Developing New Directions in Fundamental Physics
Nov 4-6 2020
Working Group on Pion and Muon physics

Universality of weak interactions and rare pion decays

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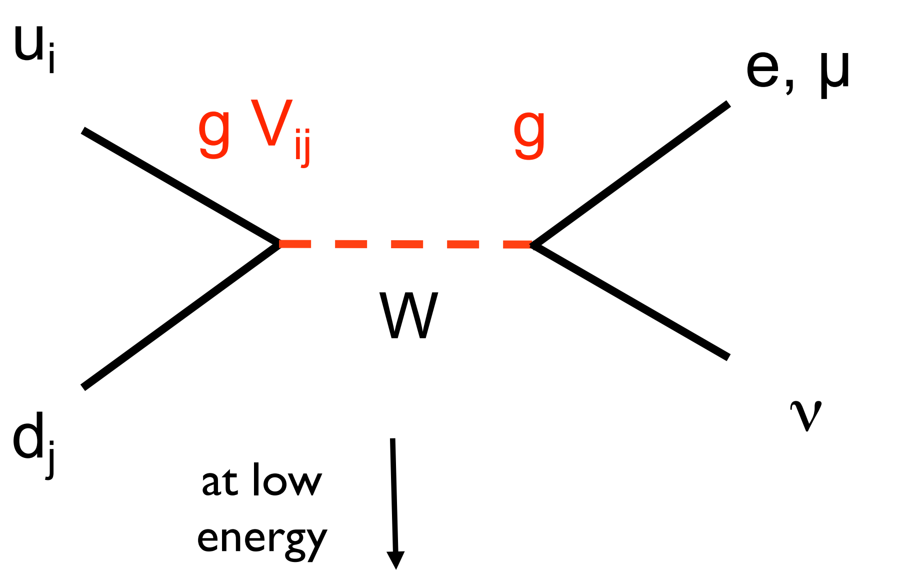
Universality of weak interactions

- CC processes the SM are mediated by W exchange between L-handed fermions $\Rightarrow$ exhibit universality relations



$G_F^{(\beta)} \sim g^2 V_{ij}/M_W^2 \sim G_F^{(\mu)}\, V_{ij}$

Lepton universality

$$[G_F]_e/[G_F]_\mu = 1$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Cabibbo universality
(Quark-Lepton universality)

- Rare pion decays offer a theoretically ‘pristine’ way to test the SM universality and probe BSM effects

$\pi \to e\nu$

$\pi^\pm \to \pi^0 e^\pm \nu$

See Martin Hoferichter’s talk

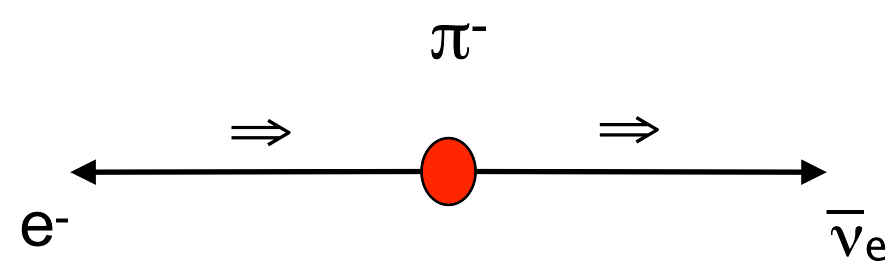
Outline

- The Standard Model baseline: theoretical status of
  - $R_{e/\mu}^{(\pi)} = \Gamma(\pi \to e\nu(\gamma)) / \Gamma(\pi \to \mu\nu(\gamma))$
  - $\Gamma(\pi^{\pm} \to \pi^0 e^{\pm}\nu(\gamma))$
- Sensitivity to BSM physics
  - Light and weakly coupled particles
  - UV new physics: lepton and Cabibbo universality

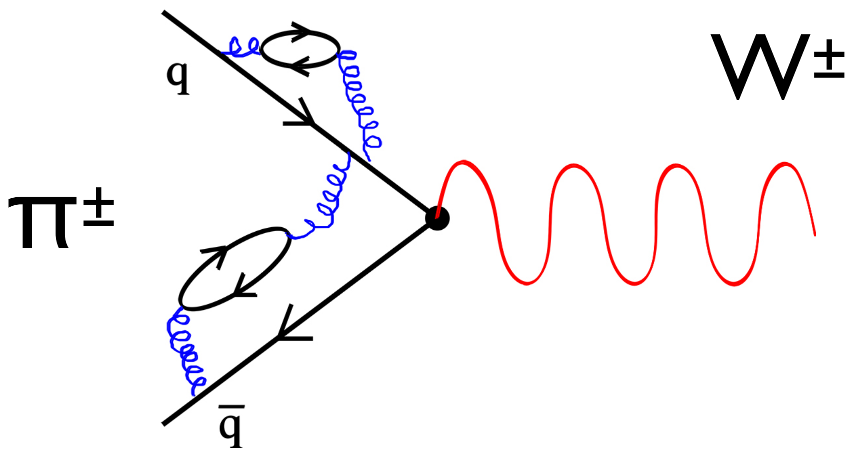
The SM baseline

$R_{e/\mu}^{(\pi)} = \Gamma(\pi \to e\nu(\gamma)) / \Gamma(\pi \to \mu\nu(\gamma))$ in the SM

- Helicity suppressed the SM (V-A structure), zero if $m_e \to 0$



- Despite involving a hadron, this ratio can be predicted with high precision. Why?



Theoretical analysis of $R_{e/\mu}(\pi,K)$

P = (π,K)

$$R_{e/\mu}^{(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2$$

- $F_{\pi,K}$ drops in the e/μ ratio → hadronic structure dependence appears only through EM corrections
- Organize calculation in EFT (ChPT): $p^2 \sim \frac{m_\pi^2, m_K^2, m_\ell^2}{(4\pi F_\pi)^2}$

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- $F_{\pi,K}$ drops in the e/μ ratio → hadronic structure dependence appears only through EM corrections
- Organize calculation in EFT (ChPT): $p^2 \sim \frac{m_\pi^2, m_K^2, m_\ell^2}{(4\pi F_\pi)^2}$
- NLO correction ↔ point-like mesons (Kinoshita 59)

No contact (LEC): contribution cancels in the ratio !

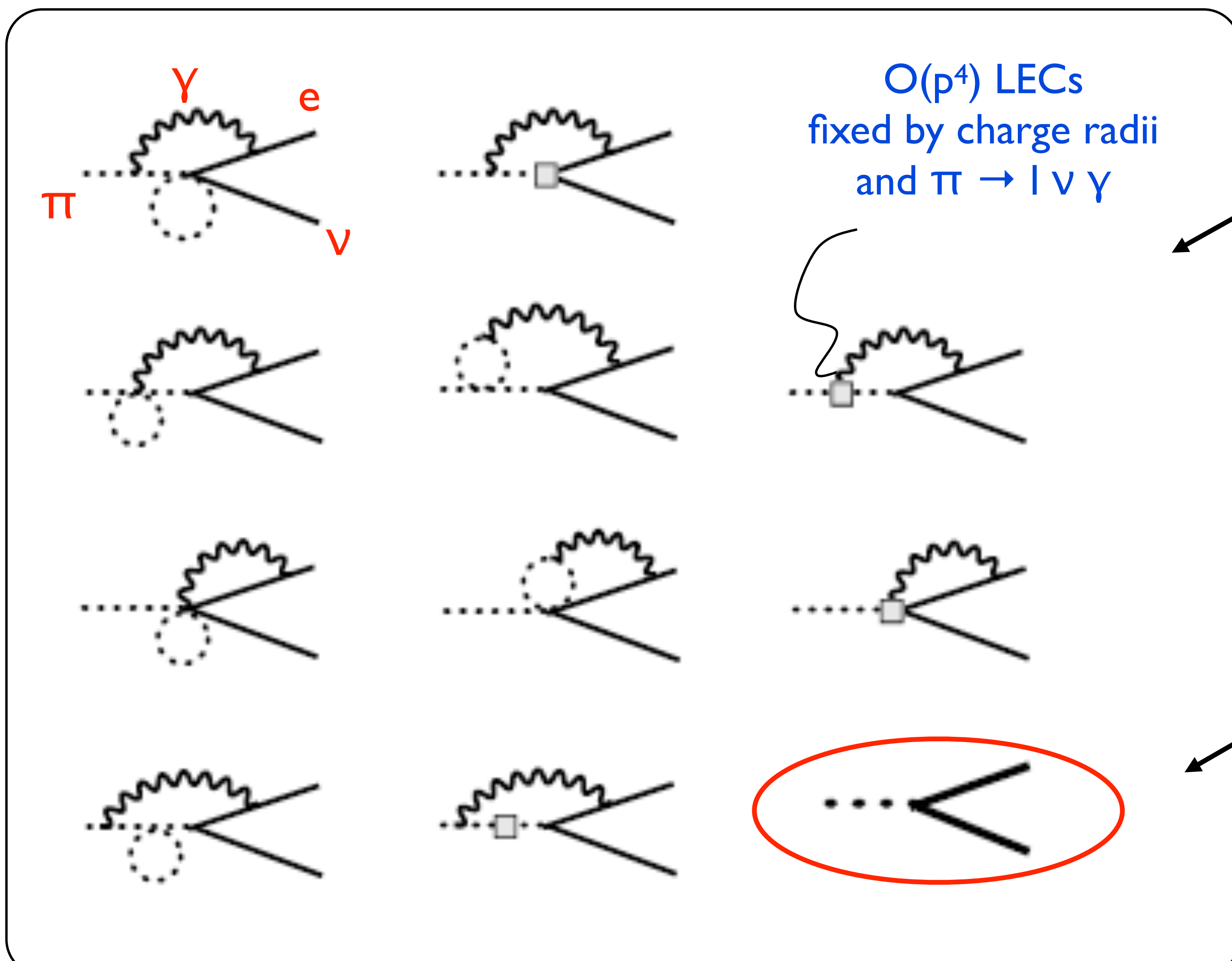
$$\Delta_{e^2p^2} \sim -3\alpha/\pi \log m_\mu/m_e \sim -3.7\%.$$

Theoretical analysis of $R_{e/\mu}(\pi,K)$

P = (π,K)

$$R_{e/\mu}^{(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2 \times \left[1 + \Delta_{e^2p^2}^{(P)} + \Delta_{e^2p^4}^{(P)} + \ldots\right]$$

- Structure dependence appears at NNLO in ChPT!



1) One- and two-loop diagrams ⇒ model-independent single and double logs

2) $O(e^2p^4)$ Low Energy Constant (LEC):

- Same for π and K

- Estimated within large-N_C inspired resonance model (satisfying QCD s.d. constraints). Small contribution to final result

3) Real photon emission

Theoretical analysis of $R_{e/\mu}(\pi,K)$

P = (π,K)

$$R_{e/\mu}^{(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2 \times \left[1 + \Delta_{e^2p^2}^{(P)} + \Delta_{e^2p^4}^{(P)} + \ldots\right]$$

Theory

$$R_{e/\mu}^{(\pi)} = 1.2352(1) \times 10^{-4}$$

$$R_{e/\mu}^{(K)} = 2.477(1) \times 10^{-5}$$

VC-Rosell 0707.3439, PRL

Experiment

$$R_{e/\mu}^{(\pi)} = 1.2327(23) \times 10^{-4}$$

$$R_{e/\mu}^{(K)} = 2.488(10) \times 10^{-5}$$

PIENU Coll. , PRL 2015
PDG 2020

NA62 Coll. , PLB 2013

Theory result provides robust baseline for new physics searches.
Might be further improved in the next decade by lattice QCD

Pion beta decay

- Decay rate

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu(\gamma)) = \frac{G_\mu^2 |V_{\rm ud}|^2 m_{\pi^+}^5 \left|f_+^\pi(0)\right|^2}{64\pi^3}(1 + {\rm RC}_\pi) I_\pi\,,$$

Pion beta decay

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- Phase space

$$I_\pi = 7.376(1) \times 10^{-8}$$

$$\sim \left(\frac{m_{\pi^+} - m_{\pi^0}}{m_{\pi^+}}\right)^5$$

Pion beta decay

- Decay rate

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu(\gamma)) = \frac{G_\mu^2 |V_{\rm ud}|^2 m_{\pi^+}^5 \, \left| f_+^\pi(0) \right|^2}{64\pi^3} (1 + \mathrm{RC}_\pi) I_\pi \,,$$

- Phase space

$$I_\pi = 7.376(1) \times 10^{-8} \qquad \sim \left(\frac{m_{\pi^+} - m_{\pi^0}}{m_{\pi^+}} \right)^5$$

- Vector form factor at t=0, controlled by isospin and its breaking

$$\langle \pi^0(p_0) | \bar{d} \gamma_\mu u | \pi^+(p_+) \rangle = \sqrt{2}\, f_+(t) \; (p_+ + p_0)_\mu \qquad t = (p_+ - p_0)^2$$

$$f_+(0) = 1 - \frac{1}{(4\pi F_\pi)^2} \frac{\left(M_{K^+}^2 - M_{K_0}^2\right)^2_{\rm QCD}}{24 M_K^2} = 1 + O\left(\frac{m_u - m_d}{\Lambda_{\rm QCD}} \right)^2$$

Behrends-Sirlin 1962

VC-Neufeld-Pichl hep-ph/0209226, EPJC

Pion beta decay

- Decay rate

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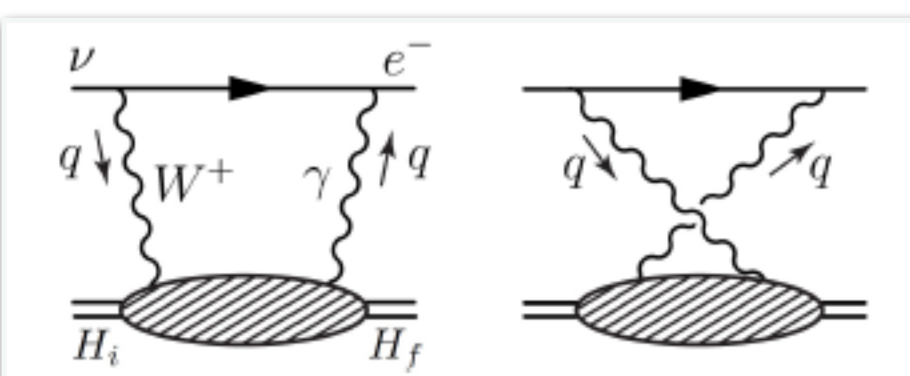
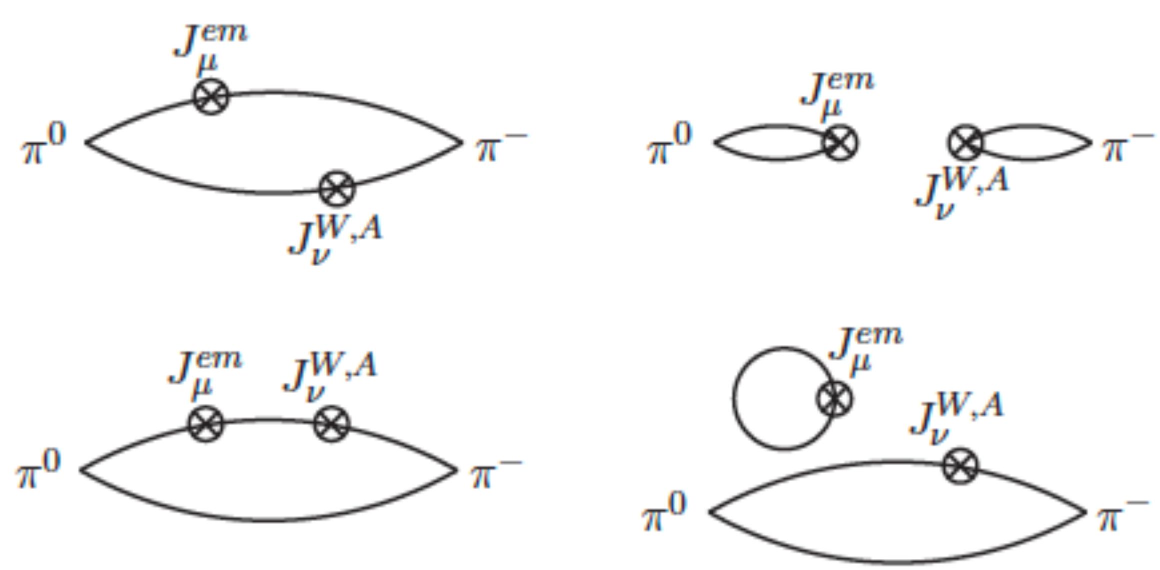
- Radiative corrections: ChPT to $O(e^2p^2)$ → Lattice QCD

$${\rm RC}_\pi = 0.0342(10) \quad ({\rm ChPT}) \longrightarrow {\rm RC}_\pi = 0.0332(1)_{\gamma W}(3)_{HO} \quad ({\rm LQCD})$$

