

Tests of lepton flavor universality and CKM unitarity

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Developing New Directions in Fundamental Physics 2020

TRIUMF virtual meeting

- Precision-frontier tests of SM
- **Unitarity** of CKM matrix

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- **Lepton flavor universality**: $\ell = \{e, \mu, \tau\}$ only differ in masses, e.g., in charged current

$$\mathcal{L} = -i \frac{g_2}{\sqrt{2}} \bar{\ell}_i \gamma^\mu P_L \nu_j W_\mu (\delta_{ij} + \epsilon_{ij})$$

$\leftrightarrow \epsilon_{ij} = 0$ in SM

- This talk: review of current status and future prospects

Determination of V_{ud} from superallowed β decays

- Master formula [Hardy, Towner 2018](#)

$$|V_{ud}|^2 = \frac{2984.432(3) \text{ s}}{\mathcal{F}t(1 + \Delta_R^V)}$$

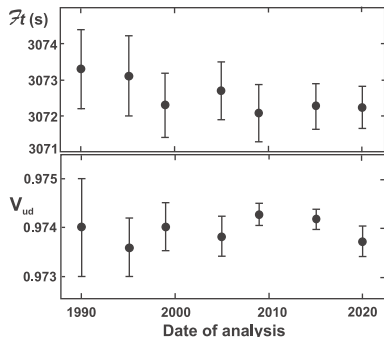
with (universal) radiative corrections Δ_R^V

- Value of V_{ud} crucially depends on Δ_R^V :

Ref.	Δ_R^V
Marciano, Sirlin 2006	0.02361(38)
Seng, Gorchtein, Patel, Ramsey-Musolf 2018	0.02467(22)
Czarnecki, Marciano, Sirlin 2019	0.02426(32)
Seng, Feng, Gorchtein, Jin 2020	0.02477(24)
Hayen 2020	0.02474(31)

- Possibly further nuclear corrections

[Miller, Schwenk 2008, 2009](#), [Seng, Gorchtein, Ramsey-Musolf 2018](#),
[Gorchtein 2018](#)



[Hardy, Towner 2020](#)

Determination of V_{ud} from neutron decay

- Master formula [Czarnecki, Marciano, Sirlin 2018](#)

$$|V_{ud}|^2 \tau_n (1 + 3g_A^2)(1 + \Delta_{RC}) = 5100.1(7) \text{ s}$$

with radiative corrections Δ_{RC}

\hookrightarrow need lifetime τ_n and asymmetry $\lambda = g_A/g_V$

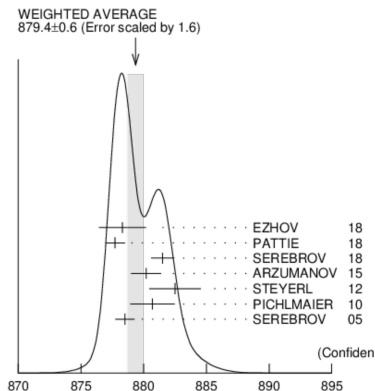
- PDG average only considers bottle experiments, latest [Pattie et al. 2018](#)

$$\tau_n = 877.7(7) \left(\begin{smallmatrix} +0.4 \\ -0.2 \end{smallmatrix} \right) \text{ s}$$

- Latest asymmetry measurement [Märkisch et al. 2019](#)

$$\lambda = g_A/g_V = -1.27641(56)$$

- Consistent with, but less precise than V_{ud} from superallowed β decays



PDG 2020

Determination of V_{ud} from pion β decay

- Master formula [Cirigliano, Knecht, Neufeld, Pichl 2003](#)
[Czarnecki, Marciano, Sirlin 2020](#), [Feng et al. 2020](#)

$$\Gamma(\pi^+ \rightarrow \pi^0 e^+ \nu_e(\gamma)) = \frac{G_F^2 |V_{ud}|^2 M_\pi^5 |f_+^\pi(0)|^2}{64\pi^3} \times (1 + \Delta_{RC}^{\pi\ell}) I_{\pi\ell}$$

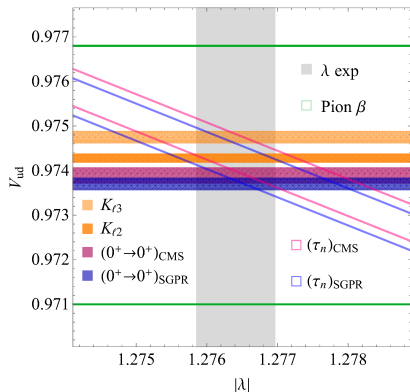
with radiative corrections $\Delta_{RC}^{\pi\ell}$

