The Nuclear Delta Force and The Two-Particle-Rotor Model

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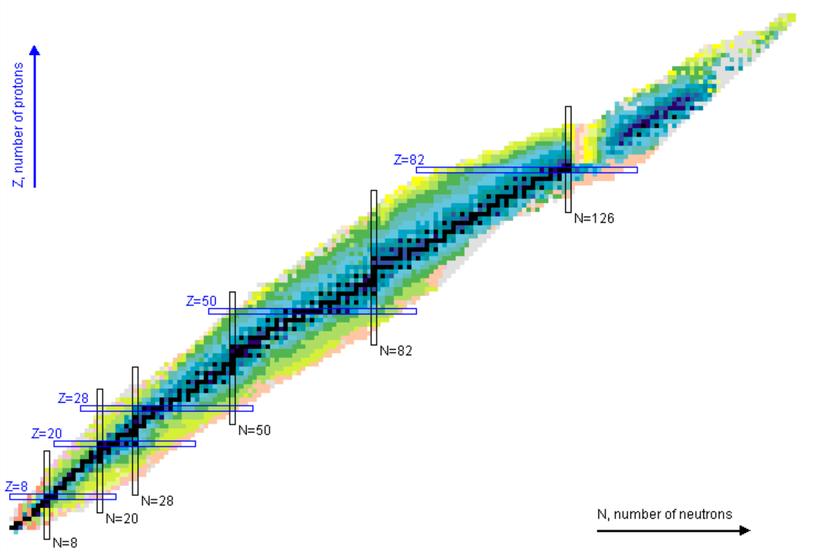
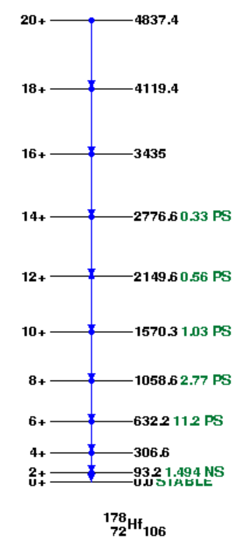
Chart of Nuclides



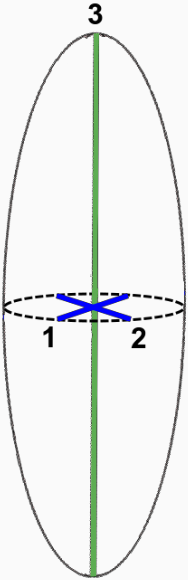
Observables



Extracted from the NNDC On-Line Data Service from the ENSDF database.

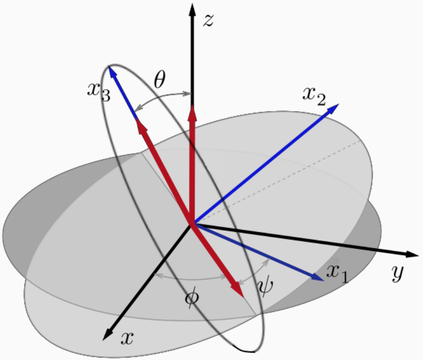
Collective Degrees of Freedom

Quadrupole Deformation



Rotor Wavefunction

$$|RMK\rangle = \frac{1}{N} D^{R}_{MK}(\phi, \theta, \psi)$$



Schematic diagram describing the relation between the fixed and rotating frames through the euler angles.

Applying the Quadrupole Symmetry to the Rotor Wavefunction

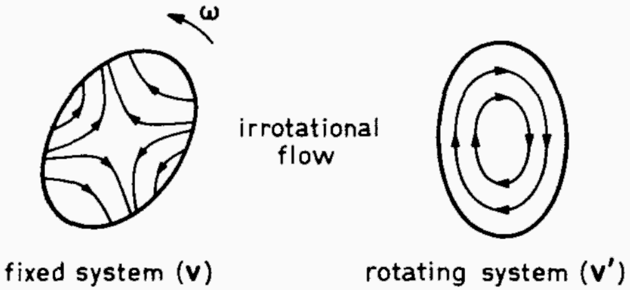
The characteristic symmetry elements are the π rotations about each axis as well as the identity element:

$$\hat{S} = \hat{R}(\mathbf{e}_1, \pi) + \hat{R}(\mathbf{e}_2, \pi) + \hat{R}(\mathbf{e}_3, \pi) + \hat{I}.$$

Applying this operator to $|RMK\rangle$ and normalizing, we obtain the rotor wavefunction:

$$|\Psi\rangle = \frac{1 + (-1)^K}{2} \frac{1}{\sqrt{2(1 + \delta_{K,0})}} \sqrt{\frac{2R+1}{8\pi^2}} \left(D^R_{MK} + (-1)^R D^R_{M\bar{K}} \right).$$

Liquid Drop Model



(Left) The surface flow patterns are visualized from the fixed lab frame for a liquid drop. (Right)The surface flow patterns are visualized in the intrinsic rotating frame for a liquid drop[†].

$$\mathscr{I}_{33} = 0, \mathscr{I}_{11} = \mathscr{I}_{22} = \frac{3}{4}\mathscr{I}_0$$

[†] A. Bohr and B. Mottelson, Nuclear Structure, Vol. II, W. A. Benjamin Inc., New York, Amsterdam, 1975