Sirlin 1978
VC-Neufeld-Pichl 2002, EPJC
Desxotes-Genon Moussallam 2005, EPJC
Passera et al., 2011

Feng, Gorchtein, Jin, Ma, Seng , 2003.09798, PRL





Pion beta decay

- Decay rate

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu(\gamma)) = \frac{G_\mu^2 |V_{\rm ud}|^2 m_{\pi^+}^5 \, \left| f_+^\pi(0) \right|^2}{64\pi^3} (1 + {\rm RC}_\pi) I_\pi \,,$$

- Current extraction of V_{ud}

$$V_{ud} = 0.9739(28)_{\rm exp}(1)_{\rm th}$$

- 0.3% uncertainty dominated by BR = $1.036(6) \times 10^{-8}$

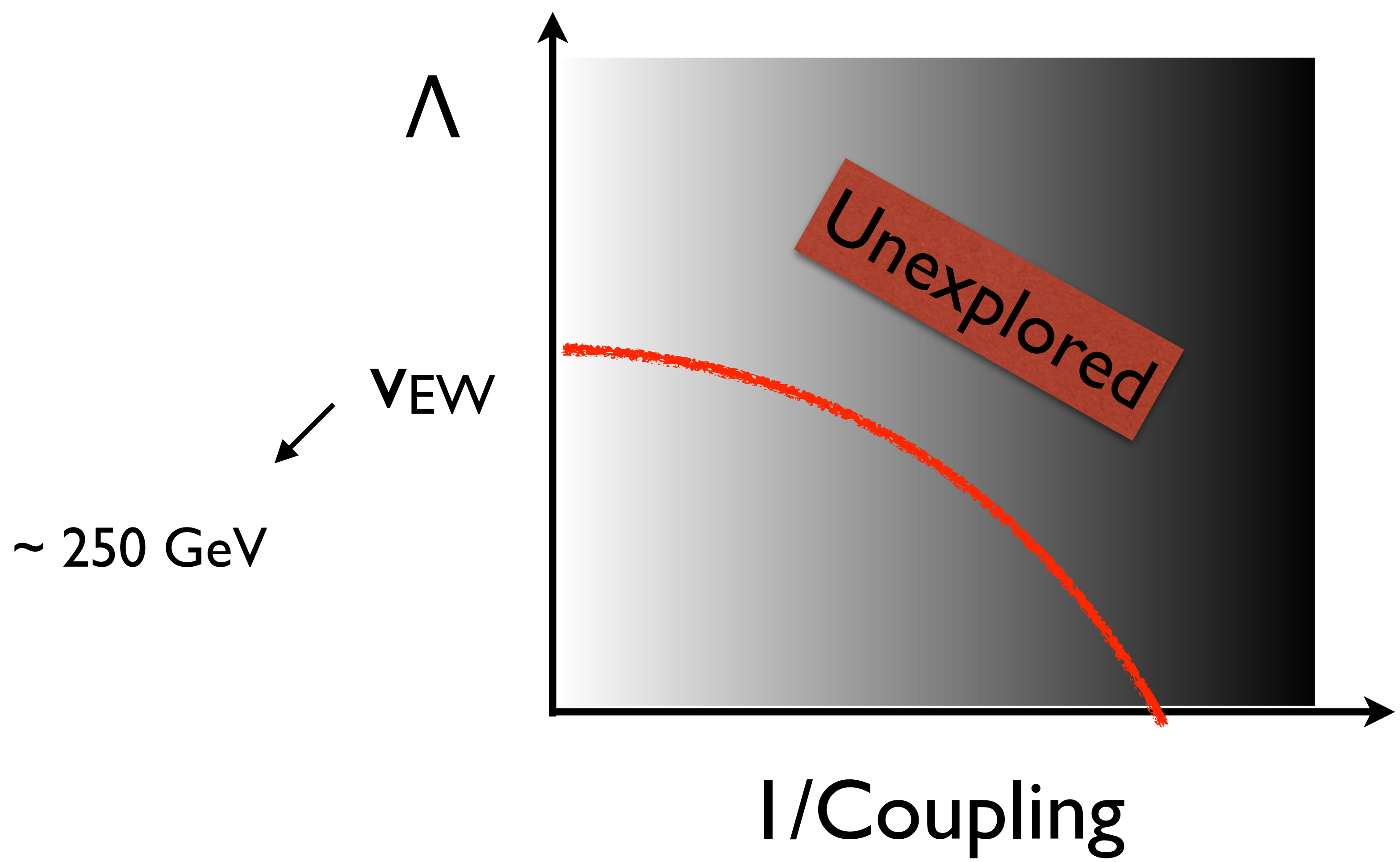
PIBETA Coll. , hep-ex/031230, PRL

(Will discuss impact on Cabibbo universality tests in 2nd part of the talk)

Probing BSM physics with rare pion decays

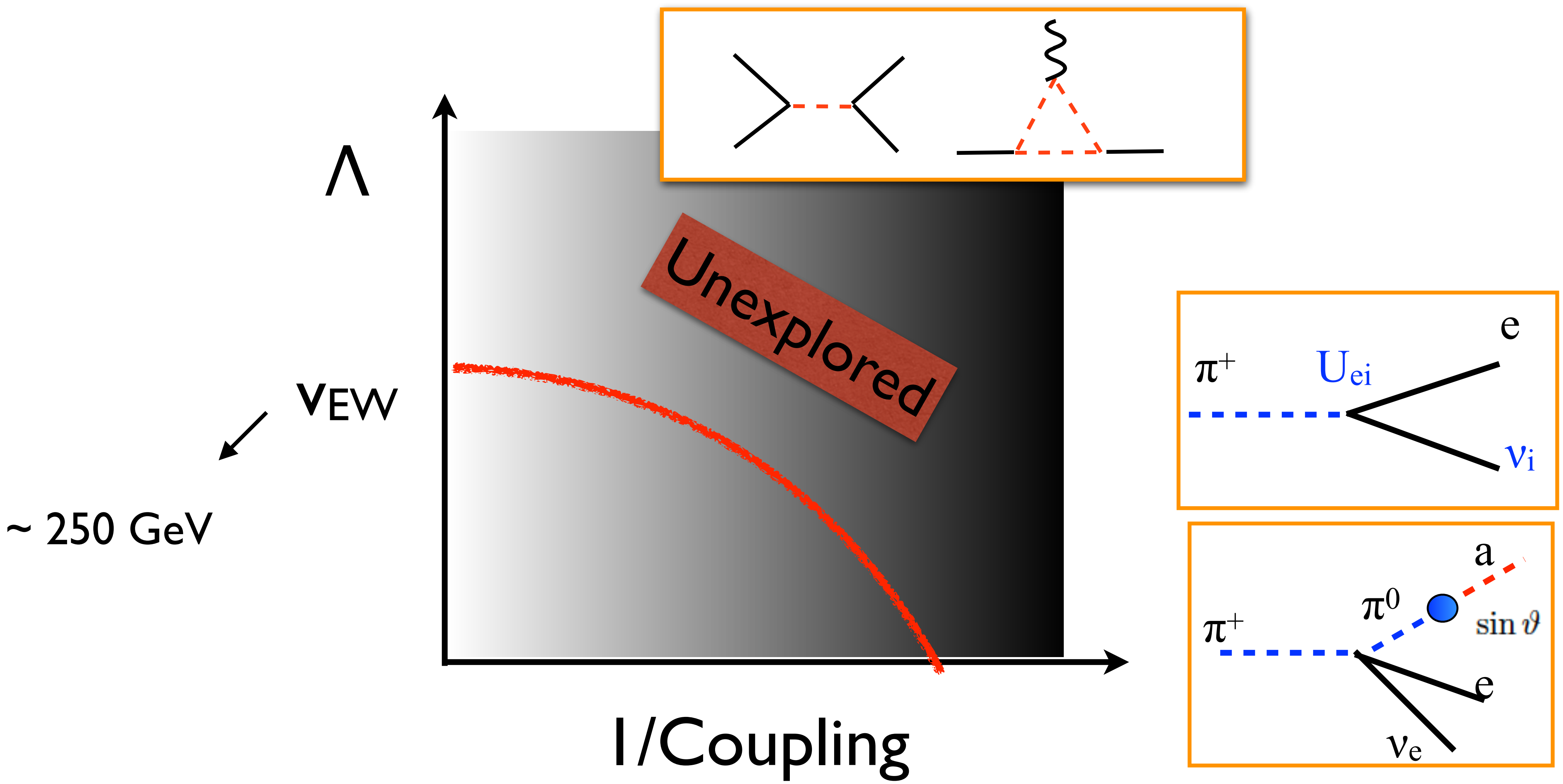
BSM sensitivity

- What kind of new physics are rare pion decays probing?
- Light and weakly coupled? Heavy?



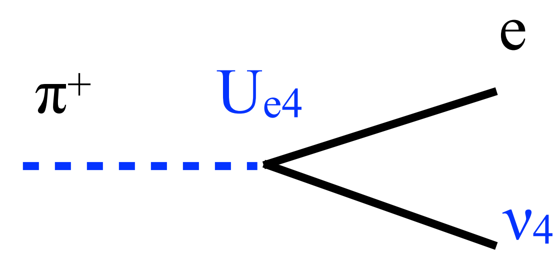
BSM sensitivity

- What kind of new physics are rare pion decays probing?
- Light and weakly coupled? Heavy? Both!



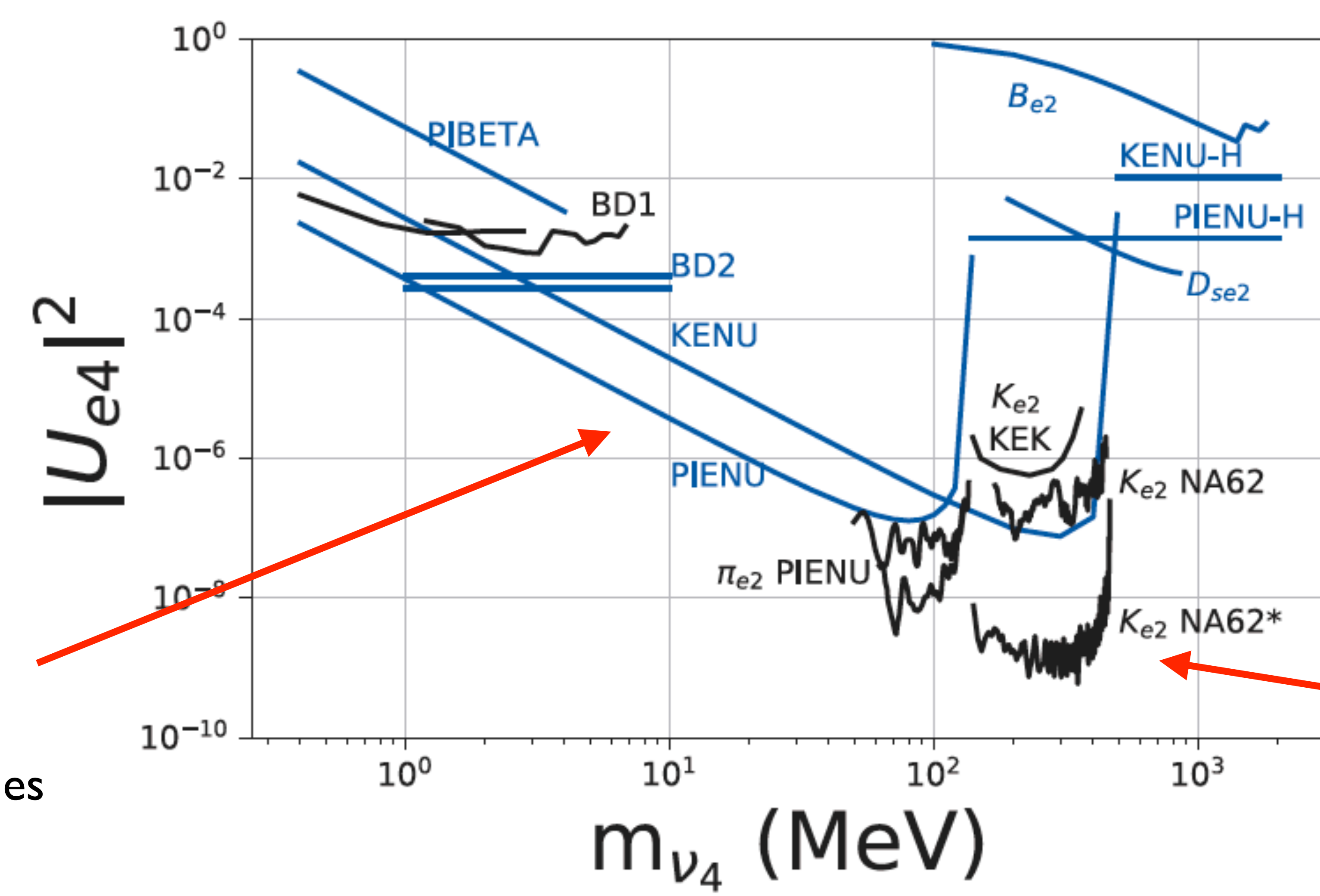
Sterile neutrinos

- Sensitivity to sterile neutrino mass & mixing
- $\pi \rightarrow e\nu_4$ provides strongest bounds for $m_{\nu 4} \sim 1\text{-}140$ MeV



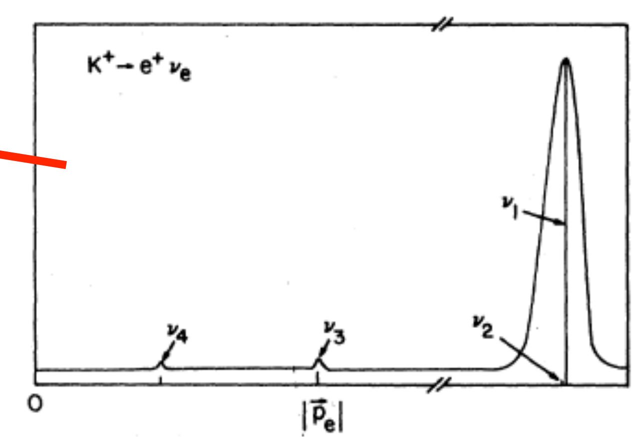
$$\nu_\ell = \sum_{i=1}^{3+n_s} U_{\ell i}\, \nu_i$$