\hookrightarrow need branching fraction and pion life time

- Resulting V_{ud} extracted from [PIBETA 2004](#) still with relatively large error compared to other β decays

$$V_{ud} = 0.9739(29)$$

- Good consistency among all β -decay constraints, including mirror decays [Hayen 2020](#)



[Crivellin, MH 2020](#)

Indirect determination of V_{ud} from kaon decays: $K_{\ell 2}$

- Kaon-decay constraints derived **assuming**

CKM unitarity

- $K_{\ell 2}$ decays: $K \rightarrow \ell \nu_\ell$

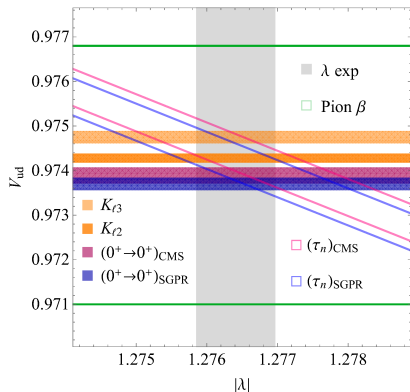
$$\frac{V_{us}}{V_{ud}} \frac{F_K}{F_\pi} = \left(\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu (\gamma) M_\pi)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu (\gamma) M_K)} \right)^{1/2} \times \frac{1 - \frac{m_\mu^2}{M_\pi^2}}{1 - \frac{m_\mu^2}{M_K^2}} \left(1 - \frac{\Delta_{RC}^K - \Delta_{RC}^\pi}{2} \right)$$

↪ typically consider the ratio over $\pi_{\ell 2}$

- Need input for:

- F_K/F_π
- Isospin-breaking corrections

↪ lattice QCD, ChPT



Crivellin, MH 2020

Indirect determination of V_{ud} from kaon decays: $K_{\ell 3}$

- **$K_{\ell 3}$ decays:** $K \rightarrow \pi \ell \nu_\ell$

$$\Gamma(K \rightarrow \pi \ell \nu_\ell(\gamma)) = \frac{C_K^2 G_F^2 |V_{us}|^2 M_K^5 |f_+^{K\pi}(0)|^2}{192\pi^3} \times (1 + \Delta_{RC}^{K\ell}) I_{K\ell}$$

$\hookrightarrow \ell = \mu, e$ and two charge channels

- Need input for:

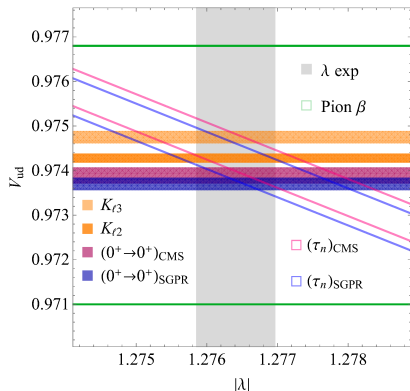
- $K\pi$ form factor $f_+^{K\pi}(0)$
- Radiative corrections

\hookrightarrow lattice QCD, ChPT

- Tensions between

- $K_{\ell 2}$ and $K_{\ell 3}$ decays
- Kaon decays and β decays

\hookrightarrow apparent violation of CKM unitarity



Crivellin, MH 2020

V_{ud} tension as a sign for the violation of lepton flavor universality?

- Let us parameterize the **W couplings** as $\mathcal{L} = -i\frac{g_2}{\sqrt{2}}\bar{\ell}_i\gamma^\mu P_L\nu_j W_\mu(\delta_{ij} + \epsilon_{ij})$
- Modifies Fermi constant in **muon decay**

$$\frac{1}{\tau_\mu} = \frac{(G_F^{\mathcal{L}})^2 m_\mu^5}{192\pi^3} (1 + \Delta q)(1 + \epsilon_{ee} + \epsilon_{\mu\mu})^2$$

\hookrightarrow measured Fermi constant $G_F = G_F^{\mathcal{L}}(1 + \epsilon_{ee} + \epsilon_{\mu\mu})$

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- All β -decay observables affected according to

$$V_{ud} \rightarrow V_{ud}^\beta = V_{ud}^{\mathcal{L}}(1 - \varepsilon_{\mu\mu})$$

where $V_{ij}^{\mathcal{L}}$ fulfill CKM unitarity

- Construct ratio [Crivellin, MH 2020](#)

$$R(V_{us}) \equiv \frac{V_{us}^{K_{\mu 2}}}{V_{us}^\beta} \equiv \frac{V_{us}^{K_{\mu 2}}}{\sqrt{1 - (V_{ud}^\beta)^2 - |V_{ub}|^2}} = 1 - \left(\frac{V_{ud}}{V_{us}}\right)^2 \varepsilon_{\mu\mu} + \mathcal{O}(\varepsilon^2)$$

\hookrightarrow LFUV effect enhanced by $(V_{ud}/V_{us})^2 \sim 20!$

V_{ud} tension as a sign for the violation of lepton flavor universality?