Axial Hamiltonian

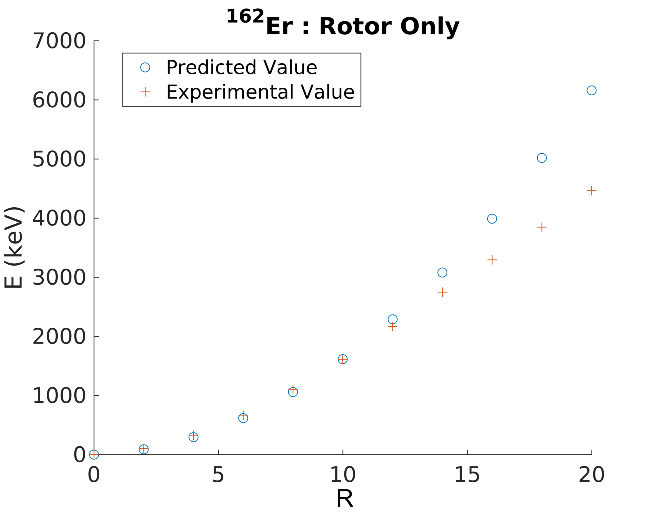
$$H_{Rot} = \frac{1}{2}\left[\frac{\hat{R}_1^2}{\mathscr{I}_{11}} + \frac{\hat{R}_2^2}{\mathscr{I}_{22}} + \frac{\hat{R}_3^2}{\mathscr{I}_{33}}\right]$$

But, $\mathscr{I}_{11} = \mathscr{I}_{22} = \frac{3}{4}\mathscr{I}_0$ and $\mathscr{I}_{33} = 0 \implies R_3 = 0$

$$H_{Rot} = \frac{2}{3}\left[\frac{\hat{R}^2}{\mathscr{I}_o}\right]$$

$$H_{Rot}|\Psi\rangle = \frac{2\hbar^2}{3\mathscr{I}_o} R(R+1)|\Psi\rangle$$

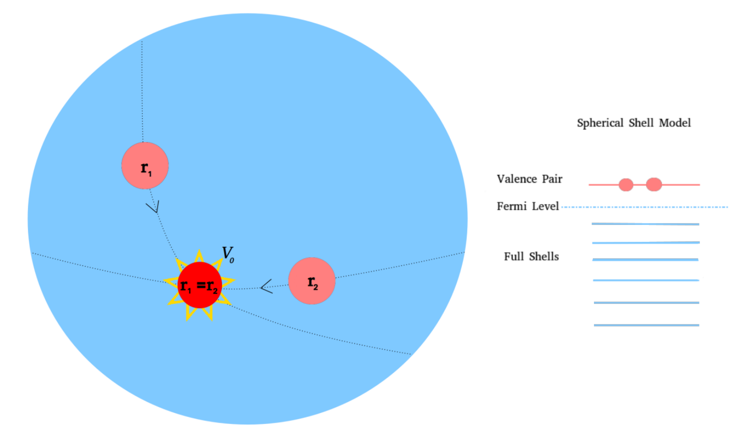
Axial Rotor



Data extracted using the NNDC On-Line Data Service from the ENSDF database.

Modeling Single-Nucleon Interaction

Nuclear Delta Force



$$H = H_{Spherical} + H_\delta$$

$$E = (E_1 + E_2)\langle \Psi' | \Psi \rangle + \langle \Psi' | V_\delta \delta(\mathbf{r}_2 - \mathbf{r}_1) | \Psi \rangle$$

Wavefunction

$$|\Psi\rangle \triangleq \frac{1-(-1)^{j_1+j_2-J}}{2}|\mathscr{R}\rangle|JQ\rangle$$

where

$$|JQ\rangle = \sum_{q_1,q_2}\langle j_1 q_1 j_2 q_2|JQ\rangle|j_1 q_1\rangle|j_2 q_2\rangle$$

and

$$|\mathscr{R}(r_1)\rangle|\mathscr{R}(r_2)\rangle \triangleq |\mathscr{R}\rangle$$

Nuclear Delta Force

$$H_\delta = V_\delta \delta(\mathbf{r}_2 - \mathbf{r}_1) = V_\delta \delta(r_2 - r_1)\delta(\theta_2 - \theta_1)\delta(\phi_2 - \phi_1)$$

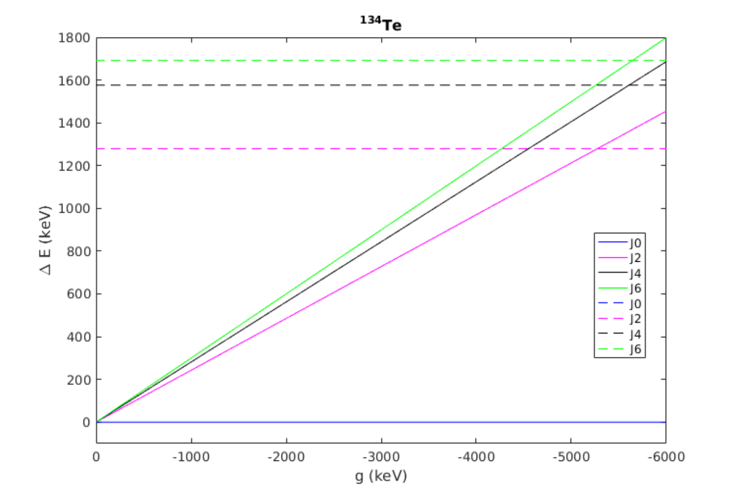
Applying this, we derive

$$\langle \Psi' | H_\delta | \Psi \rangle = V_\delta \langle \mathscr{R}' | \delta(r_2 - r_1) | \mathscr{R} \rangle \langle J' Q' | \delta(\theta_2 - \theta_1)\delta(\phi_2 - \phi_1) | JQ \rangle$$

where

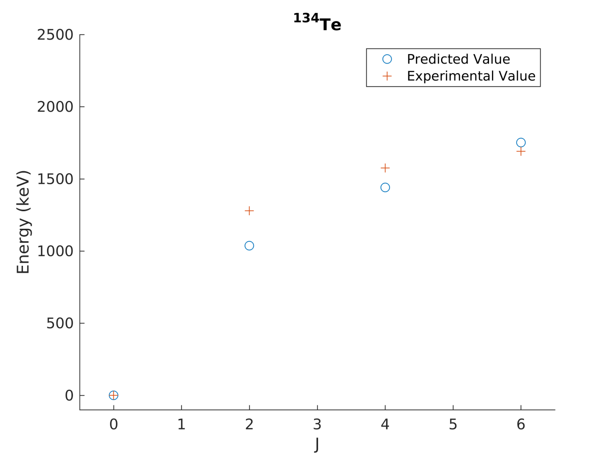
$$V_\delta \langle \mathscr{R}' | \delta(r_2 - r_1) | \mathscr{R} \rangle \triangleq g$$

Comparison to Data



Data extracted using the NNDC On-Line Data Service from the ENSDF database.