Bryman and Shrock, 1904.06787, 1909.11198, PRD



$R_{e/\mu}(\pi)$ assuming bounds on $U_{\mu 4}$ from peak searches

Peak searches

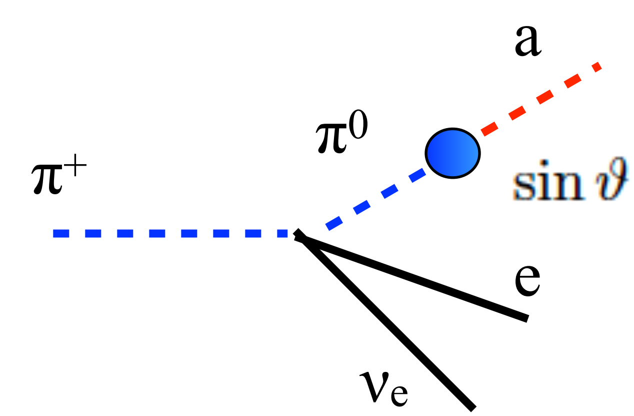


Shrock 1980-81

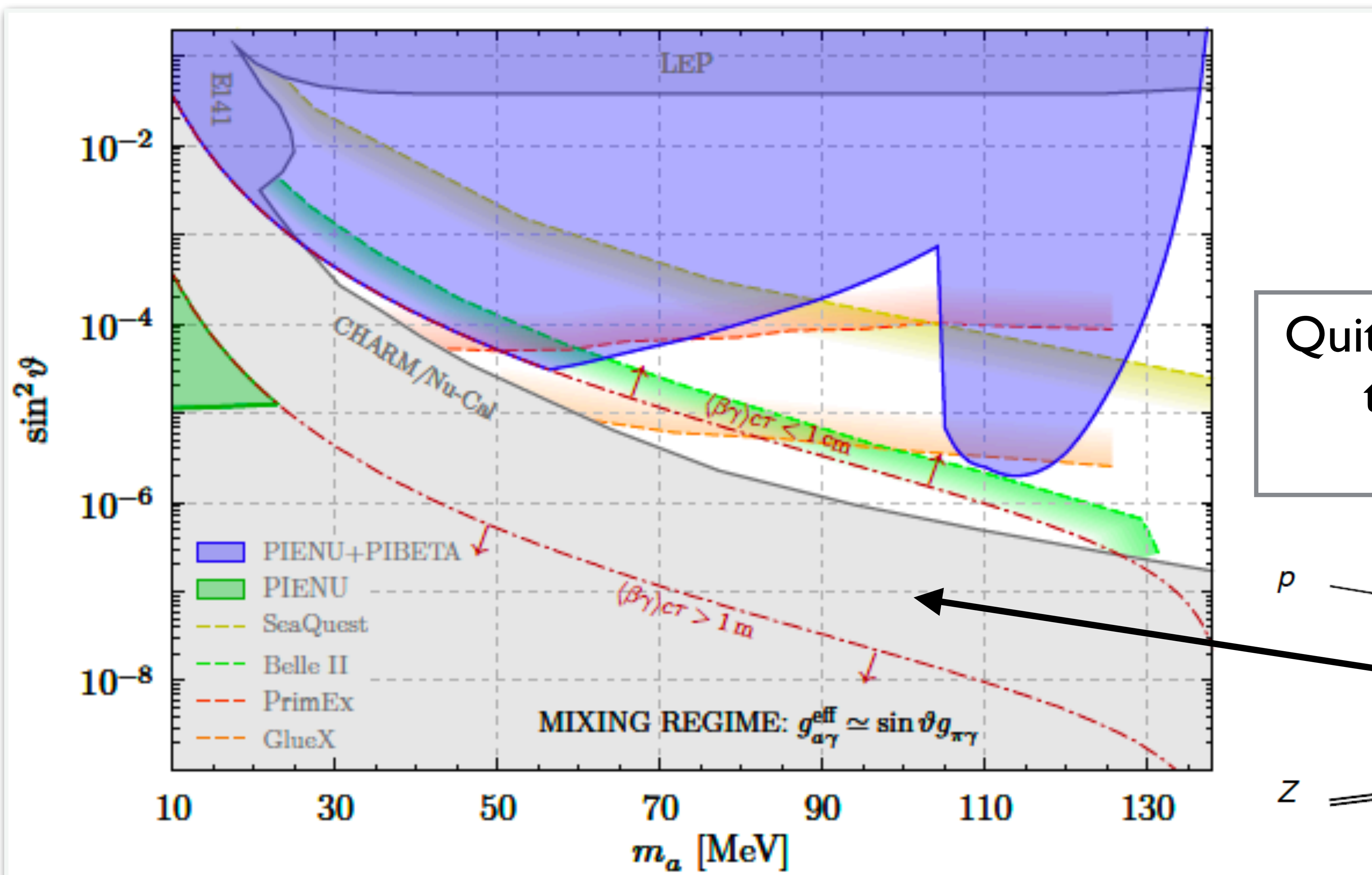
See talk by D. Bryman

Axion-like particles

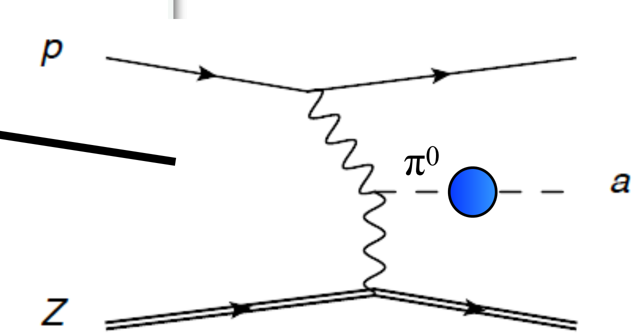
- a-π^0 mixing induces the decay $\pi^+ \to a e \nu$
- Would affect E_{cal} distribution in PIENU and the $\gamma\gamma$ opening angle distribution in PIBETA



Altmanshofer-Gori-Robinson 1909.00005



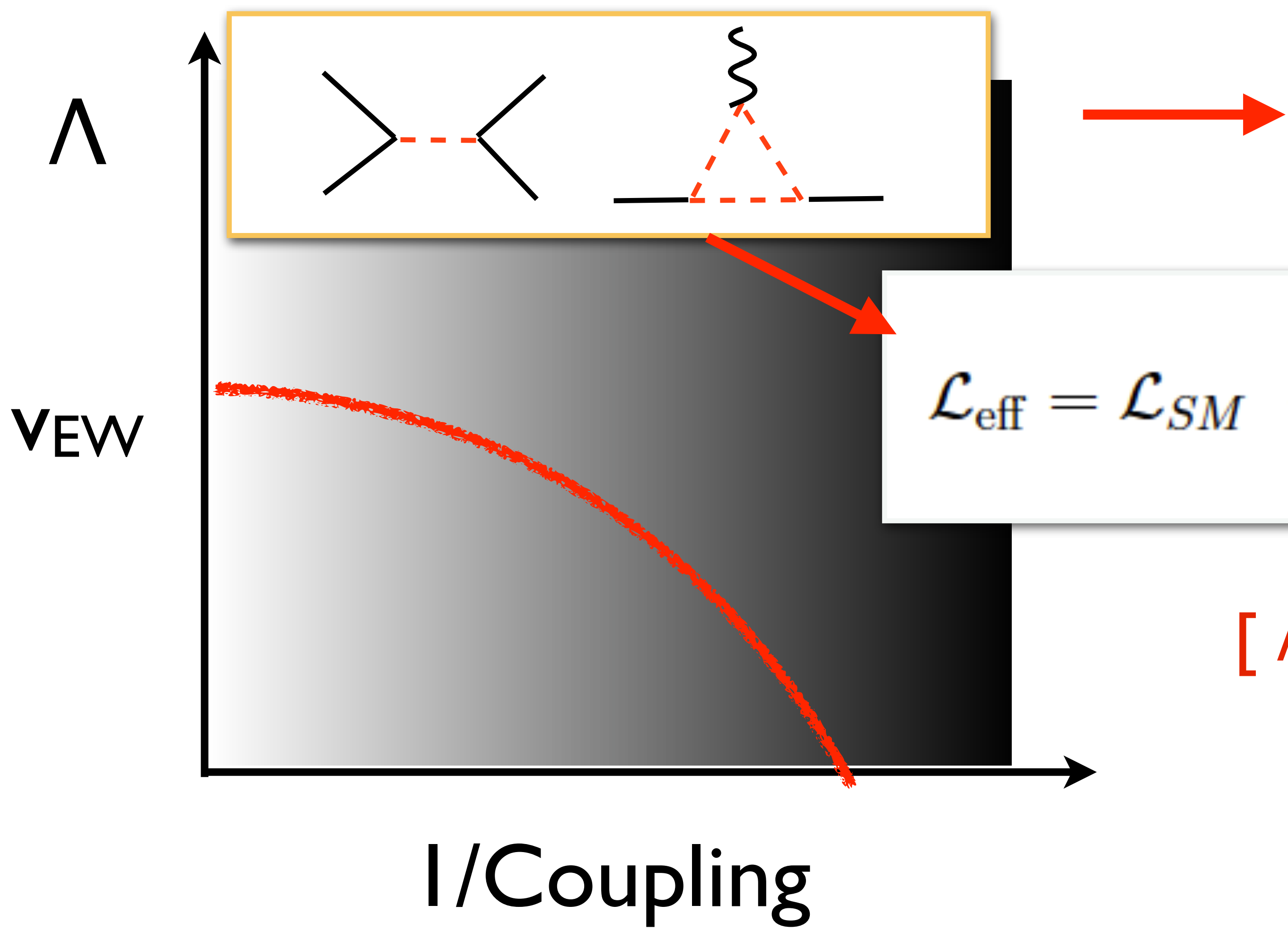
Quite complementary to beam dump experiments



Sensitivity to UV new physics

See talk by D. Bryman

- Many models: charged Higgs, leptoquarks, LRSM, SUSY, VLL, …
- Their effect captured by 'low-energy' effective theory at E << Λ



$$\mathcal{L}_{\rm eff} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

[$\Lambda \leftrightarrow M_{\rm Heavy}$] $C_i\ [g_{\rm BSM},\ M_a/M_b]$

Low-energy effective Lagrangian (I)

VC, Gonzalez-Alonso, Jenkins 0908.1754, NPB

VC, Graesser, Gonzalez-Alonso 1210.4553, JHEP

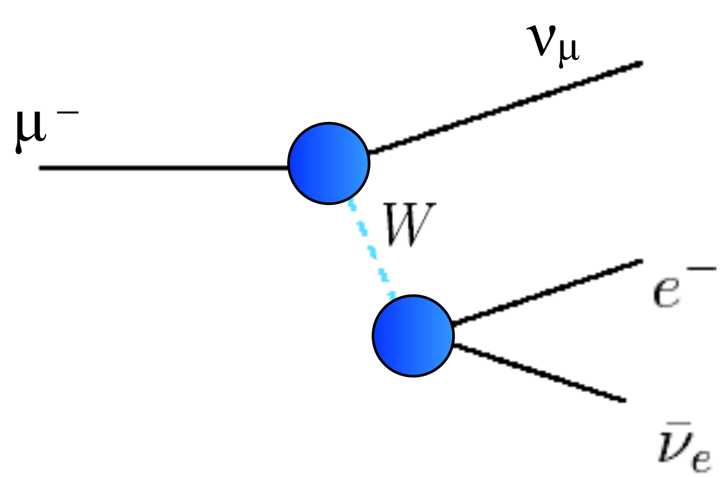
Leptonic interactions

$$\mathcal{L}_{CC}^{(\mu)} = -\frac{G_F^{(0)}}{\sqrt{2}}\left(1+\epsilon_L^{(\mu)}\right)\bar{e}\gamma^\rho(1-\gamma_5)\nu_e\cdot\bar{\nu}_\mu\gamma_\rho(1-\gamma_5)\mu \; + \; \ldots$$

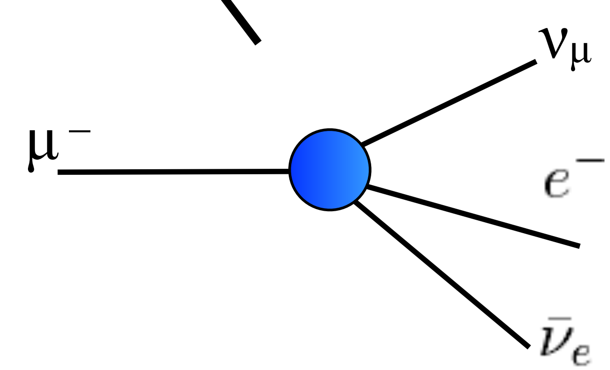
$$G_F^{(\mu)} = G_F^{(0)}\left(1+\epsilon_L^{(\mu)}\right)$$

$$\epsilon_L^{(\mu)} = \epsilon_{W\ell}^{ee} + \epsilon_{W\ell}^{\mu\mu} + \epsilon_{4\ell}$$

$\varepsilon_i \sim (v/\Lambda)^2$



Vertex corrections



4-fermion contact interaction

Low-energy effective Lagrangian (2)

VC, Gonzalez-Alonso, Jenkins 0908.1754, NPB

VC, Graesser, Gonzalez-Alonso 1210.4553, JHEP

Semi-leptonic interactions

$$\begin{aligned}\mathcal{L}_{\mathrm{CC}} = & -\frac{G_F^{(0)} V_{ud}}{\sqrt{2}} \times \Big[\left(\delta^{ab} + \epsilon_L^{ab}\right) \bar{e}_a \gamma_\mu (1-\gamma_5) \nu_b \cdot \bar{u} \gamma^\mu (1-\gamma_5) d \\ & + \epsilon_R^{ab} \, \bar{e}_a \gamma_\mu (1-\gamma_5) \nu_b \cdot \bar{u} \gamma^\mu (1+\gamma_5) d \\ & + \epsilon_S^{ab} \, \bar{e}_a (1-\gamma_5) \nu_b \cdot \bar{u} d \\ & - \epsilon_P^{ab} \, \bar{e}_a (1-\gamma_5) \nu_b \cdot \bar{u} \gamma_5 d \\ & + \epsilon_T^{ab} \, \bar{e}_a \sigma_{\mu\nu} (1-\gamma_5) \nu_b \cdot \bar{u} \sigma^{\mu\nu} (1-\gamma_5) d \Big] + \mathrm{h.c.}\end{aligned}$$

$\varepsilon_i \sim (v/\Lambda)^2$

Low-energy effective Lagrangian (2)

VC, Gonzalez-Alonso, Jenkins 0908.1754, NPB

VC, Graesser, Gonzalez-Alonso 1210.4553, JHEP

Semi-leptonic interactions

$$\mathcal{L}_{\rm CC} = -\frac{G_F^{(0)} V_{ud}}{\sqrt{2}} \times \Big[\left(\delta^{ab} + \epsilon_L^{ab}\right) \bar{e}_a \gamma_\mu (1-\gamma_5) \nu_b \cdot \bar{u} \gamma^\mu (1-\gamma_5) d$$

$$+ \epsilon_R^{ab} \bar{e}_a \gamma_\mu (1-\gamma_5) \nu_b \cdot \bar{u} \gamma^\mu (1+\gamma_5) d$$

$$+ \epsilon_S^{ab} \bar{e}_a (1-\gamma_5) \nu_b \cdot \bar{u} d$$

$$- \epsilon_P^{ab} \bar{e}_a (1-\gamma_5) \nu_b \cdot \bar{u} \gamma_5 d$$

$$+ \epsilon_T^{ab} \bar{e}_a \sigma_{\mu\nu} (1-\gamma_5) \nu_b \cdot \bar{u} \sigma^{\mu\nu} (1-\gamma_5) d \Big] + \text{h.c.}$$

$\varepsilon_i \sim (v/\Lambda)^2$

$$+ \quad \epsilon_i \longrightarrow \tilde{\epsilon}_i \qquad (1-\gamma_5)\nu_\ell \overset{**}{\longrightarrow} (1+\gamma_5)\nu_\ell$$

← Interference with SM suppressed by m_ν/E: quadratic sensitivity to $\tilde{\varepsilon}_i$

Low-energy effective Lagrangian (2)

VC, Gonzalez-Alonso, Jenkins 0908.1754, NPB

VC, Graesser, Gonzalez-Alonso 1210.4553, JHEP

Semi-leptonic interactions

$$\mathcal{L}_{CC} = -\frac{G_F^{(0)} V_{ud}}{\sqrt{2}} \times \Big[\left(\delta^{ab} + \epsilon_L^{ab}\right) \bar{e}_a \gamma_\mu (1-\gamma_5) \nu_b \cdot \bar{u} \gamma^\mu (1-\gamma_5) d$$

