

- *PRELIMINARY inclusive lattice* us *V+A results*
  - $N = 4$, $C = 0.7\,GeV^2$, $\Gamma[K_{\mu 2}]$: $|V_{us}| = 0.2258(10)_{exp}(13)_{th}$
  - $N = 4$, $C = 0.7\,GeV^2$, B_K^τ: $|V_{us}| = 0.2241(14)_{exp}(13)_{th}$

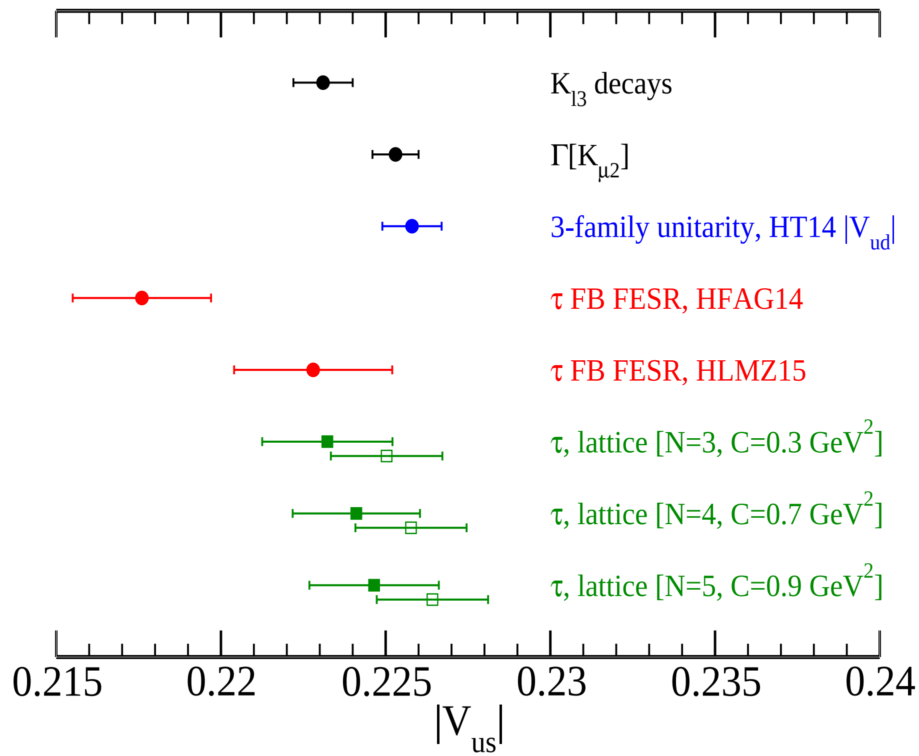
- Advantages of lattice-based vs. FB FESR approach
  - K essentially saturates $J = 0$, A contribution $\Rightarrow |V_{us}|$ determinations possible with or without K pole
  - Reduced expt'l error *without theory error blowup*
  - Theory side: lattice in place of OPE $\Rightarrow$ theory errors systematically improvable

- Stability e.g.



$N = 3, 4, 5$, B_K^τ choice for K pole contribution

Comparison to $|V_{us}|$ from other sources



SUMMARY

- Old 3σ low inclusive FB τ FESR $|V_{us}|$ problem resolved
  - Alternate, no-assumptions implementation: $|V_{us}|$ higher by $\sim$ 0.0020, compatible with other determinations
  - Near-term improvements feasible through improvements in us exclusive mode BFs
  - Highly favorable theoretical error situation
  - However, for competitive $|V_{us}|$ need improvements to old ALEPH higher-multiplicity, low-statistics data [unlikely in the near-term]

- Advantage of new lattice-inclusive us $V+A$ τ approach
  - Theory:
    - Lattice in place of OPE; no us $J=0$ subtraction; improvement through increased statistics
    - Parasitic on lattice a_μ effort (major effort in lattice community)
  - Spectral integrals:
    - Theory errors still small for weights strongly suppressing higher multiplicity contributions
    - Strong K, $K\pi$ dominance of spectral integral
    - Significant experimental improvements possible through just improved $K\pi$ BFs, distributions