Comparison to Data



Two protons in the $1g_{\frac{7}{2}}$ shell. The coupling constant value obtained from a least squares fit was 6969keV.

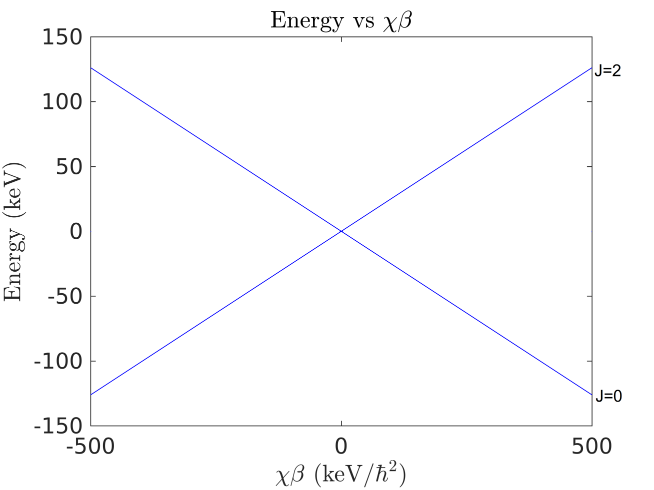
Data extracted using the NNDC On-Line Data Service from the ENSDF database.

Two Coupled Nucleons in an Axially-Deformed Potential

$$H = H_{spherical} + H_\beta$$

$$H_\beta = \pm\chi\beta\left[Y_{20}(\Omega_1) + Y_{20}(\Omega_2)\right]$$

Model

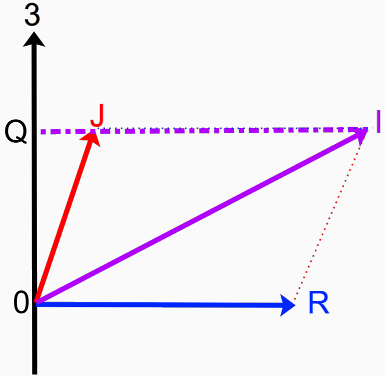


Example of two $j = \frac{3}{2}$ particles

Interaction of Single-Particle and Collective Degrees of Freedom

Wavefunction for Two Identical Nucleons Coupled to an Axial Rotor

$$|IMj_1j_2JQ\rangle = \frac{1-(-1)^{j_1+j_2-J}}{2}\sqrt{\frac{2I+1}{16\pi^2(1+\delta_{Q,0})}}\left[D^I_{MQ}|JQ\rangle + (-1)^{I-J}D^I_{M\bar{Q}}|J\bar{Q}\rangle\right]$$



Model

$$H = H_{Rotor} + H_{\delta} + H_{\beta}$$

Rotor Hamiltonian

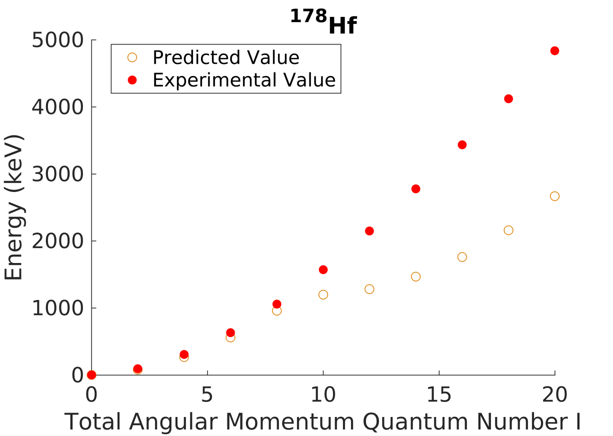
$$H_{Rotor} = \frac{2}{3}\left[\frac{\mathbf{R}^2}{\mathscr{I}_o}\right]$$

$$H_{Rotor} = \frac{2}{3\mathscr{I}_0}\left[\mathbf{I} - \mathbf{J}\right]^2$$

$$H_{Rotor} = \frac{2\hbar^2}{3\mathscr{I}_0}\left[\hat{I}^2 - \hat{I}_3^2 + \hat{J}^2 - \hat{J}_3^2\right] + \frac{4\hbar^2}{3\mathscr{I}_0}\left[\hat{I}_{+1}\hat{J}_{-1} + \hat{I}_{-1}\hat{J}_{+1}\right]$$

$$H_{Rotor} = H_{Diagonal} + H_{Coriolis}$$

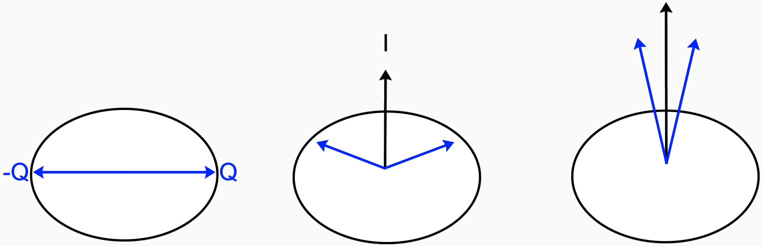
Coriolis Effect



Two $1g_{\frac{11}{2}}$ protons taken as the valence particles, modeled with the rotor Hamiltonian.