Observable	Measurement	Constraint $\times 10^3$
$\frac{K \rightarrow \pi \mu \bar{\nu}}{K \rightarrow \pi e \bar{\nu}} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	1.0010(25)	1.0(2.5)
$\frac{K \rightarrow \mu \nu}{K \rightarrow e \nu} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	0.9978(18)	-2.2(1.8)
$\frac{\pi \rightarrow \mu \nu}{\pi \rightarrow e \nu} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	1.0010(9)	1.0(9)
$\frac{\tau \rightarrow \mu \nu \bar{\nu}}{\tau \rightarrow e \nu \bar{\nu}} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	1.0018(14)	1.8(1.4)
$\frac{W \rightarrow \mu \bar{\nu}}{W \rightarrow e \bar{\nu}} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	0.9960(100)	-4(10)
$\frac{B \rightarrow D^{(*)} \mu \nu}{B \rightarrow D^{(*)} e \nu} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	0.9890(120)	-11(12)
$R(V_{us}) \simeq 1 - \left(\frac{V_{ud}}{V_{us}}\right)^2 \varepsilon_{\mu\mu}$	0.9891(33) (SGPR)	0.58(17)
	0.9927(39) (CMS)	0.39(21)

- Most stringent constraint on $\varepsilon_{\mu\mu}$ thanks to **CKM enhancement**
- Could explain tension between β decays and kaon decays, but not between $K_{\ell 2}$ and $K_{\ell 3}$ (right-handed currents?)
- Best constraint on $\varepsilon_{\mu\mu} - \varepsilon_{ee}$ from

$$R_{e/\mu}^{\pi} = \frac{\Gamma(\pi \rightarrow e \nu_e(\gamma))}{\Gamma(\pi \rightarrow \mu \nu_{\mu}(\gamma))}$$

Searching for the violation of lepton flavor universality in $R_{e/\mu}^{\pi,K}$

- SM prediction at two loops [Cirigliano, Rosell 2007](#)

$$R_{e/\mu}^{\pi} = 1.2352(1) \times 10^{-4}$$

$$R_{e/\mu}^K = 2.477(1) \times 10^{-5}$$

↪ see [talk by Vincenzo Cirigliano](#)

- Experimental status [PDG 2020](#)

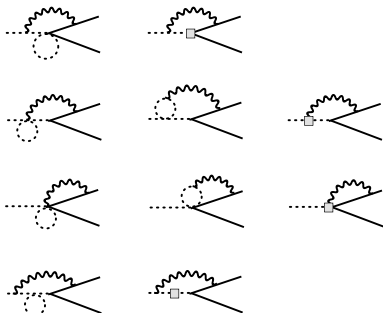
$$R_{e/\mu}^{\pi} = 1.2327(23) \times 10^{-4}$$

$$R_{e/\mu}^K = 2.488(9) \times 10^{-5}$$

↪ in both cases theory far ahead!

- Future improvements from PEN, PiENU

($R_{e/\mu}^{\pi}$) and J-PARC E36 ($R_{e/\mu}^K$)



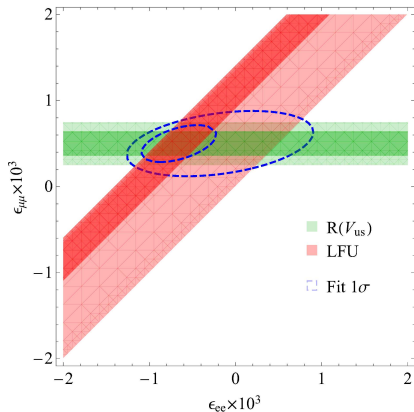
[Cirigliano, Rosell 2007](#)

Projections in $\epsilon_{ee}-\epsilon_{\mu\mu}$ plane

- Potential improvements for $R(V_{us})$
 - Radiative corrections for β decays
 - Improved measurements of τ_n and g_A
 - New data on $K_{\ell 3}$ decays

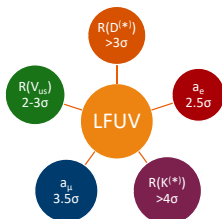
↪ assumed $\sqrt{2}$ due to neutron decay
- Potential improvements for $\epsilon_{\mu\mu} - \epsilon_{ee}$
 - **Factor 3 from PEN/PiENU**
 - Factor 3 for τ decays from Belle II

↪ would probe ϵ_{ee} below $\mathcal{O}(10^{-3})$

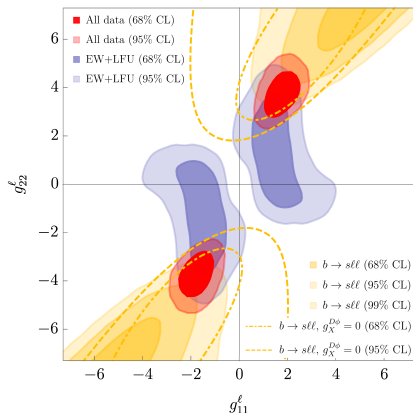


Crivellin, MH 2020

Correlations with B anomalies?