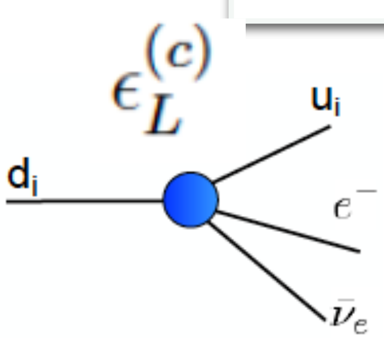
$$+ \epsilon_R^{ab} \; \bar{e}_a \gamma_\mu (1-\gamma_5) \nu_b \cdot \bar{u} \gamma^\mu (1+\gamma_5) d$$

$$+ \epsilon_S^{ab} \; \bar{e}_a (1-\gamma_5) \nu_b \cdot \bar{u} d$$

$$- \epsilon_P^{ab} \; \bar{e}_a (1-\gamma_5) \nu_b \cdot \bar{u} \gamma_5 d$$

$$+ \epsilon_T^{ab} \; \bar{e}_a \sigma_{\mu\nu} (1-\gamma_5) \nu_b \cdot \bar{u} \sigma^{\mu\nu} (1-\gamma_5) d \Big] + \text{h.c.}$$

$\varepsilon_i \sim (v/\Lambda)^2$

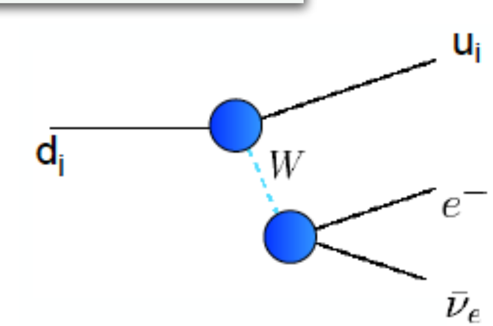


$$\epsilon_L = \epsilon_L^{(v)} + \epsilon_L^{(c)}$$

Vertex + contact 4-Fermi

$$[\epsilon_L^{(v)}]^{ab} = \epsilon_{W\ell}^{ab} + \epsilon_{Wq}$$

W-lepton and W-quark vertex corrections



Low-energy effective Lagrangian (2)

VC, Gonzalez-Alonso, Jenkins 0908.1754, NPB

VC, Graesser, Gonzalez-Alonso 1210.4553, JHEP

$$\frac{G_F^{(\mu)} V_{ud}}{\sqrt{2}} \left(1 - \epsilon_L^{(\mu)}\right)$$

Semi-leptonic interactions

$$\begin{aligned}
\mathcal{L}_{CC} = & -\frac{G_F^{(0)} V_{ud}}{\sqrt{2}} \times \Big[\left(\delta^{ab} + \epsilon_L^{ab}\right) \bar{e}_a \gamma_\mu (1-\gamma_5) \nu_b \cdot \bar{u} \gamma^\mu (1-\gamma_5) d \\
& + \epsilon_R^{ab} \, \bar{e}_a \gamma_\mu (1-\gamma_5) \nu_b \cdot \bar{u} \gamma^\mu (1+\gamma_5) d \\
& + \epsilon_S^{ab} \, \bar{e}_a (1-\gamma_5) \nu_b \cdot \bar{u} d \\
& - \epsilon_P^{ab} \, \bar{e}_a (1-\gamma_5) \nu_b \cdot \bar{u} \gamma_5 d \\
& + \epsilon_T^{ab} \, \bar{e}_a \sigma_{\mu\nu} (1-\gamma_5) \nu_b \cdot \bar{u} \sigma^{\mu\nu} (1-\gamma_5) d \Big] + \text{h.c.}
\end{aligned}$$

$\varepsilon_i \sim (v/\Lambda)^2$

Beta decays sensitive to

$$\epsilon_L^{ee} - \epsilon_L^{(\mu)} = -\epsilon_{W\ell}^{\mu\mu} + \epsilon_{Wq} + [\epsilon_L^{(c)}]^{ee} - \epsilon_{4\ell}$$

$R_{e/\mu}^{(\pi)}$ BSM master formula

$$r_\pi \equiv \frac{R_{e/\mu}^{(\pi)}}{\left[R_{e/\mu}^{(\pi)}\right]^{\mathrm{SM}}} = 0.9980(18)$$

$R_{e/\mu}^{(\pi)}$ BSM master formula

(taking into account that ν flavor is not observed)

$$\frac{R_{e/\mu}^{(\pi)}}{\left[R_{e/\mu}^{(\pi)}\right]^{\rm SM}} = \frac{\left[\left|1+\epsilon_L^{ee}-\epsilon_R^{ee}-\frac{B_0}{m_e}\epsilon_P^{ee}\right|^2+\left|\frac{B_0}{m_e}\epsilon_P^{e\mu}\right|^2+\left|\frac{B_0}{m_e}\epsilon_P^{e\tau}\right|^2+\sum_\alpha\left|\frac{B_0}{m_e}\tilde{\epsilon}_P^{e\alpha}\right|^2\right]}{\left[\left|1+\epsilon_L^{\mu\mu}-\epsilon_R^{\mu\mu}-\frac{B_0}{m_\mu}\epsilon_P^{\mu\mu}\right|^2+\left|\frac{B_0}{m_\mu}\epsilon_P^{\mu e}\right|^2+\left|\frac{B_0}{m_\mu}\epsilon_P^{\mu\tau}\right|^2+\sum_\alpha\left|\frac{B_0}{m_\mu}\tilde{\epsilon}_P^{\mu\alpha}\right|^2\right]}$$

$$r_\pi \equiv \frac{R_{e/\mu}^{(\pi)}}{\left[R_{e/\mu}^{(\pi)}\right]^{\rm SM}} = 0.9980(18)$$

$R_{e/\mu}(\pi)$ BSM master formula

(taking into account that ν flavor is not observed)

$$\frac{R_{e/\mu}^{(\pi)}}{\left[R_{e/\mu}^{(\pi)}\right]^{SM}} = \frac{\left[\left|1+\epsilon_L^{ee}-\epsilon_R^{ee}-\frac{B_0}{m_e}\epsilon_P^{ee}\right|^2+\left|\frac{B_0}{m_e}\epsilon_P^{e\mu}\right|^2+\left|\frac{B_0}{m_e}\epsilon_P^{e\tau}\right|^2+\sum_\alpha\left|\frac{B_0}{m_e}\tilde{\epsilon}_P^{e\alpha}\right|^2\right]}{\left[\left|1+\epsilon_L^{\mu\mu}-\epsilon_R^{\mu\mu}-\frac{B_0}{m_\mu}\epsilon_P^{\mu\mu}\right|^2+\left|\frac{B_0}{m_\mu}\epsilon_P^{\mu e}\right|^2+\left|\frac{B_0}{m_\mu}\epsilon_P^{\mu\tau}\right|^2+\sum_\alpha\left|\frac{B_0}{m_\mu}\tilde{\epsilon}_P^{\mu\alpha}\right|^2\right]}$$

- BSM axial-current contribution $\epsilon_A \equiv \epsilon_L - \epsilon_R$

$$-1.9\times 10^{-3} < \epsilon_A^{ee} - \epsilon_A^{\mu\mu} < -0.1\times 10^{-3}$$

$\Lambda_A \sim 5.5$ TeV

$R_{e/\mu}^{(\pi)}$ BSM master formula

(taking into account that ν flavor is not observed)

$$\frac{R_{e/\mu}^{(\pi)}}{\left[R_{e/\mu}^{(\pi)}\right]^{\rm SM}} = \frac{\left[\left|1+\epsilon_L^{ee}-\epsilon_R^{ee}-\frac{B_0}{m_e}\epsilon_P^{ee}\right|^2+\left|\frac{B_0}{m_e}\epsilon_P^{e\mu}\right|^2+\left|\frac{B_0}{m_e}\epsilon_P^{e\tau}\right|^2+\sum_\alpha\left|\frac{B_0}{m_e}\tilde{\epsilon}_P^{e\alpha}\right|^2\right]}{\left[\left|1+\epsilon_L^{\mu\mu}-\epsilon_R^{\mu\mu}-\frac{B_0}{m_\mu}\epsilon_P^{\mu\mu}\right|^2+\left|\frac{B_0}{m_\mu}\epsilon_P^{\mu e}\right|^2+\left|\frac{B_0}{m_\mu}\epsilon_P^{\mu\tau}\right|^2+\sum_\alpha\left|\frac{B_0}{m_\mu}\tilde{\epsilon}_P^{\mu\alpha}\right|^2\right]}$$

- BSM pseudoscalar contribution: not helicity suppressed!

$$B_0(\mu) \equiv \frac{M_\pi^2}{m_u(\mu)+m_d(\mu)}$$

$$B_0/m_e = 3.6 \times 10^3$$

@ μ = 2 GeV

- Non-interfering \`wrong' neutrino flavor & R-handed neutrino contributions

- LFU violation ↔ $[\varepsilon_P]^{\alpha\beta} \neq \kappa\, m_\alpha$

$R_{e/\mu}^{(\pi)}$ BSM master formula

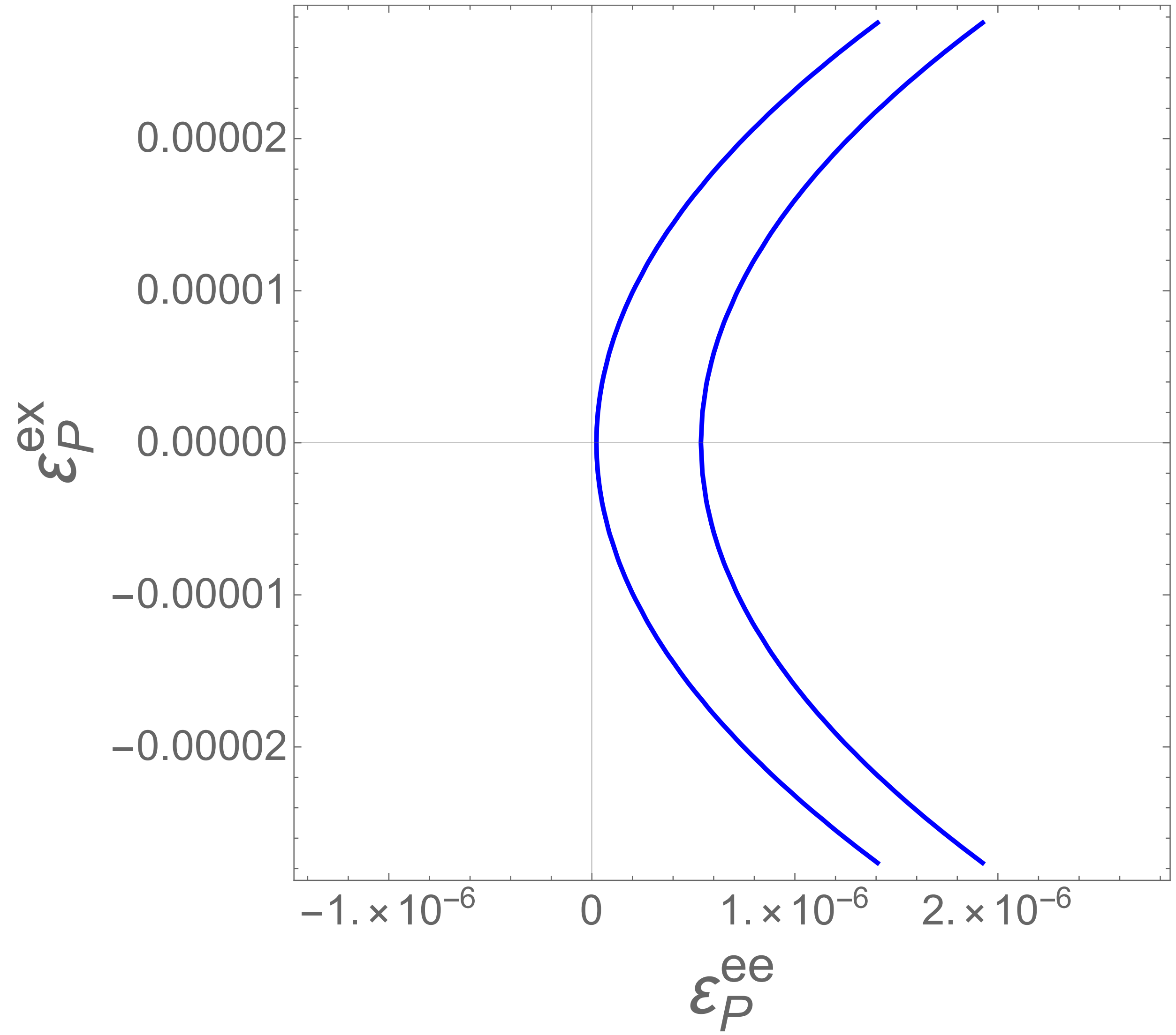
(taking into account that ν flavor is not observed)

$$\frac{R_{e/\mu}^{(\pi)}}{\left[R_{e/\mu}^{(\pi)}\right]^{SM}} = \frac{\left[\left|1+\epsilon_L^{ee}-\epsilon_R^{ee}-\frac{B_0}{m_e}\epsilon_P^{ee}\right|^2+\left|\frac{B_0}{m_e}\epsilon_P^{e\mu}\right|^2+\left|\frac{B_0}{m_e}\epsilon_P^{e\tau}\right|^2+\sum_\alpha\left|\frac{B_0}{m_e}\tilde{\epsilon}_P^{e\alpha}\right|^2\right]}{\left[\left|1+\epsilon_L^{\mu\mu}-\epsilon_R^{\mu\mu}-\frac{B_0}{m_\mu}\epsilon_P^{\mu\mu}\right|^2+\left|\frac{B_0}{m_\mu}\epsilon_P^{\mu e}\right|^2+\left|\frac{B_0}{m_\mu}\epsilon_P^{\mu\tau}\right|^2+\sum_\alpha\left|\frac{B_0}{m_\mu}\tilde{\epsilon}_P^{\mu\alpha}\right|^2\right]}$$

- Single operator analysis ($\varepsilon_P^{ex}=0$) gives impressive constraint

$$\epsilon_P^{ee} < 5.4 \times 10^{-7}$$

$\Lambda_P \sim 330$ TeV



$R_{e/\mu}(\pi)$ BSM master formula

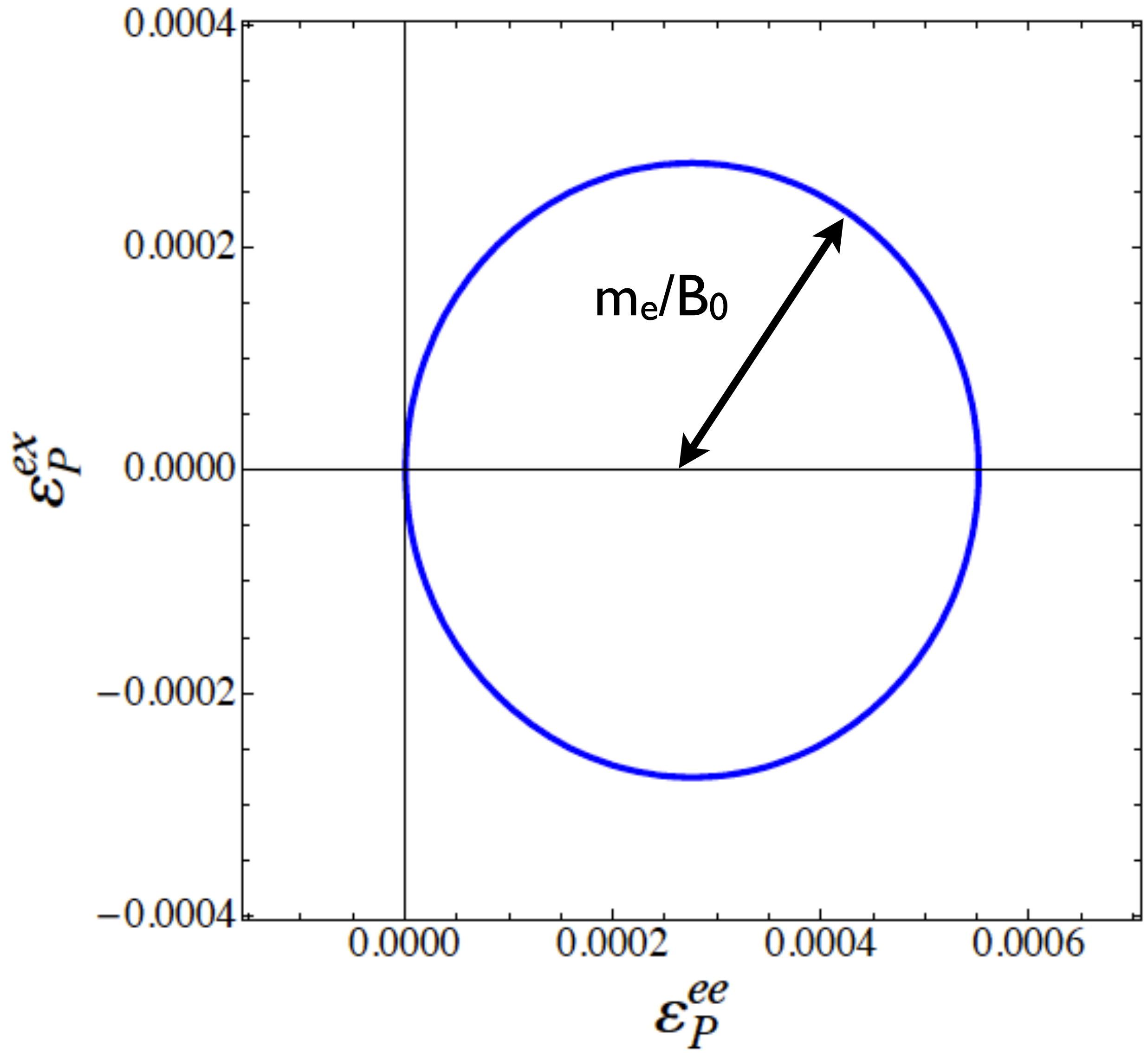
(taking into account that ν flavor is not observed)