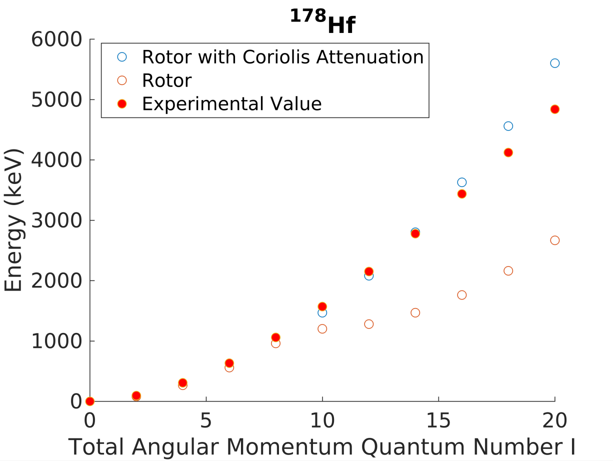
Data extracted using the NNDC On-Line Data Service from the ENSDF database.

Coriolis Force



For larger total angular momentum I, the Coriolis force breaks the pair, and aligns its projections with the rotor angular momentum.

Coriolis Attenuation



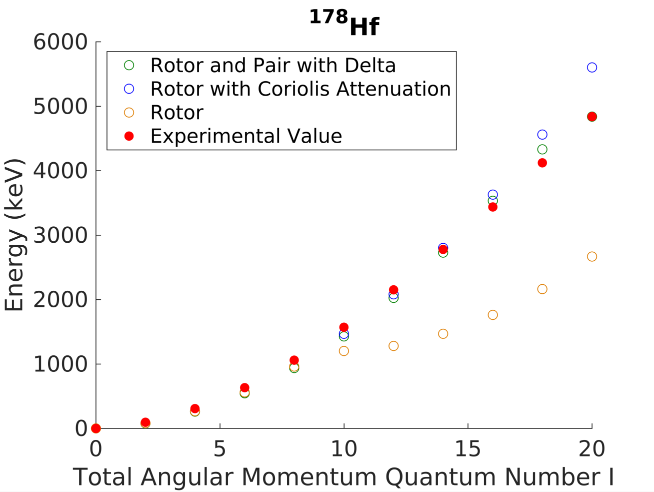
Two $1g_{\frac{11}{2}}$ protons taken as the valence particles, modeled with the rotor Hamiltonian including heavy Coriolis attenuation.

Data extracted using the NNDC On-Line Data Service from the ENSDF database.

Model

$$H = H_{Diagonal} + H_{Coriolis} + H_{\delta} + H_{\beta}$$

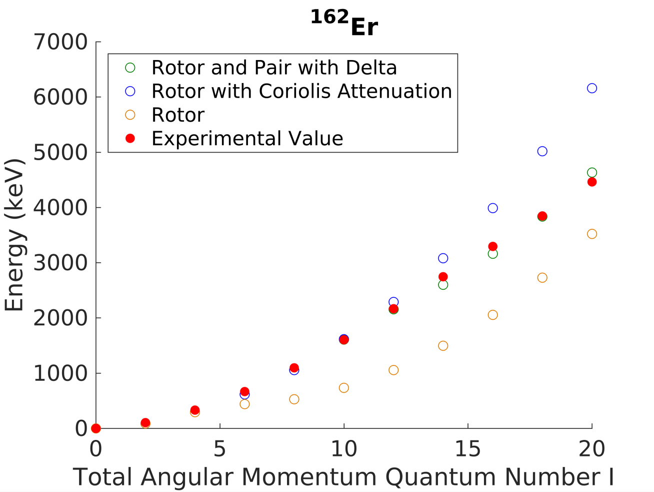
Comparison to Data



Two $1g_{\frac{11}{2}}$ protons taken as the valence particles.

$\chi\beta = 500 keV/\hbar^2$, $g = -8000 keV$, $\mathscr{I}_0^{-1} = 1/20 keV/\hbar^2$

Comparison to Data

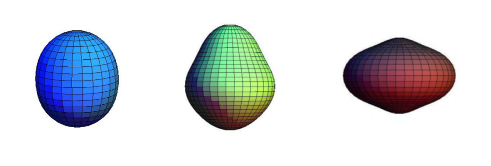


Two $2f_{\frac{7}{2}}$ neutrons taken as the valence particles.

$\chi\beta = 500keV/\hbar^2$, $g = -6000keV$, $\mathscr{I}_0^{-1} = 1/22keV/\hbar^2$

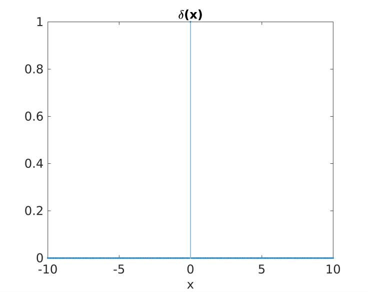
Summary and Future Work

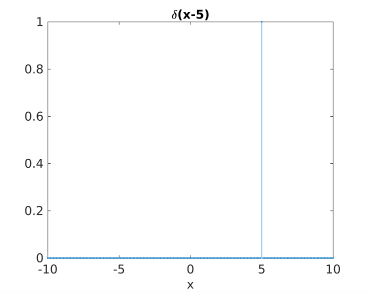
1. Pair breaking Coriolis force is dampened by pairing Nuclear Delta Force between the valence nucleons.
2. The model shows that the single particle degrees of freedom are of comparable impact with the collective degrees of freedom.
3. The wavefunction may be analyzed for the system.
4. New deformations to be explored : Triaxial for the quadrupole, octupole, hexadecapole, etc.



Questions?