- Modified W couplings \Rightarrow expect **modified Z couplings** from $SU(2)_L$ invariance
- Example: heavy W', Z' (“**vector triplet**”)
- W' for β decay tension
- Z' can explain $b \rightarrow sll$ anomalies
- Via $SU(2)_L$: constraints from electroweak precision observables



Capdevila, Crivellin, Manzari, Montull 2020

$g_{11/22}^l$: electron/muon coupling

- For $K_{\ell 2}$ and $\pi_{\ell 2}$ decays one uses the ratio

$$\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu(\gamma))}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu(\gamma))} = \left(\frac{V_{us}}{V_{ud}} \frac{F_K}{F_\pi} \right)^2 \frac{M_K}{M_\pi} \left(\frac{1 - \frac{m_\mu^2}{M_K^2}}{1 - \frac{m_\mu^2}{M_\pi^2}} \right)^2 \left(1 + \Delta_{RC}^K - \Delta_{RC}^\pi \right)$$

to cancel uncertainties and extract V_{us}/V_{ud}

- Can do the same for $K_{\ell 3}$ and $\pi_{\ell 3}$! [Czarnecki, Marciano, Sirlin 2020](#)
- Pion β decay not competitive for V_{us} , but combined with $K_{\ell 3}$ decays improvement by a factor 2–3 would suffice to obtain a competitive value of V_{us}/V_{ud}

- Generalize master formula to include **effective operators** not present in SM

$$\Gamma(\pi^+ \rightarrow \pi^0 e^+ \nu_e(\gamma)) = \frac{G_F^2 |V_{ud}|^2}{192\pi^3 M_\pi^3} (1 + \Delta_{RC}^{\pi\ell}) \int_{m_e^2}^{(M_\pi - M_{\pi^0})^2} ds \lambda^{3/2}(s) \left(1 + \frac{m_e^2}{2s}\right) \left(1 - \frac{m_e^2}{s}\right)^2$$

$$\times \left[|V(s)|^2 + |A(s)|^2 + \frac{4(s - m_e^2)^2}{9sm_e^2} |T(s)|^2 + \frac{3m_e^2(M_\pi^2 - M_{\pi^0}^2)^2}{(2s + m_e^2)\lambda(s)} (|S(s)|^2 + |P(s)|^2) \right]$$

with $V(s)$, $A(s)$, ... depending on Wilson coefficients C_V , C_A , ...

- Tensor:** $T(s) = \frac{3s}{2s+m_e^2} \frac{m_e}{M_\pi} c_T B_T^\pi(s)$
 \leftrightarrow suppressed by electron mass and tensor form factor
- Scalar:** more competitive constraints, but still not at the same level as other β decays [Falkowski, Gonzáles-Alonso, Naviliat-Cuncic 2020](#)
- More on rare pion decays in [talk by Vincenzo Cirigliano](#)

Testing Lepton Flavor Universality and CKM Unitarity with Rare Pion Decays

LOI for Snowmass 2020 Discussion

August 27, 2020

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Abstract

We describe the physics motivation and concept of a next-generation experiment to measure the charged-pion branching ratio to electrons vs. muons, $R_{e/\mu}$, which is extremely sensitive to new physics at high mass scales. The proposed detector system will also measure pion beta decay, $\pi^+ \rightarrow \pi^0 e^+ \nu(\gamma)$, and other rare decays to high precision. Order of magnitude improvements in sensitivity to these reactions will probe lepton universality at an unprecedented level, determine V_{ud} in a theoretically pristine manner and test CKM unitarity at the quantum loop level.

- Tensions between β decays and kaon decays point to the **apparent violation of CKM unitarity**
- Could be interpreted as a hint for the **violation of lepton flavor universality**
 \hookrightarrow relation to B anomalies and $(g - 2)_{\mu, e}$?
- $R(V_{us})$ ideal probe of LFU due to **CKM enhancement**
- Interesting parameter space probed by forthcoming results on $R_{e/\mu}^{\pi}$
- Next-generation PiENu experiment could
 - bring experimental error for $R_{e/\mu}^{\pi}$ even closer to the theoretical uncertainty
 - improve pion β decay and extraction of V_{us}/V_{ud}