$$\frac{R^{(\pi)}_{e/\mu}}{\left[R^{(\pi)}_{e/\mu}\right]^{SM}} = \frac{\left[\left|1+\epsilon_L^{ee}-\epsilon_R^{ee}-\frac{B_0}{m_e}\epsilon_P^{ee}\right|^2+\left|\frac{B_0}{m_e}\epsilon_P^{e\mu}\right|^2+\left|\frac{B_0}{m_e}\epsilon_P^{e\tau}\right|^2+\sum_\alpha\left|\frac{B_0}{m_e}\tilde{\epsilon}_P^{e\alpha}\right|^2\right]}{\left[\left|1+\epsilon_L^{\mu\mu}-\epsilon_R^{\mu\mu}-\frac{B_0}{m_\mu}\epsilon_P^{\mu\mu}\right|^2+\left|\frac{B_0}{m_\mu}\epsilon_P^{\mu e}\right|^2+\left|\frac{B_0}{m_\mu}\epsilon_P^{\mu\tau}\right|^2+\sum_\alpha\left|\frac{B_0}{m_\mu}\tilde{\epsilon}_P^{\mu\alpha}\right|^2\right]}$$

- In general, allowed region is an annulus of radius m_e/B_0 and thickness $(m_e/B_0) \times \delta r_\pi \sim 5\times10^{-7}$
- Marginalize w.r.t. ε_P^{ex}

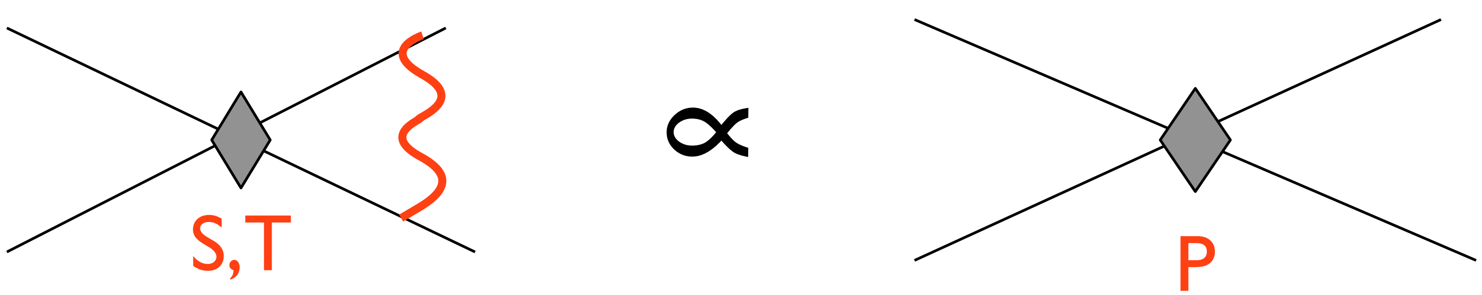
$$\epsilon_P^{ee} < 5.5 \times 10^{-4}$$

$\Lambda_P \sim 10$ TeV



$R_{e/\mu}(\pi)$ and scalar / tensor operators

- $R_{e/\mu}(\pi)$ constrains *any* new physics that induces ε_A and ε_P at low E
- Notably, S and T operators at high scale Λ induce via radiative corrections the P operator at the low scale μ



Voloshin '92
Campbell-Maybury '05
Herczeg 95

$$\gamma_{SP}\,\epsilon_S + \gamma_{TP}\,\epsilon_T < \frac{5.5\times 10^{-4}}{\log(\Lambda/\mu)}$$

$$\gamma_{SP} = \frac{15}{72}\frac{\alpha_1}{\pi} \approx 6.7\times 10^{-4}$$
$$\gamma_{TP} = -\frac{9}{2}\frac{\alpha_2}{\pi} - \frac{15}{2}\frac{\alpha_1}{\pi} \approx -7.3\times 10^{-2}$$

- Using marginalized bound on ε_P^{ee} with $\Lambda\sim 1$ TeV results in $|\varepsilon_S| < 0.1$ and $|\varepsilon_T| < 10^{-3}$. Single operator bound is 10^3 stronger!

$R_{e/\mu}^{(\pi)}$ and scalar / tensor operators

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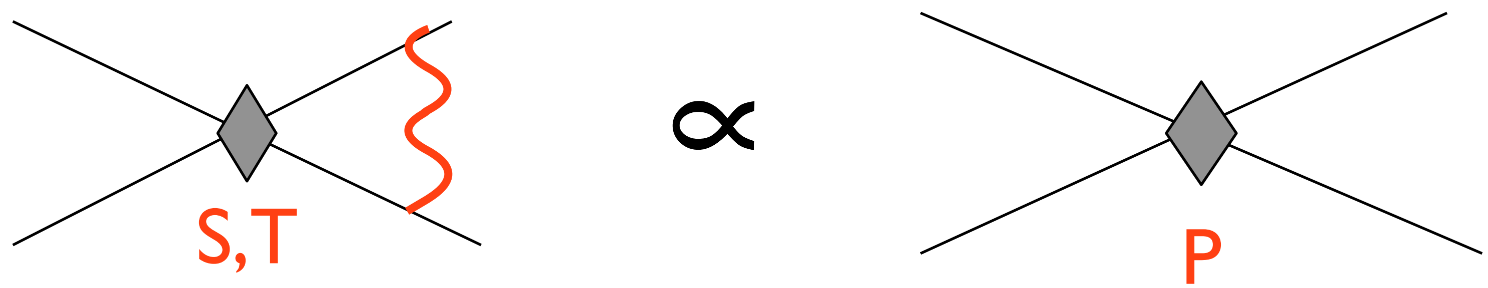

Voloshin '92
Campbell-Maybury '05
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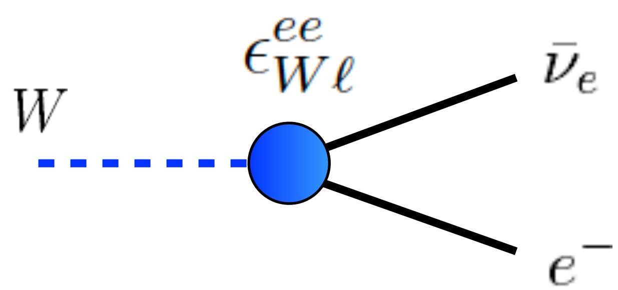
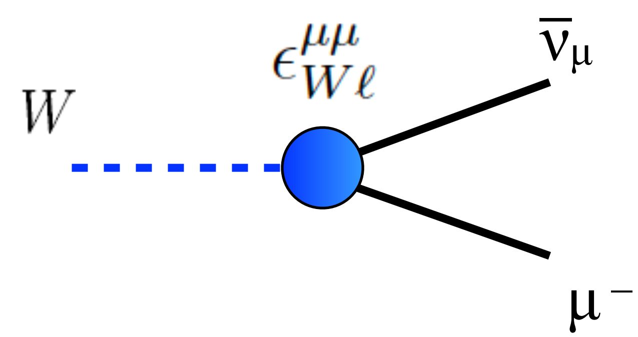
$R_{e/\mu}^{(\pi)}$ is a powerful probe of *all* charged-current operator structures!

- Using marginalized bound on ε_P^{ee} with $\Lambda\sim 1$ TeV results in $|\varepsilon_S| < 0.1$ and $|\varepsilon_T| < 10^{-3}$. Single operator bound is 10^3 stronger!

$R_{e/\mu}(\pi)$ vs other probes of LFU

- Comparison possible within a given class of models
- Instructive example: LFU violation in vertex corrections





Probed by decays of W boson, tau lepton, B, K, π mesons

$R_{e/\mu}(\pi)$ vs other probes of LFU

Crivellin-Hoferichter 2002.07184, PRL

Notation dictionary $\epsilon_{W\ell}^{aa} \to \varepsilon_{aa} \qquad a = e, \mu$

$(\varepsilon_{\mu\mu} - \varepsilon_{ee}) \times 10^3$

Observable	Measurement	Constraint
$\frac{K\to\pi\mu\bar{\nu}}{K\to\pi e\bar{\nu}} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	1.0010(25) [77]	1.0(2.5)
$\frac{K\to\mu\nu}{K\to e\nu} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	0.9978(18) [3, 78, 79]	−2.2(1.8)
$\frac{\pi\to\mu\nu}{\pi\to e\nu} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	1.0010(9) [3, 80–82]	1.0(9)
$\frac{\tau\to\mu\nu\bar{\nu}}{\tau\to e\nu\bar{\nu}} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	1.0018(14) [3, 32]	1.8(1.4)
$\frac{W\to\mu\bar{\nu}}{W\to e\bar{\nu}} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	0.9960(100) [83, 84]	−4(10)
$\frac{B\to D^{(*)}\mu\nu}{B\to D^{(*)}e\nu} \simeq 1 + \varepsilon_{\mu\mu} - \varepsilon_{ee}$	0.9890(120) [85]	−11(12)

$R_{e/\mu}(\pi)$ gives strongest constraint on ee − μμ combination

See Martin Hoferichter's talk

Probing Cabibbo universality

$$\Gamma_k = (G_F^{(\mu)})^2 \times |\bar{V}_{ij}|^2 \times |M_{\rm had}|^2 \times (1+\delta_{RC}) \times F_{\rm kin}$$

In the SM, these are the elements of the CKM matrix.

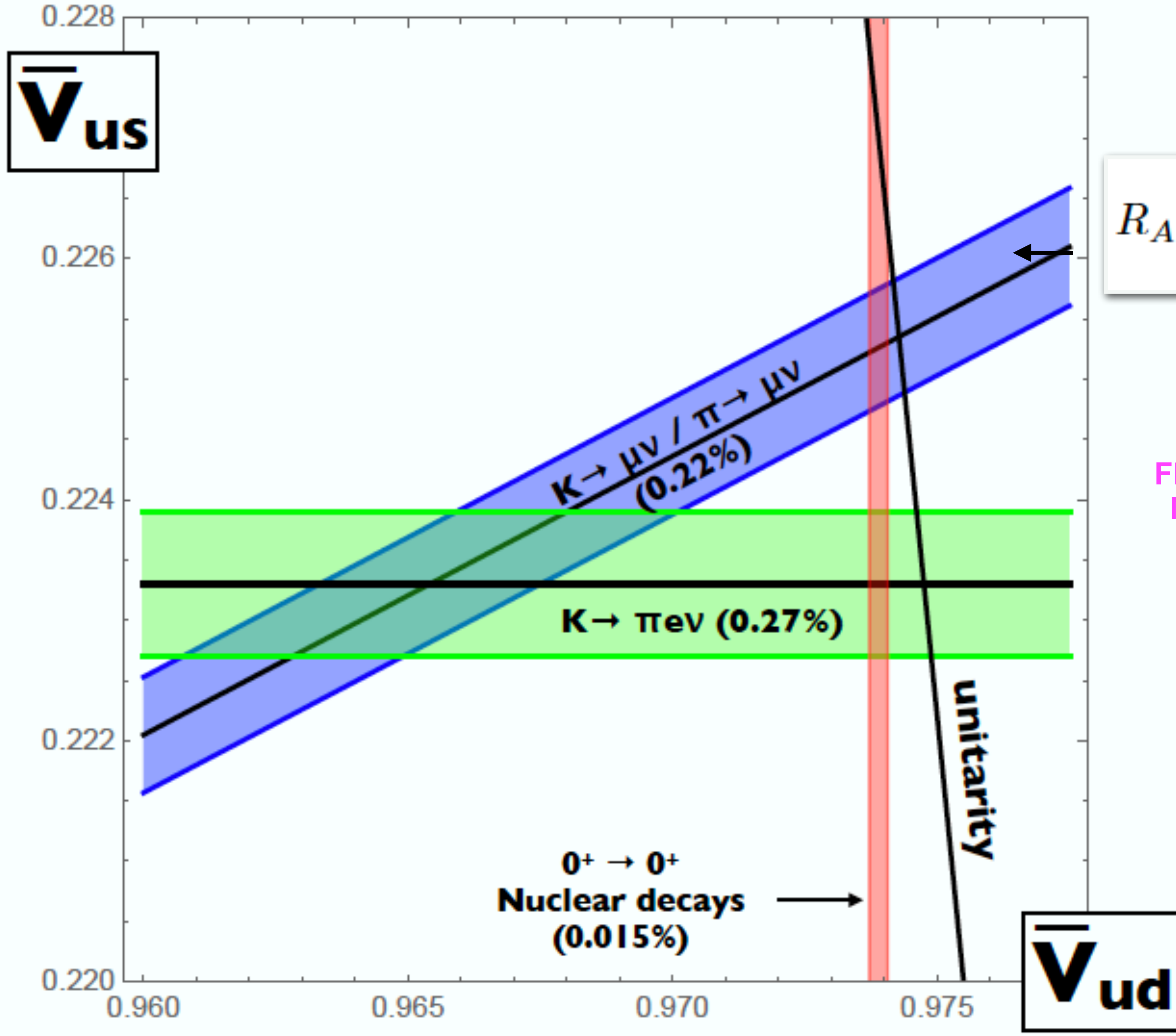
Beyond the SM, they pick up channel-dependent corrections

$$|\bar{V}_{ud}|^2 + |\bar{V}_{us}|^2 + |\bar{V}_{ub}|^2 = 1 + \Delta_{\rm CKM}(\epsilon_i)$$

See Martin Hoferichter's talk for discussion of various inputs to the CKM analysis

The Cabibbo angle anomaly

$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 - 1$$



Marciano,
hep-ph/0402299, PRL

$$R_A = \frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}$$

Flavianet WG,
1005.2323, EPJC
FLAG report, 1902.08191
MILC Coll.,1809.02827.