Dirac Delta





$$\int_a^b f(x)\delta(x - x_0)dx = f(x_0) \qquad x_0 \in [a, b]$$

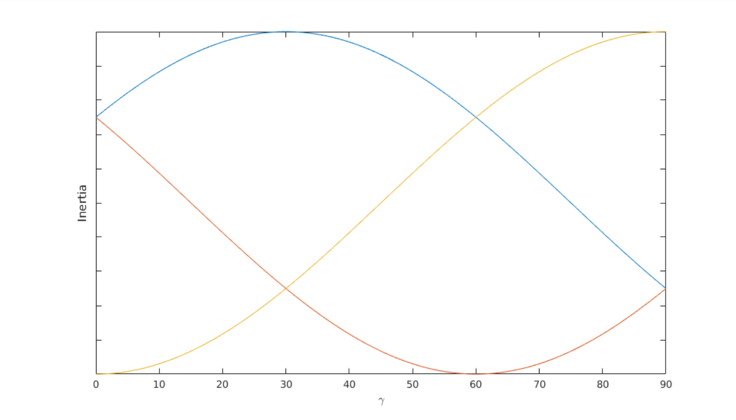
If we extend this to 3D in spherical coordinates,

$$\delta(x - x_0) \to \delta(\mathbf{r}_2 - \mathbf{r}_1)$$

$$\delta(\mathbf{r}_2 - \mathbf{r}_1) = \delta(r_2 - r_1)\delta(\theta_2 - \theta_1)\delta(\phi_2 - \phi_1)$$

Moment of Inertia

$$\mathscr{I}_{kk} = 4D\beta^2 \sin^2(\gamma - k \cdot 120)$$



The yellow curve corresponds to the 3 axis, the red curve to the 2 axis, and the blue curve to the 1 axis.

Parametrization of Nuclear Shape

$$R(\theta, \phi) = R_o \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi) \right]$$

1. $\lambda = 0 \rightarrow$ Compression
2. $\lambda = 1 \rightarrow$ Centre of Mass Shift

Triaxial and Axial Cases

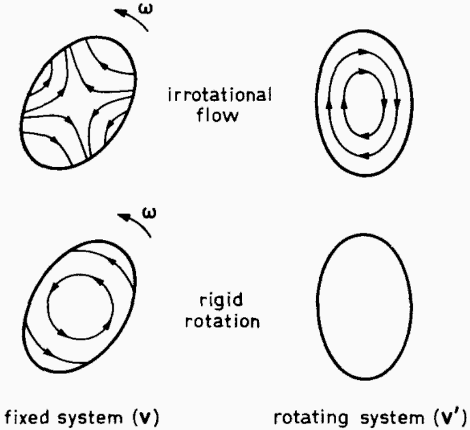
$$\text{Axial} \rightarrow \gamma = 0^o$$

$$R(\theta, \phi, \beta, \gamma) = R_o \left[1 + \beta Y_{20}(\theta, \phi)\right]$$

$$\text{Triaxial} \rightarrow \gamma = 90^o$$

$$R(\theta, \phi, \beta, \gamma) = R_o \left[1 + \frac{1}{\sqrt{2}} \beta \left(Y_{2-2}(\theta, \phi) + Y_{22}(\theta, \phi)\right)\right]$$

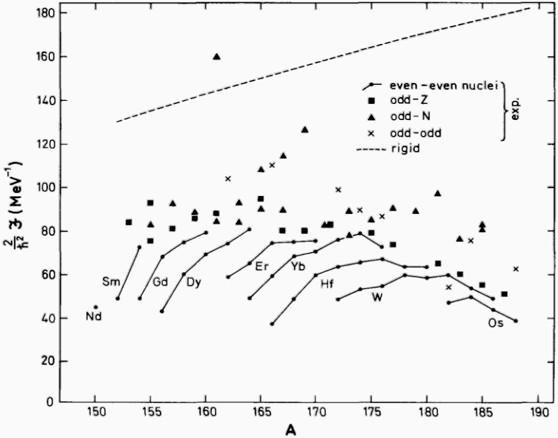
Liquid Drop Model



A. Bohr and B. Mottelson, Nuclear Structure, Vol. II, W. A. Benjamin Inc., New York, Amsterdam, 1975

Rigid Body Moment of Inertia

$$\mathscr{I}_{Rig,i} = \frac{M}{5}\left[R_j^2 + R_k^2\right]$$



Nuclear Delta Force

$$ME = V_\delta \langle \Psi' | \delta(\mathbf{r}_2 - \mathbf{r}_1) | \Psi \rangle$$
$$= g \langle J' M' | \delta(\theta_2 - \theta_1) \delta(\phi_2 - \phi_1) | JM \rangle$$

$$\delta(\theta_2 - \theta_1)\delta(\phi_2 - \phi_1) = \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} Y^*_{\lambda,\mu}(\theta_2, \phi_2) Y_{\lambda,\mu}(\theta_1, \phi_1)$$

Model

Using

$$H_\beta = \pm\chi\beta\left[Y_{20}(\Omega_1) + Y_{20}(\Omega_2)\right]$$

We derive

$$\langle\Psi'|H_\beta|\Psi\rangle = \pm\,\chi\beta\frac{1+(-1)^{j_1'+j_2'-J'}}{2}\,\frac{1+(-1)^{j_1+j_2-J}}{2}\sqrt{\frac{5}{4\pi}}\sqrt{2J+1}$$
$$\left[\delta_{j_2j_2'}\sqrt{2j_1+1}\langle JQ20|J'Q'\rangle\langle j_1\tfrac{1}{2}20|j_1'\tfrac{1}{2}\rangle\begin{Bmatrix} j_1 & j_2 & J \\ J' & 2 & j_1' \end{Bmatrix}(-1)^{J+j_1'+j_2}+\right.$$
$$\left.\delta_{j_1j_1'}\sqrt{2j_2+1}\langle JQ20|J'Q'\rangle\langle j_2\tfrac{1}{2}20|j_2'\tfrac{1}{2}\rangle\begin{Bmatrix} j_2 & j_1 & J \\ J' & 2 & j_2' \end{Bmatrix}(-1)^{J+j_1+j_2'}.\right]$$

Transition Rates

$$T_{B\lambda} = \frac{2}{\epsilon_0 \hbar} \frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2} \left[\frac{E_\gamma}{\hbar c}\right]^{2\lambda+1} \frac{1}{2J+1} \left|\langle J' \,||\mathcal{M}_{B\lambda}||\, J\rangle\right|^2.$$

For electric quadrupole transitions, we derive

$$T_{E2} = |\langle R'|e(r_1)^2 r_1^2 + e(r_2)^2 r_2^2 |R\rangle|^2 \frac{2E_\gamma^5}{75\epsilon_0 \hbar^6 c^5 (2J+1)}$$

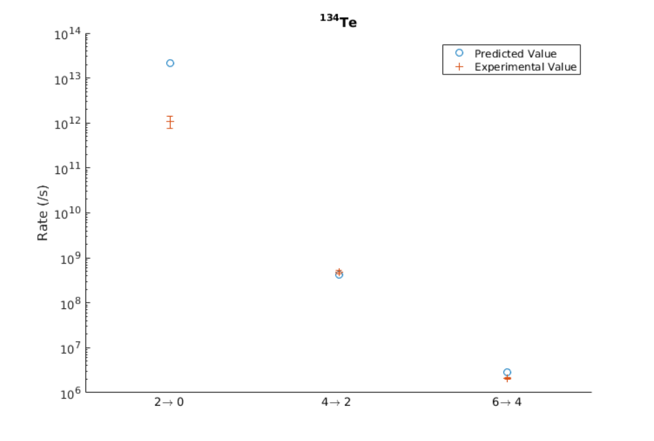
$$\Bigg| \frac{1-(-1)^{j_1+j_2-J}}{2} \frac{1-(-1)^{j_1'+j_2-J'}}{2} (-1)^{s_1+j_1+l_1'+J+j_2+j_1'}$$

$$\sqrt{2l_1+1}\sqrt{2j_1+1}\sqrt{2J+1}\sqrt{2j_1'+1}\langle l_1 020|l_1`0\rangle \begin{Bmatrix} l_1 & s_1 & j_1 \\ j_1' & 2 & l_1' \end{Bmatrix} \begin{Bmatrix} j_1 & j_2 & J \\ J' & 2 & j_1' \end{Bmatrix}$$

$$+ \frac{1-(-1)^{j_1+j_2-J}}{2} \frac{1-(-1)^{j_1+j_2'-J'}}{2} (-1)^{s_2+j_2+l_2'+J'+j_2+j_1}$$

$$\sqrt{2l_2+1}\sqrt{2j_2+1}\sqrt{2J+1}\sqrt{2j_2'+1}\langle l_2 020|l_2`0\rangle \begin{Bmatrix} l_2 & s_2 & j_2 \\ j_2' & 2 & l_2' \end{Bmatrix} \begin{Bmatrix} j_2 & j_1 & J \\ J' & 2 & j_2' \end{Bmatrix} \Bigg|^2$$

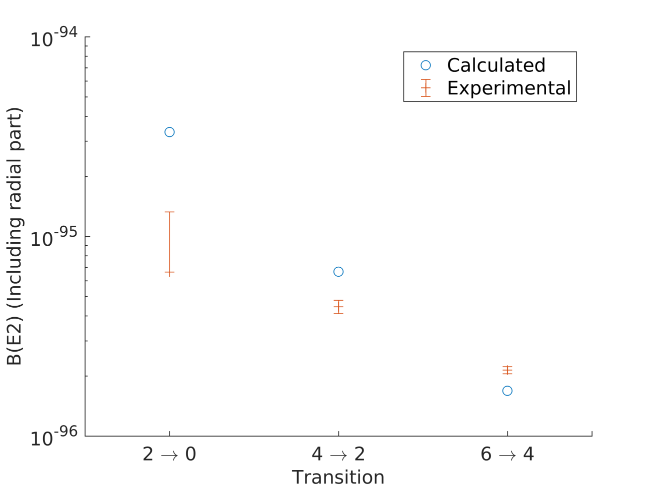
Comparison to Data



Transition rates using two protons in the $1g\frac{7}{2}$ shell.

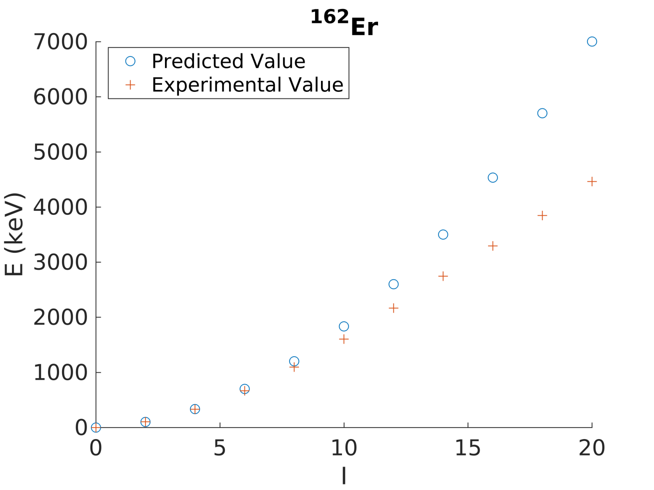
Data extracted using the NNDC On-Line Data Service from the ENSDF database.