Hardy-Towner,
1411.5987, PRC 2020

Radiative Corrections
fo V_{ud} from
Czarnecki-Marciano-
Sirlin
1907.06737, PRD

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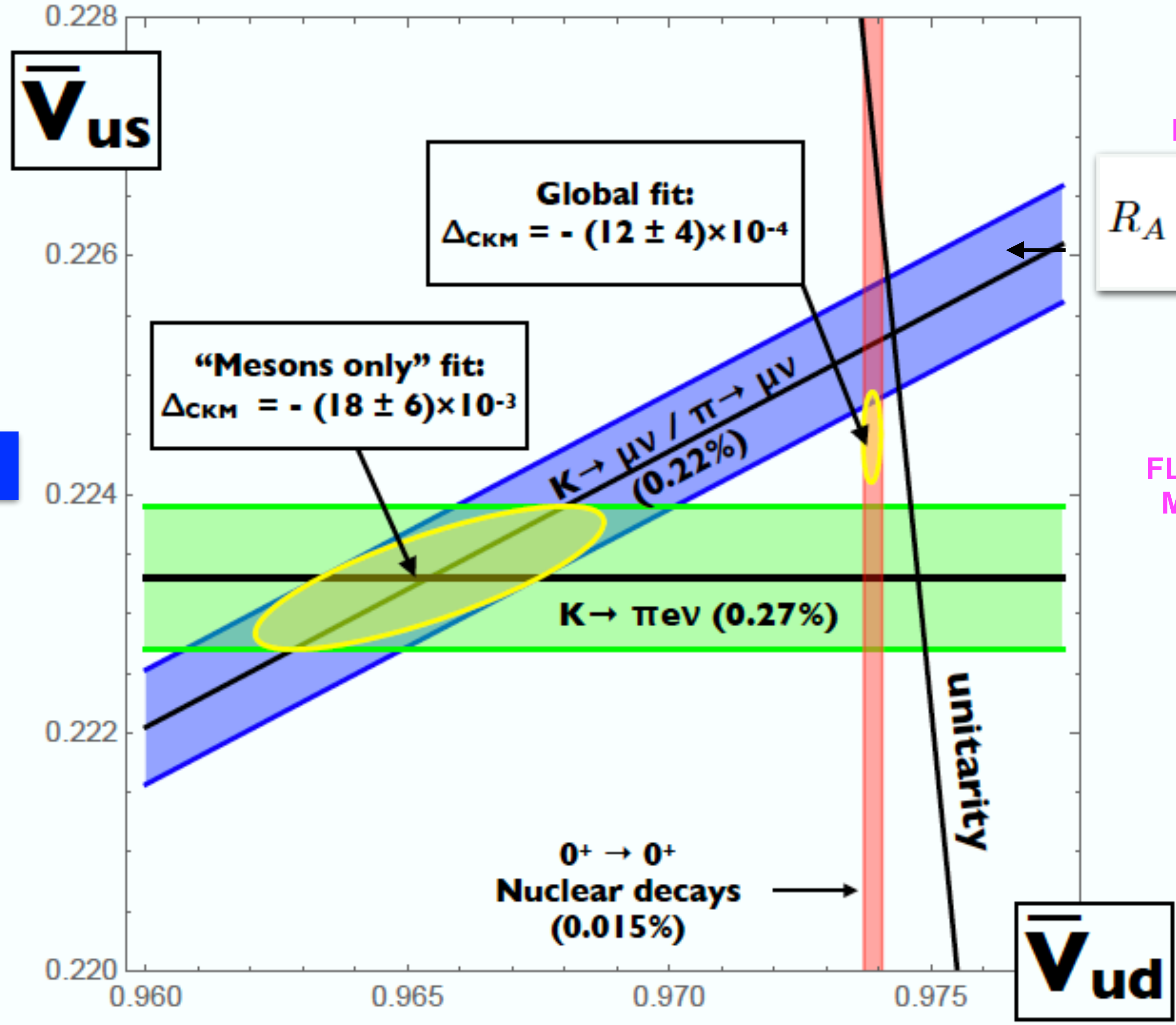

Two anomalies

Marciano, hep-ph/0402299, PRL

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Flavianet WG, 1005.2323, EPJC
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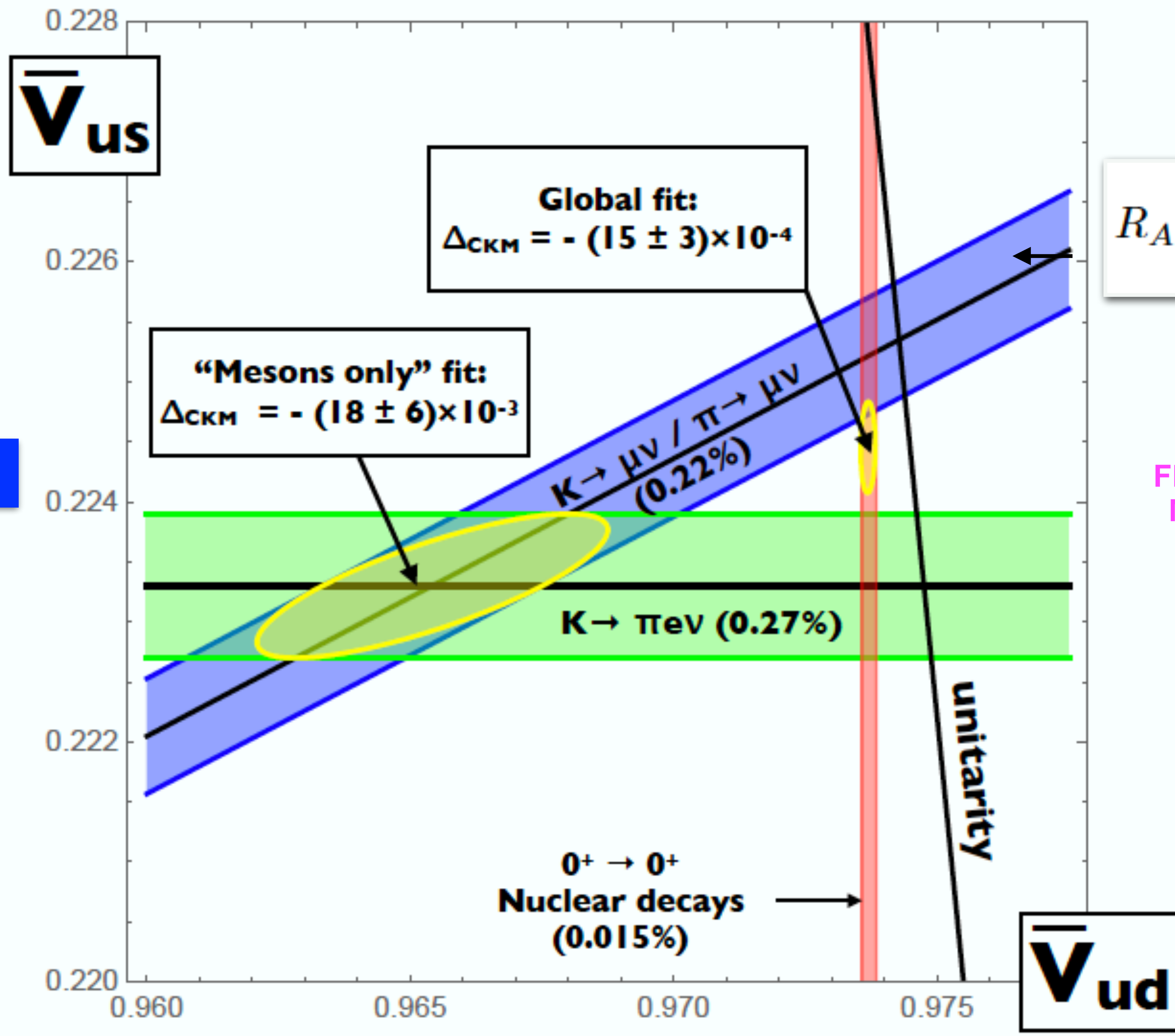

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Flavianet WG, 1005.2323, EPJC
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Hardy-Towner, 1411.5987, PRC 2020

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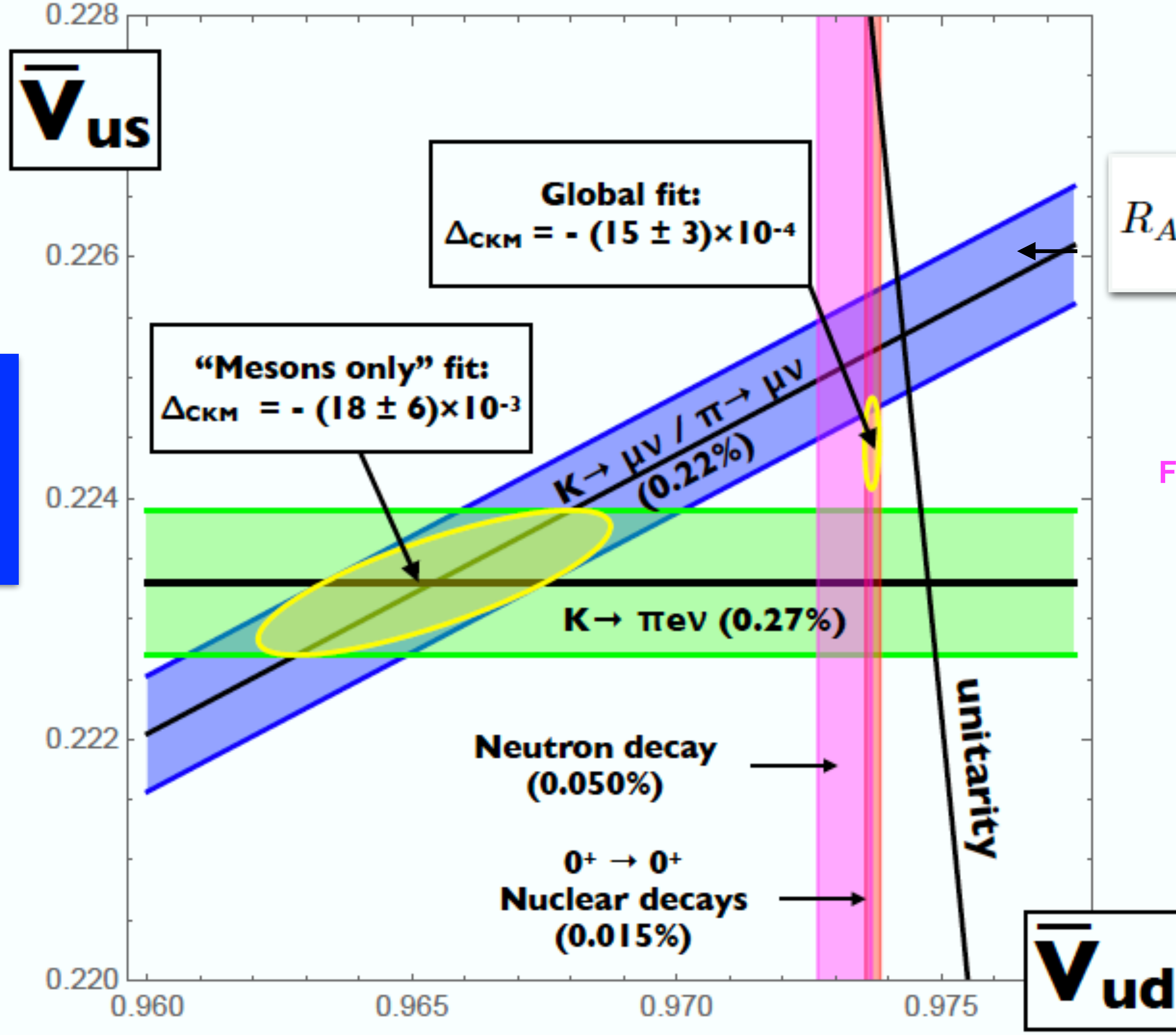


Neutron decay will soon provide a competitive extraction of V_{ud}

Marciano, hep-ph/0402299, PRL

$$R_A = \frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}$$

Flavianet WG, 1005.2323, EPJC
FLAG report, 1902.08191
MILC Coll.,1809.02827.

Hardy-Towner, 1411.5987, PRC 2020

Radiative Corrections fo V_{ud} from Seng et al, 1807.10197, PRL

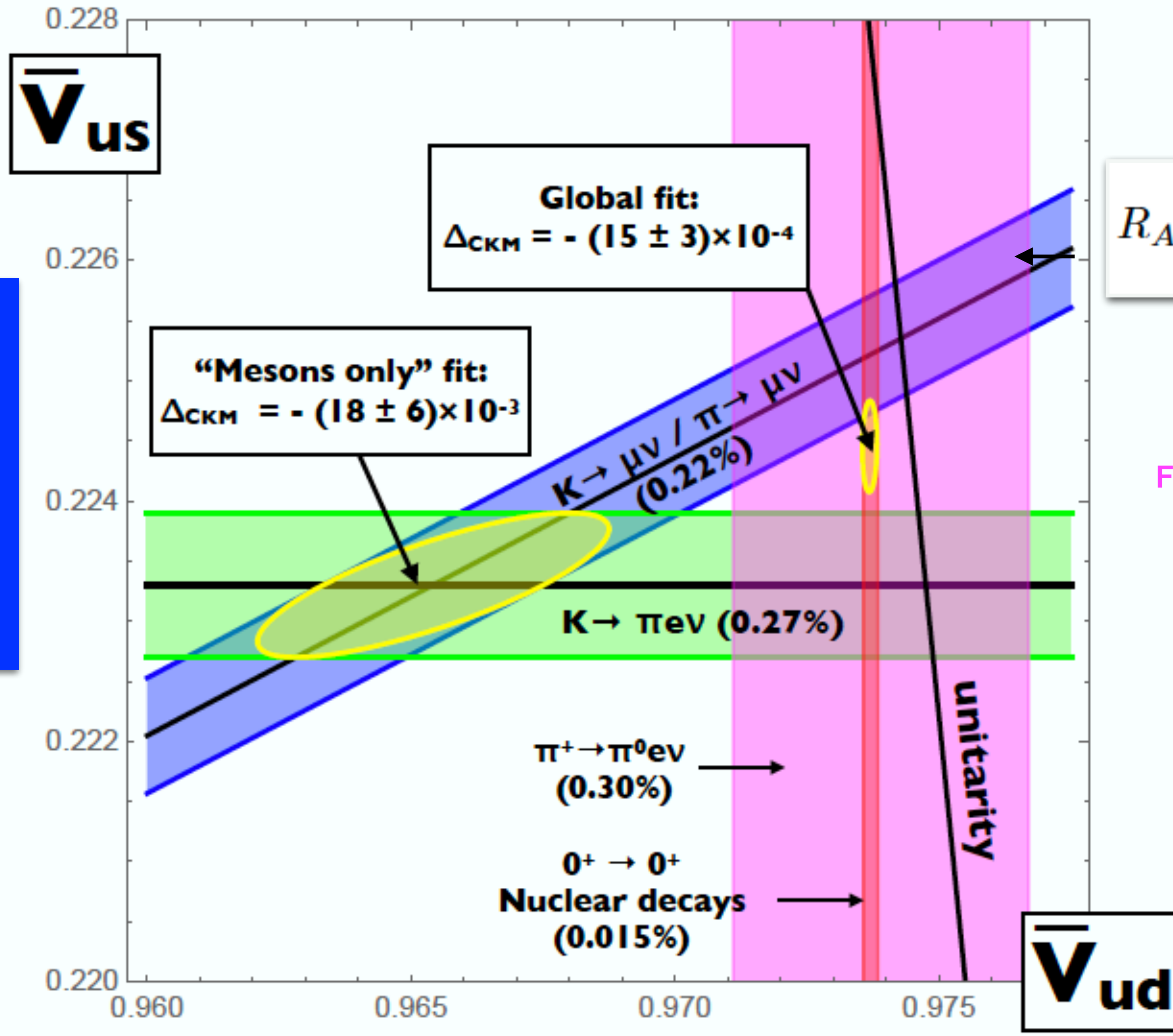
n lifetime from PDG,, g_A from PERKEO3

Impact of pion beta decay (1)

$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 - 1$$

Need 20x improvement in BR for a competitive extraction of V_{ud}

Ambitious, long term goal?



Marciano, hep-ph/0402299, PRL

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Flavianet WG, 1005.2323, EPJC
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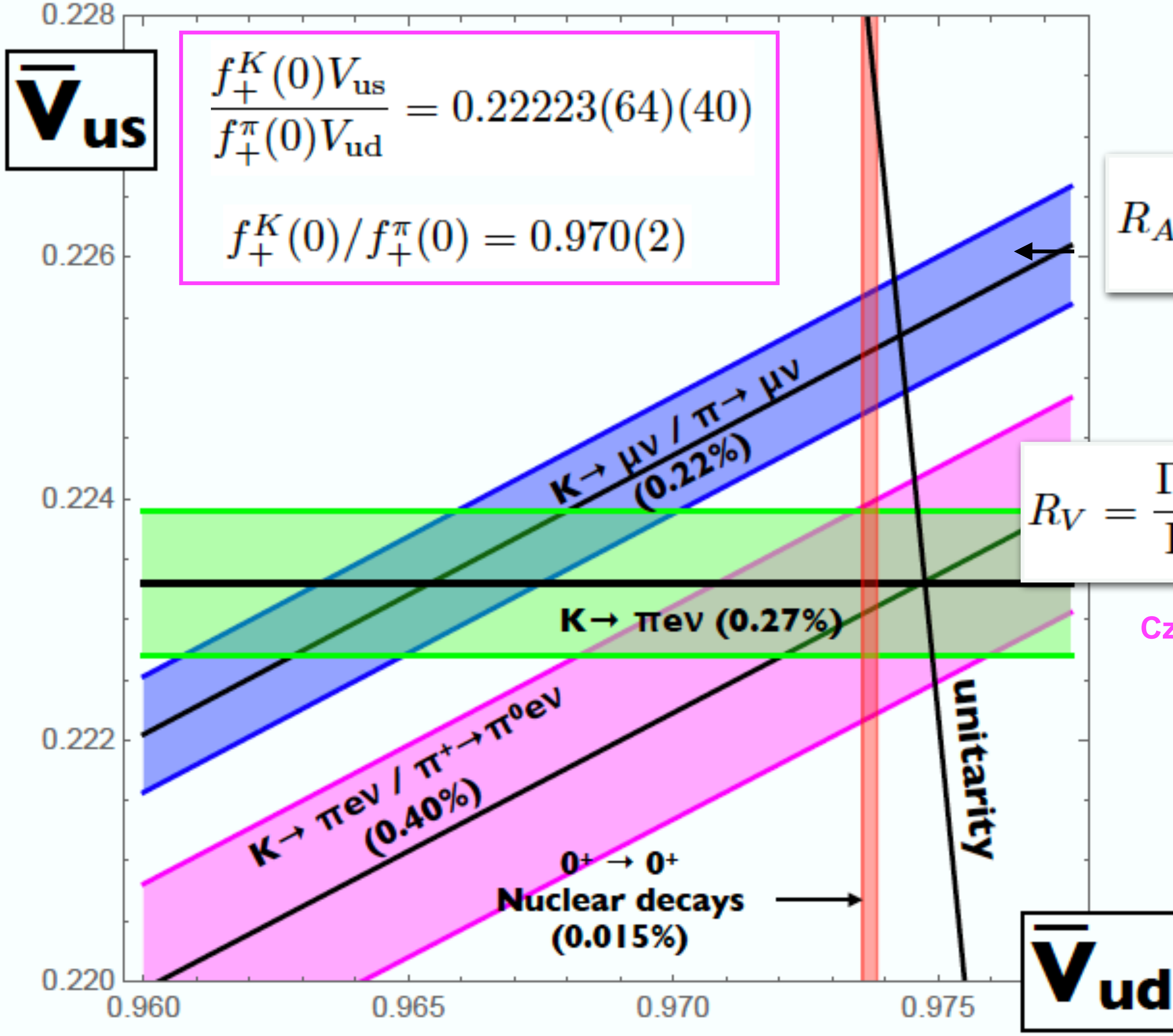
MILC Coll.,1809.02827.

Hardy-Towner, 1411.5987, PRC 2020

Radiative Corrections fo V_{ud} from Seng et al, 1807.10197, PRL

Impact of pion beta decay (2)

$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 - 1$



Marciano, hep-ph/0402299, PRL

$$R_A = \frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}$$

$$R_V = \frac{\Gamma(K_L \to \pi^{\pm}e^{\mp}\nu(\gamma))}{\Gamma(\pi^+ \to \pi^0 e^+\nu(\gamma))}$$

Czarnecki, Marciano, Sirlin 1911.04685, PRD

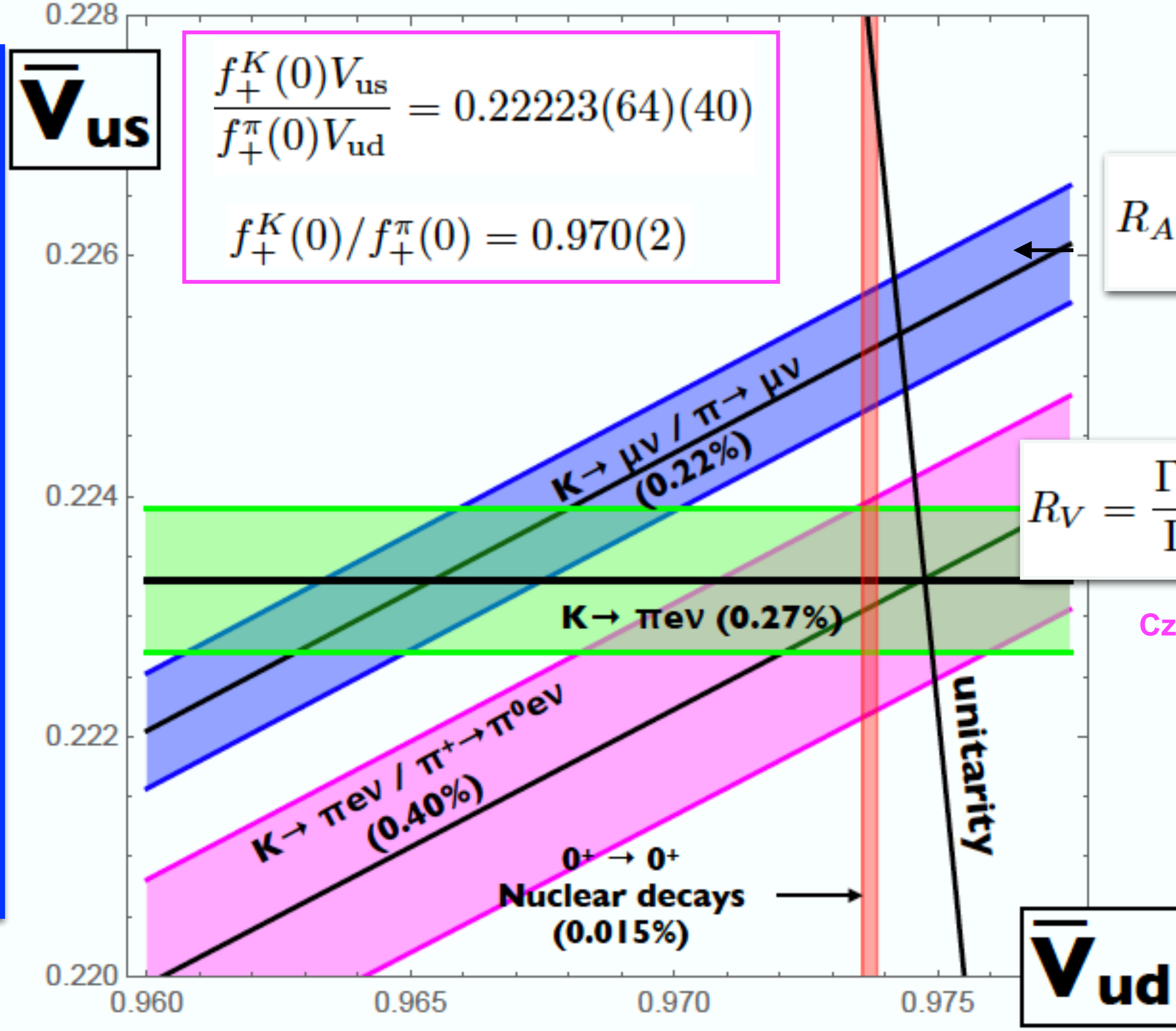
Impact of pion beta decay (2)

$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 - 1$$

R_V is nearly competitive with R_A. Hints to yet another tension!

3x improvement in PIBETA BR, along with modest improvement in K form factor and BRs, would lead to competitive V_{us}/V_{ud} @ 0.2%.

Realistic short term goal?



Marciano, hep-ph/0402299, PRL

$$R_A = \frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}$$

$$R_V = \frac{\Gamma(K_L \to \pi^\pm e^\mp \nu(\gamma))}{\Gamma(\pi^+ \to \pi^0 e^+ \nu(\gamma))}$$

Czarnecki, Marciano, Sirlin 1911.04685, PRD

Standard Model explanations?

- K-π vector form factor normalization: $f_+^K(0)$: 0.970(2) → 0.961(4)

$$\Gamma(K_L \to \pi^{\mp} e^{\pm} \nu(\gamma)) = \frac{G_\mu^2 |V_{us}|^2 m_{K_L}^5 \, |f_+^K(0)|^2}{192\pi^3} (1 + \mathrm{RC}_K) I_K$$

Czarnecki, Marciano, Sirlin 1911.04685, PRD

- Radiative corrections
  - K →πeν, K →πμν (improvable in lattice QCD)
  - Neutron decay (improvable in lattice QCD)
  - Nuclear decays: improvable with EFT + ab-initio calculations

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Czarnecki, Marciano, Sirlin 1911.04685, PRD

- Radiative corrections
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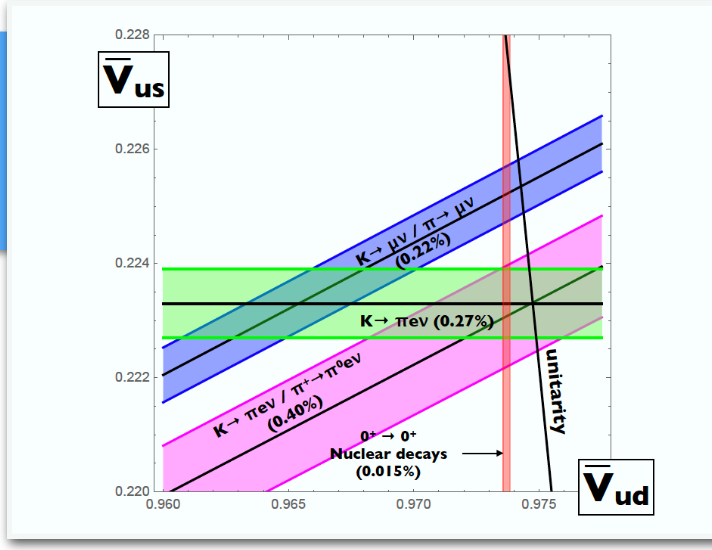
  - Neutron decay (improvable in lattice QCD)
  - Nuclear decays: improvable with EFT + ab-initio calculations

- Most robust theoretical input:
  - R_A : RC + isospin breaking in ChPT and LQCD
  - Pion beta decay: RC in LQCD

VC-Neufeld, 1102.0563, PLB
Di Carlo et al., 1904.08731, PRD

Feng et al. 2003.09798, PRL

BSM explanations?



- General case

$$|\bar{V}_{ud}|^2_i = |V_{ud}|^2 \left(1 + \sum_\alpha C_{i\alpha} \epsilon_\alpha\right)$$

$$|\bar{V}_{us}|^2_j = |V_{us}|^2 \left(1 + \sum_\alpha C_{j\alpha} \epsilon_\alpha\right)$$

Channel-dependent, extracted CKM elements

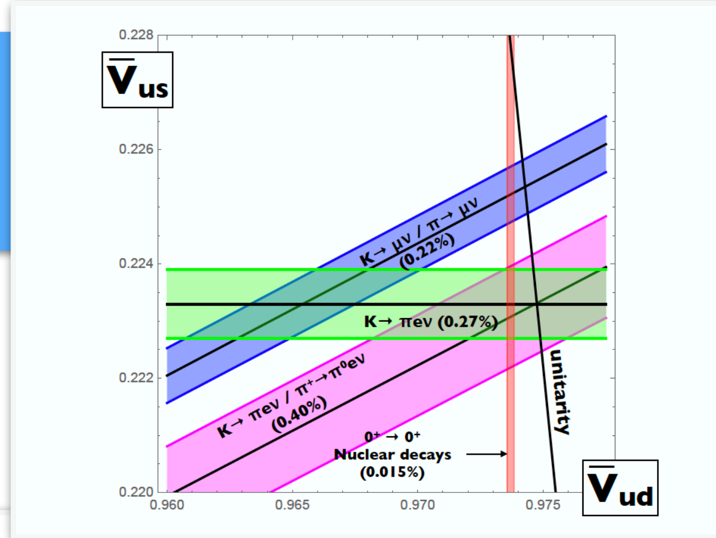
Elements of the unitary CKM matrix

Known coefficients

BSM effective coupligs

Find set of ε's so that V_{ud} and V_{us} bands meet on the unitarity circle

BSM explanations?



- ‘Turn on’ only vertex corrections to leptons

$$|\bar{V}_{ud}|^2_{0^+\to 0^+} = |V_{ud}|^2 \left(1 - 2\,\epsilon^{\mu\mu}_{W\ell}\right)$$

$$|\bar{V}_{ud}|^2_{n\to pe\bar{\nu}} = |V_{ud}|^2 \left(1 - 2\,\epsilon^{\mu\mu}_{W\ell}\right)$$

$$|\bar{V}_{us}|^2_{Ke3} = |V_{us}|^2 \left(1 - 2\,\epsilon^{\mu\mu}_{W\ell}\right)$$

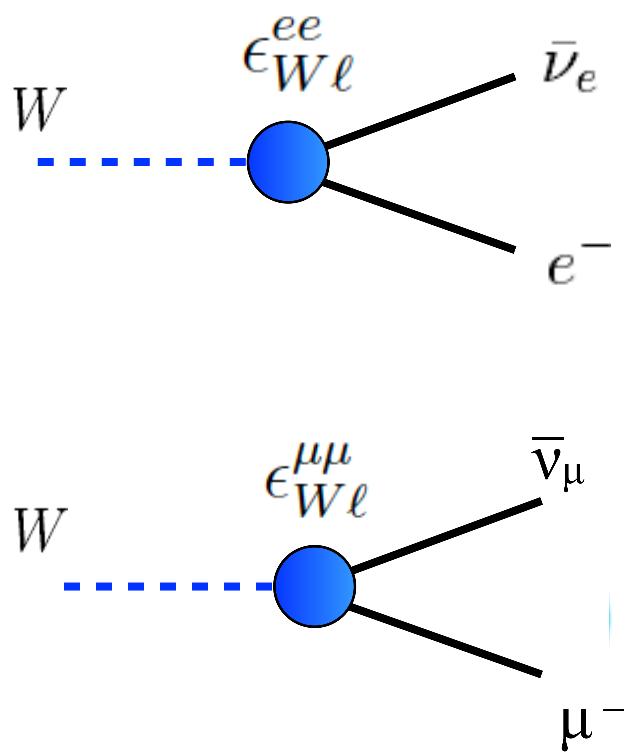
Relevant for R_V

$$|\bar{V}_{ud}|^2_{\pi e3} = |V_{ud}|^2 \left(1 - 2\,\epsilon^{\mu\mu}_{W\ell}\right)$$

$$|\bar{V}_{us}|^2_{K\mu 2} = |V_{us}|^2 \left(1 - 2\,\epsilon^{ee}_{W\ell}\right)$$

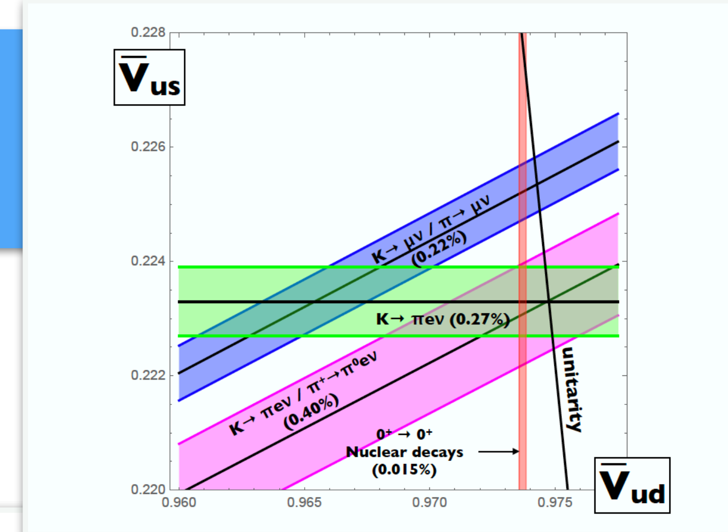
Relevant for R_A

$$|\bar{V}_{ud}|^2_{\pi\mu 2} = |V_{ud}|^2 \left(1 - 2\,\epsilon^{ee}_{W\ell}\right)$$



- R_V and R_A unchanged
- Shift the V_{ud} vertical band to the left
- No resolution of Kl3 vs Kl2 and R_V vs R_A tension

BSM explanations?