B(E2)

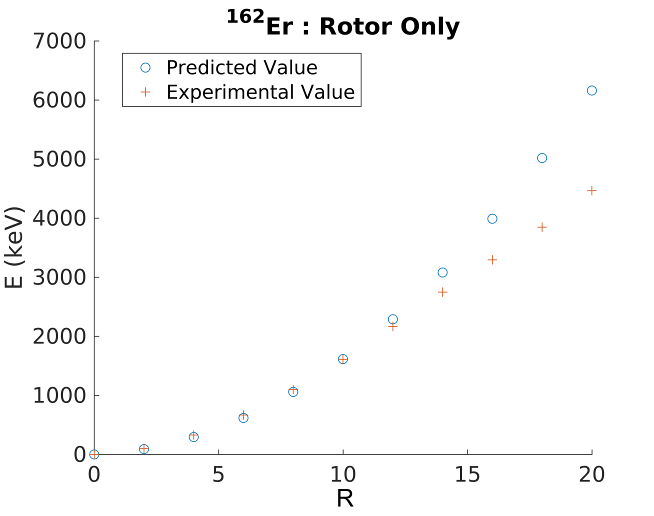


Calculated
Experimental
10^{-94}
10^{-95}
10^{-96}
B(E2) (Including radial part)
$2 \to 0$
$4 \to 2$
$6 \to 4$
Transition

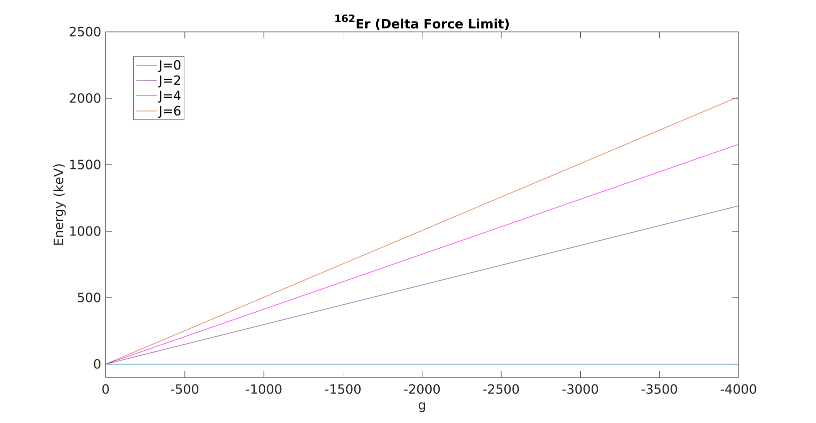
Coriolis Attenuation



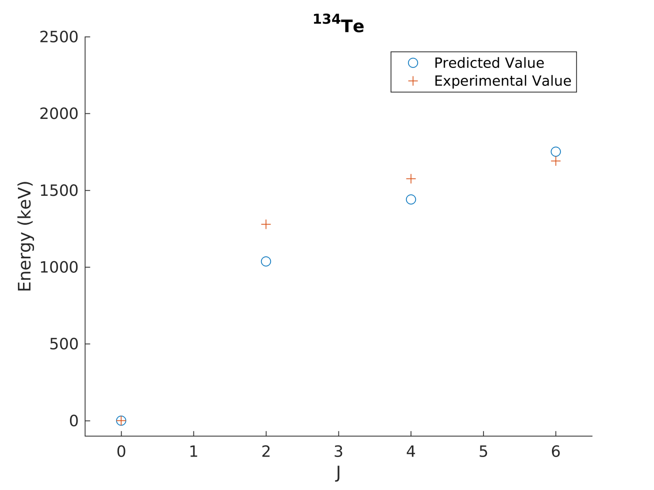
Limiting Cases : Rotor



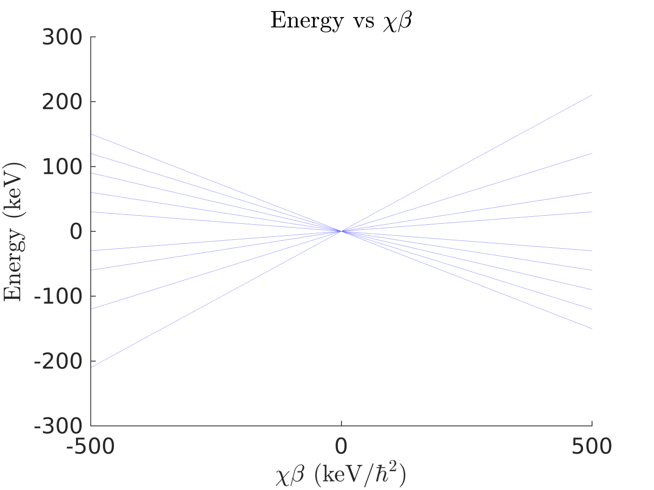
Limiting Cases : Delta



Limiting Cases : Delta



Limiting Cases : Deformation



Diagonal

$$\delta_{II'}\,\delta_{MM'}\,\delta_{JJ'}\,\frac{\delta_{QQ'}+\delta_{Q\bar{Q}'}(-1)^{I-J}}{\sqrt{1+\delta_{Q'0}}\sqrt{1+\delta_{Q0}}}\,\frac{1-(-1)^{j_1+j_2-J}}{2}\,\frac{1-(-1)^{j_1'+j_2'-J}}{2}$$

$$\frac{2\hbar^2}{3\mathscr{I}_0}\left[I(I+1)-Q^2+J(J+1)-Q^2\right]$$

Delta

$$\delta_{II'}\,\delta_{MM'}\,\delta_{JJ'}\,\frac{\delta_{QQ'}+\delta_{Q\bar{Q}'}(-1)^{I-J}}{\sqrt{1+\delta_{Q'0}}\sqrt{1+\delta_{Q0}}}\,\frac{1-(-1)^{j_1+j_2-J}}{2}\,\frac{1-(-1)^{j_1'+j_2'-J}}{2}(-1)^{s_1+s_2+2j_1+j_2+j_2'+J+l_1'+l_2}$$

$$(g)\,\frac{1}{4\pi}\,\sqrt{2l_1+1}\sqrt{2l_2'+1}\sqrt{2j_1+1}\sqrt{2j_2+1}\sqrt{2j_1'+1}\sqrt{2j_2'+1}\sqrt{2J+1}$$

$$\sum_{\lambda}\Big[(2\lambda+1)\begin{Bmatrix} l_2' & s_2 & j_2' \\ j_2 & \lambda & l_2 \end{Bmatrix}\begin{Bmatrix} l_1 & s_1 & j_1 \\ j_1' & \lambda & l_1' \end{Bmatrix}\begin{Bmatrix} j_1' & j_2' & J \\ j_2 & j_1 & \lambda \end{Bmatrix}$$

$$\langle l_2' 0 \lambda 0 | l_2 0\rangle \langle l_1 0 \lambda 0 | l_1' 0\rangle\Big]$$