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Relevant for R_V

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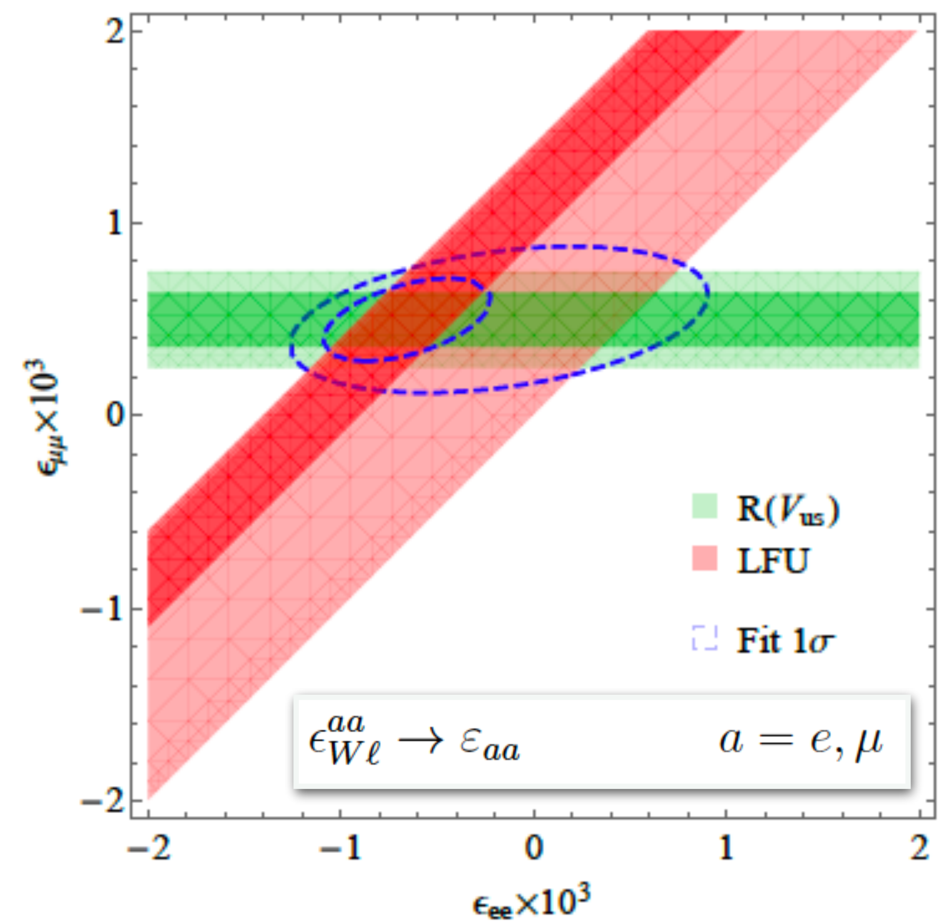
$$|\bar{V}_{ud}|^2_{\pi e3} = |V_{ud}|^2\left(1 - 2\,\epsilon^{\mu\mu}_{W\ell}\right)$$

Relevant for R_A

$$|\bar{V}_{us}|^2_{K\mu 2} = |V_{us}|^2\left(1 - 2\,\epsilon^{ee}_{W\ell}\right)$$

$$|\bar{V}_{ud}|^2_{\pi\mu 2} = |V_{ud}|^2\left(1 - 2\,\epsilon^{ee}_{W\ell}\right)$$

Crivellin-Hoferichter 2002.07184, PRL



- R_V and R_A unchanged
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- Connection with PIENU

$$r_\pi = 1 + 2\left(\epsilon^{ee}_{W\ell} - \epsilon^{\mu\mu}_{W\ell}\right)$$

(and other LFU probes)

BSM explanations?

- Right-handed currents (in the ‘ud’ and ‘us' sectors)

Grossman-Passemar-Schacht
1911.07821 JHEP
Alioli et al 1703.04751, JHEP

$$|\bar{V}_{ud}|^2_{0^+\to 0^+} = |V_{ud}|^2 \left(1 + 2\,\epsilon_R\right)$$

$$|\bar{V}_{ud}|^2_{n\to pe\bar{\nu}} = |V_{ud}|^2 \left(1 + 2\,\epsilon_R\right)$$

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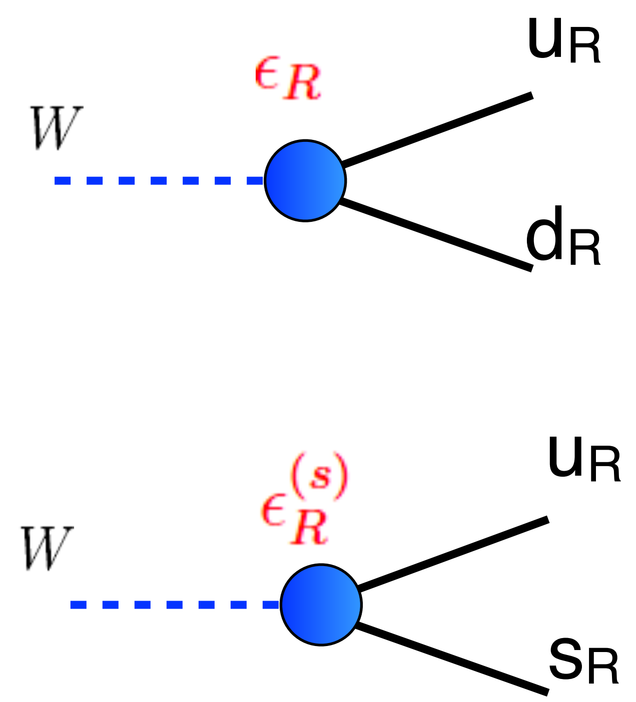
Relevant for R_A

$$|\bar{V}_{ud}|^2_{\pi\mu 2} = |V_{ud}|^2 \left(1 - 2\,\epsilon_R\right)$$



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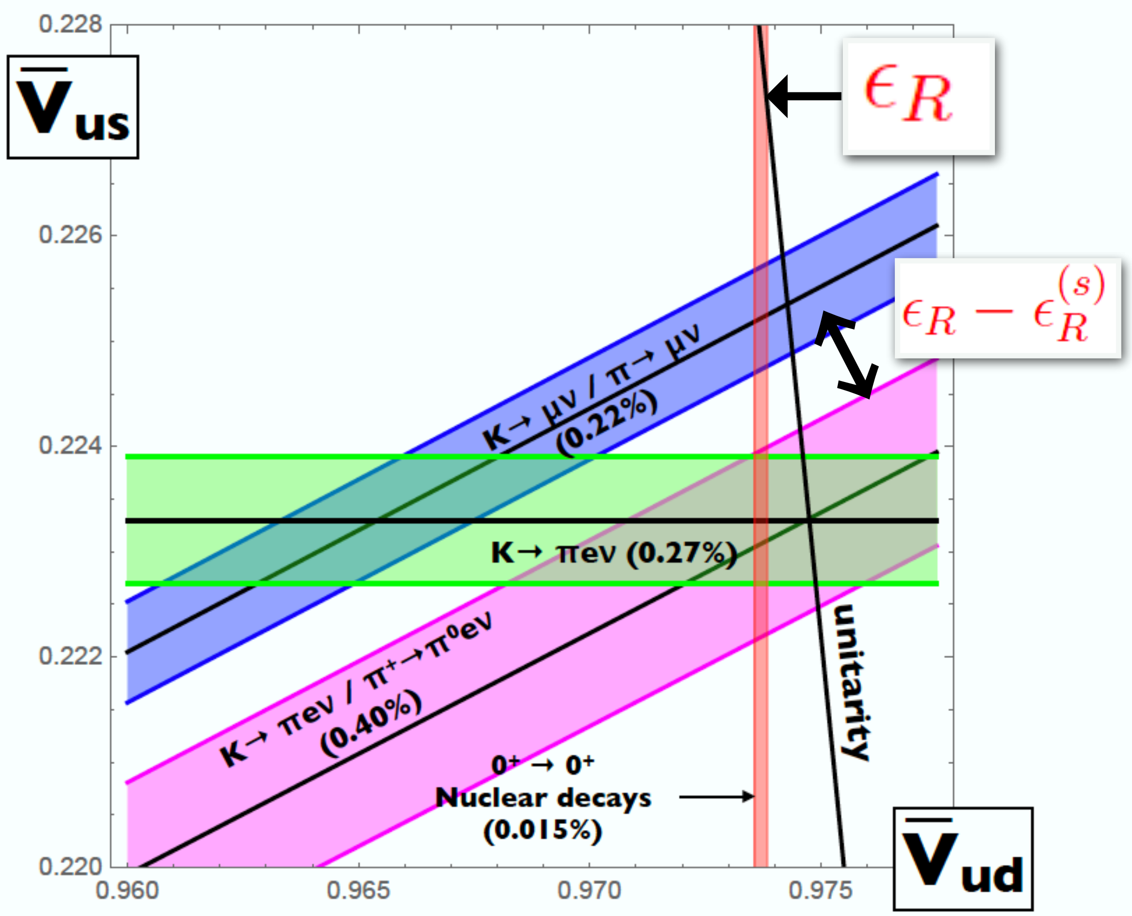


- R_V, R_A, V_{ud} and V_{us} bands shift in correlated way, can resolve all tensions!
- Points to $\epsilon_R^{(s)} \sim 0.4(2)\%$ and $\epsilon_R \sim 0.10(2)\%$, consistent with other probes of ϵ_R (g_A, LHC tails…)

Conclusions & Outlook

- Rare pion decays enable stringent tests of the universality of charged current weak interactions, probing new physics from very high scale as well as light and weakly coupled particles

- 10x improvement in PIENU will probe very high effective scales, up to $\Lambda_P \sim$ 30-1000 TeV and $\Lambda_A \sim$ 30 TeV

- 3x improvement in PIBETA will shed light on the Cabibbo angle anomaly. If anomaly persist, PIBETA will provide key input to disentangle the underlying BSM physics up to $\Lambda \sim$ 10-20 TeV

- 20x improvement in PIBETA would provide the ultimate determination of V_{ud}

- Theory framework is robust and will improve over the next decade, mostly through QCD+QED lattice calculations of neutron and K decays

Backup

Parameterization of NNLO corrections to $R_{e/\mu}^{(\pi,K)}$

P = (π,K)

$$R_{e/\mu}^{(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2 \times \left[1 + \Delta_{e^2p^2}^{(P)} + \Delta_{e^2p^4}^{(P)} + \ldots\right]$$

$$\Delta_{e^2p^4}^{(P)} = \frac{\alpha}{\pi}\frac{m_\mu^2}{m_\rho^2}\left(c_2^{(P)} \log\frac{m_\rho^2}{m_\mu^2} + c_3^{(P)} + c_4^{(P)}(m_\mu/m_P)\right) + \frac{\alpha}{\pi}\frac{m_P^2}{m_\rho^2}\tilde{c}_2^{(P)} \log\frac{m_\mu^2}{m_e^2}$$

	$(P = \pi)$	$(P = K)$
$\tilde{c}_2^{(P)}$	0	$(7.84 \pm 0.07_\gamma) \times 10^{-2}$
$c_2^{(P)}$	$5.2 \pm 0.4_{L_9} \pm 0.01_\gamma$	$4.3 \pm 0.4_{L_9} \pm 0.01_\gamma$
$c_3^{(P)}$	$-10.5 \pm 2.3_m \pm 0.53_{L_9}$	$-4.73 \pm 2.3_m \pm 0.28_{L_9}$
$c_4^{(P)}(m_\mu)$	$1.69 \pm 0.07_{L_9}$	$0.22 \pm 0.01_{L_9}$

Theoretical analysis of $R_{e/\mu}(\pi,K)$

P = (π,K) $$R_{e/\mu}^{(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2 \times \left[1 + \Delta_{e^2p^2}^{(P)} + \Delta_{e^2p^4}^{(P)} + \ ....\right]\left[1 + \Delta_{LL}\right]$$

		$(P=\pi)$	$(P=K)$
	$\Delta_{e^2p^2}^{(P)}$ (%)	-3.929	-3.786
	$\Delta_{e^2p^4}^{(P)}$ (%)	0.053 ± 0.011	0.135 ± 0.011
*	$\Delta_{e^2p^6}^{(P)}$ (%)	0.073	
**	Δ_{LL} (%)	0.055	0.055

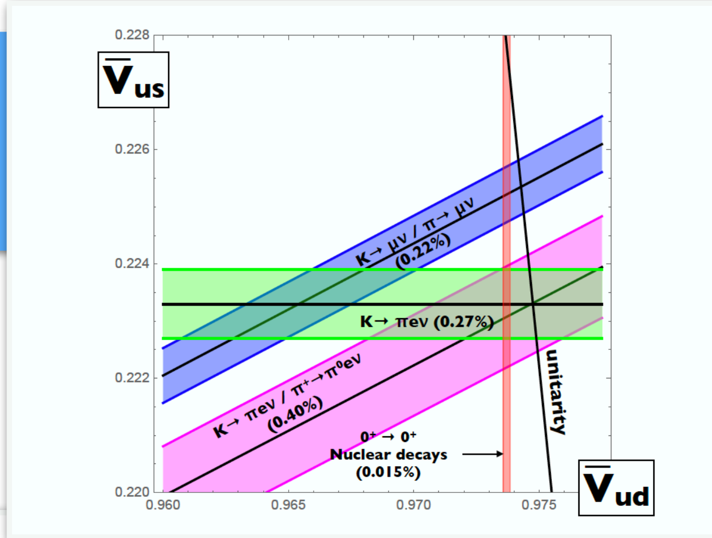
* Structure-dependent contribution to $\pi \to e\nu\gamma$, unsuppressed by helicity argument

** Contribution of higher order Leading Logarithms $O(e^{2n} p^2)$ [via RG, Marciano-Sirlin '93]

$$R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0001) \times 10^{-4}$$

$$R_{e/\mu}^{(K)} = (2.477 \pm 0.001) \times 10^{-5}$$ ← 4 x matching uncertainty [estimate of e^2p^6 effect]

BSM explanations?



- General case

$$|\bar{V}_{ud}|^2_{0^+\to 0^+} = |V_{ud}|^2 \left(1 + 2\,(\epsilon_L^{ee} + \epsilon_R - \epsilon_L^{(\mu)}) + c^S_{0^+}(Z)\;\epsilon_S^{ee}\right)$$

$$|\bar{V}_{ud}|^2_{n\to pe\bar{\nu}} = |V_{ud}|^2 \left(1 \;+\; 2\,(\epsilon_L^{ee} + \epsilon_R - \epsilon_L^{(\mu)}) + c_n^S \epsilon_S^{ee} + c_n^T \epsilon_T^{ee}\right)$$

Relevant for R_V

$$|\bar{V}_{us}|^2_{Ke3} = |V_{us}|^2 \left(1 + 2\,(\epsilon_L^{ee(s)} + \epsilon_R^{(s)} - \epsilon_L^{(\mu)})\right)$$

$$|\bar{V}_{ud}|^2_{\pi e3} = |V_{ud}|^2 \left(1 + 2\,(\epsilon_L^{ee} + \epsilon_R - \epsilon_L^{(\mu)})\right)$$

Relevant for R_A

$$|\bar{V}_{us}|^2_{K\mu 2} = |V_{us}|^2 \left(1 + 2\,(\epsilon_L^{\mu\mu(s)} - \epsilon_R^{(s)} - \epsilon_L^{(\mu)}) - 2\frac{B_0}{m_\ell}\,\epsilon_P^{\mu\mu(s)}\right)$$

$$|\bar{V}_{ud}|^2_{\pi\mu 2} = |V_{ud}|^2 \left(1 + 2\,(\epsilon_L^{\mu\mu} - \epsilon_R - \epsilon_L^{(\mu)}) - 2\frac{B_0}{m_\ell}\,\epsilon_P^{\mu\mu}\right)$$

ε_R and $\varepsilon_R^{(s)}$ are lepton-flavor universal in the Standard Model EFT
(arise from a W-u_R-d_R vertex correction)