Coriolis

$$
\begin{aligned}
&- \frac{\hbar^2}{3\mathscr{I}_0} \frac{\delta_{II'}\delta_{MM'}\delta_{JJ'}}{\sqrt{1+\delta_{Q'0}}\sqrt{1+\delta_{Q0}}} \frac{1-(-1)^{j_1+j_2-J}}{2} \frac{1-(-1)^{j_1'+j_2'-J}}{2} \\
&\Big[\delta_{Q',Q-1}\sqrt{(I+Q)(I-Q+1)}\sqrt{(J+Q)(J-Q+1)}+ \\
&\delta_{\bar{Q}',\bar{Q}-1}\sqrt{(I-Q)(I+Q+1)}\sqrt{(J-Q)(J+Q+1)}+ \\
&(-1)^{I-J}\big[\delta_{Q',\bar{Q}-1}\sqrt{(I-Q)(I+Q+1)}\sqrt{(J-Q)(J+Q+1)}+ \\
&\delta_{\bar{Q}',Q-1}\sqrt{(I+Q)(I-Q+1)}\sqrt{(J+Q)(J-Q+1)}\ \big]+ \\
&\delta_{Q',Q+1}\sqrt{(I-Q)(I+Q+1)}\sqrt{(J-Q)(J+Q+1)}+ \\
&\delta_{\bar{Q}',\bar{Q}+1}\sqrt{(I+Q)(I-Q+1)}\sqrt{(J+Q)(J-Q+1)}+ \\
&(-1)^{I-J}\big[\delta_{Q',\bar{Q}+1}\sqrt{(I+Q)(I-Q+1)}\sqrt{(J+Q)(J-Q+1)}+ \\
&\delta_{\bar{Q}',Q+1}\sqrt{(I-Q)(I+Q+1)}\sqrt{(J-Q)(J+Q+1)}\ \big]\Big]
\end{aligned}
$$

Deformation

$$
\begin{aligned}
&\pm\,\chi\beta\,\frac{\delta_{II'}\,\delta_{MM'}}{\sqrt{1+\delta_{Q'0}}\sqrt{1+\delta_{Q0}}}\,\frac{1-(-1)^{j_1+j_2-J}}{2}\,\frac{1-(-1)^{j_1'+j_2'-J}}{2}\,\sqrt{2J+1}\sqrt{\frac{5}{16\pi}}\\
&\Bigg(\delta_{QQ'}\Bigg[\delta_{j_2j_2'}\sqrt{2j_1+1}\langle JQ20|J'Q'\rangle\langle j_1\tfrac{1}{2}20|j_1'\tfrac{1}{2}\rangle\begin{Bmatrix} j_1 & j_2 & J\\ J' & 2 & j_1'\end{Bmatrix}(-1)^{J+j_1'+j_2}+\\
&\qquad\delta_{j_1j_1'}\sqrt{2j_2+1}\langle JQ20|J'Q'\rangle\langle j_2\tfrac{1}{2}20|j_2'\tfrac{1}{2}\rangle\begin{Bmatrix} j_2 & j_1 & J\\ J' & 2 & j_2'\end{Bmatrix}(-1)^{J+j_1+j_2'}+\\
&\qquad\delta_{j_2j_2'}\sqrt{2j_1+1}\langle J\bar{Q}20|J'\bar{Q}'\rangle\langle j_1\tfrac{1}{2}20|j_1'\tfrac{1}{2}\rangle\begin{Bmatrix} j_1 & j_2 & J\\ J' & 2 & j_1'\end{Bmatrix}(-1)^{J+j_1'+j_2}+\\
&\qquad\delta_{j_1j_1'}\sqrt{2j_2+1}\langle J\bar{Q}20|J'\bar{Q}'\rangle\langle j_2\tfrac{1}{2}20|j_2'\tfrac{1}{2}\rangle\begin{Bmatrix} j_2 & j_1 & J\\ J' & 2 & j_2'\end{Bmatrix}(-1)^{J+j_1+j_2'}\Bigg]+\\
&\delta_{Q\bar{Q}'}(-1)^{I-J}\Bigg[\delta_{j_2j_2'}\sqrt{2j_1+1}\langle JQ20|J'\bar{Q}'\rangle\langle j_1\tfrac{1}{2}20|j_1'\tfrac{1}{2}\rangle\begin{Bmatrix} j_1 & j_2 & J\\ J' & 2 & j_1'\end{Bmatrix}(-1)^{J+j_1'+j_2}+\\
&\qquad\delta_{j_1j_1'}\sqrt{2j_2+1}\langle JQ20|J'\bar{Q}'\rangle\langle j_2\tfrac{1}{2}20|j_2'\tfrac{1}{2}\rangle\begin{Bmatrix} j_2 & j_1 & J\\ J' & 2 & j_2'\end{Bmatrix}(-1)^{J+j_1+j_2'}+\\
&\qquad\delta_{j_2j_2'}\sqrt{2j_1+1}\langle J\bar{Q}20|J'Q'\rangle\langle j_1\tfrac{1}{2}20|j_1'\tfrac{1}{2}\rangle\begin{Bmatrix} j_1 & j_2 & J\\ J' & 2 & j_1'\end{Bmatrix}(-1)^{J+j_1'+j_2}+\\
&\qquad\delta_{j_1j_1'}\sqrt{2j_2+1}\langle J\bar{Q}20|J'Q'\rangle\langle j_2\tfrac{1}{2}20|j_2'\tfrac{1}{2}\rangle\begin{Bmatrix} j_2 & j_1 & J\\ J' & 2 & j_2'\end{Bmatrix}(-1)^{J+j_1+j_2'}\Bigg]\Bigg)
\end{aligned}
$$